Namunaviy misollar

(irratsional tenglama va tengsizliklar)

1-misol

Tenglamani yeching:

$$\sqrt{x + 1} + \sqrt{2x - 1} = 3$$

Yechish

1-qadam. Aniqlanish sohasi (AS)

Ildiz ostidagi ifodalar manfiy bo‘lmasligi kerak:

$x + 1 \geq 0 \Longrightarrow x \geq -1$
$2x - 1 \geq 0 \Longrightarrow x \geq 1/2$

Demak,
AS: x ≥ 1/2

2-qadam. Tenglamani kvadratlaymiz

$$(\sqrt{x + 1} + \sqrt{2x - 1})^2 = 3^2$$

$$x + 1 + 2x - 1 + 2\sqrt{(x + 1)(2x - 1)} = 9$$

$$3x + 2\sqrt{(x + 1)(2x - 1)} = 9$$

3-qadam. Ildizli hadni ajratamiz

$$2\sqrt{(x + 1)(2x - 1)} = 9 - 3x$$

Har ikki tomonni 2 ga bo‘lamiz:

$$\sqrt{(x + 1)(2x - 1)} = (9 - 3x)/2$$

4-qadam. Yana kvadratlaymiz

$$(x + 1)(2x - 1) = (9 - 3x)^2 / 4$$

Chap tomonni ochamiz:

$$2x^2 + x - 1 = (81 - 54x + 9x^2) / 4$$

4 ga ko‘paytiramiz:

$$8x^2 + 4x - 4 = 9x^2 - 54x + 81$$

5-qadam. Barcha hadlarni bir tomonga yig‘amiz

$$0 = x^2 - 58x + 85$$

Ya’ni:

$$x^2 - 58x + 85 = 0$$

6-qadam. Kvadrat tenglamani yechamiz

$$D = 58^2 - 4 \cdot 1 \cdot 85$$
$$D = 3364 - 340$$
$$D = 3024$$

$$\sqrt{3024} = \sqrt{144 \cdot 21} = 12\sqrt{21}$$

$$x_{1,2} = (58 \pm 12\sqrt{21})/2$$

$$x_1 = 29 + 6\sqrt{21}$$
$$x_2 = 29 - 6\sqrt{21}$$

7-qadam. AS bo‘yicha tekshirish

$x \geq 1/2$ shartga **ikkala qiymat ham mos keladi**.

javob:

$$x = 29 + 6\sqrt{21}$$
$$x = 29 - 6\sqrt{21}$$

2-misol

Tenglamani yeching:

$$\sqrt{x^2 - 5x + 6} = \sqrt{x - 2}$$

Yechish

AS:
$x^2 - 5x + 6 \geq 0$ va $x - 2 \geq 0$

Kvadratlaymiz:

$$x^2 - 5x + 6 = x - 2$$
$$x^2 - 6x + 8 = 0$$

$$(x - 2)(x - 4) = 0$$

$x = 2$ yoki $x = 4$

Tekshiruv:
$x = 2$ mos, $x = 4$ mos.

Javob

$$x = 4, x = 2$$

3-misol

Tenglamani yeching:

$$\sqrt{2x + 3} = x - 1$$

Yechish

AS:
$2x + 3 \geq 0$ va $x - 1 \geq 0 \rightarrow x \geq 1$

Kvadratlaymiz:

$$2x + 3 = (x - 1)^2$$
$$2x + 3 = x^2 - 2x + 1$$

$$x^2 - 4x - 2 = 0$$

$$x = 2 \pm \sqrt{6}$$

AS ga mos: faqat $x = 2 + \sqrt{6}$

Javob

$$x = 2 + \sqrt{6}$$

4-misol

Tengsizlikni yeching:

$$\sqrt{x - 1} \geq x - 3$$

Yechish

AS: $x \geq 1$

Agar $x - 3 \leq 0$ bo‘lsa ($x \leq 3$), tengsizlik avtomatik bajariladi.

Agar $x - 3 \geq 0$ bo‘lsa, kvadratlaymiz:

$$x - 1 \geq (x - 3)^2$$

$$x - 1 \geq x^2 - 6x + 9$$

$$x^2 - 7x + 10 \leq 0$$

$$(x - 2)(x - 5) \leq 0$$
$$2 \leq x \leq 5$$

Yechimlarni birlashtiramiz:

$$1 \leq x \leq 5$$

Javob

$$1 \leq x \leq 5$$

5-misol

Tengsizlikni yeching:

$$\sqrt{x + 2} + \sqrt{3 - x} \geq 2$$

Yechish

AS: $-2 \leq x \leq 3$

Ikkala tomon ≥ 0, kvadratlaymiz:

$$x + 2 + 3 - x + 2\sqrt{(x + 2)(3 - x)} \geq 4$$

$$2\sqrt{(x + 2)(3 - x)} \geq -1$$

Bu har doim to‘g‘ri (chap tomon ≥ 0).

Javob

$$-2 \leq x \leq 3$$

6-misol

Tenglamani yeching:

$$\sqrt{x + 5} - \sqrt{x - 1} = 2$$

Yechish

AS: $x \geq 1$

$$\sqrt{x + 5} = 2 + \sqrt{x - 1}$$

Kvadratlaymiz:

$$x + 5 = 4 + 4\sqrt{x - 1} + x - 1$$

$$4\sqrt{x-1} = 2$$
$$\sqrt{x-1} = 1/2$$
$$x = 5/4$$

Tekshiruv mos.

Javob

$$x = 5/4$$

7-misol

Tenglamani yeching:

$$\sqrt{x^2 - 4x + 3} = 2 - x$$

Yechish

AS:
$x^2 - 4x + 3 \geq 0$ va $2 - x \geq 0 \rightarrow x \leq 2$

Kvadratlaymiz:

$$x^2 - 4x + 3 = (2 - x)^2$$
$$x^2 - 4x + 3 = x^2 - 4x + 4$$

$3 = 4$ — mumkin emas.

Javob

Yechim yo‘q

8-misol

Tengsizlikni yeching:

$$\sqrt{2x - 1} \leq \sqrt{x + 2}$$

Yechish

AS: $x \geq 1/2$

Kvadratlaymiz:

$$2x - 1 \leq x + 2$$
$$x \leq 3$$

AS bilan kesishma:

$$1/2 \leq x \leq 3$$

Javob

$$1/2 \leq x \leq 3$$

9-misol

Tenglamani yeching:

$$\sqrt{x + \sqrt{x + 3}} = 2$$

Yechish

Kvadratlaymiz:

$$x + \sqrt{(x + 3)} = 4$$

$$\sqrt{(x + 3)} = 4 - x$$

Kvadratlaymiz:

$$x + 3 = (4 - x)^2$$
$$x^2 - 9x + 13 = 0$$

$$x = (9 \pm \sqrt{29})/2$$

Tekshiruv: faqat $x = (9 - \sqrt{29})/2$ mos.

Javob

$$(9 - \sqrt{29})/2$$

10-misol

Tengsizlikni yeching:

$$\sqrt{x^2 - 1} \geq x - 1$$

Yechish

AS: $|x| \geq 1$

Agar $x - 1 \leq 0$ $(x \leq 1)$, tengsizlik bajariladi va AS bilan kesishadi: $x \leq -1$ yoki $x = 1$

Agar $x - 1 \geq 0$ $(x \geq 1)$, kvadratlaymiz:

$$x^2 - 1 \geq (x - 1)^2$$

$$x^2 - 1 \geq x^2 - 2x + 1$$
$$x \geq 1$$

Javob

$x \leq -1$ yoki $x \geq 1$