

NAMUNAVIY MASALALAR

1-MASALA. Manbaning ichki qarshiligi va quvvat

EYuK $\varepsilon = 12 \text{ V}$, ichki qarshilik $r = 2 \text{ om}$ bo'lgan manbaga $R = 4 \text{ om}$ tashqi qarshilik ulanadi.

Topilsin:

- a) zanjirdagi tok I
- b) manba uchlaridagi kuchlanish U
- c) tashqi rezistorda ajralayotgan quvvat P
- d) manbaning o'zida issiqlik sifatida ajralayotgan quvvat P_r

Yechim

- a) To'liq zanjir uchun Ohm qonuni:

$$I = \varepsilon / (R + r) = 12 / (4 + 2) = 12 / 6 = 2 \text{ A}$$

- b) Manba uchlaridagi kuchlanish:

$$U = I \cdot R = 2 \cdot 4 = 8 \text{ V}$$

- c) Tashqi rezistordagi quvvat:

$$P = I^2 \cdot R = 2^2 \cdot 4 = 4 \cdot 4 = 16 \text{ W}$$

- d) Ichki qarshilikdagi quvvat:

$$P_r = I^2 \cdot r = 2^2 \cdot 2 = 4 \cdot 2 = 8 \text{ W}$$

Jami manba bergan quvvat: $P_{\text{jam}} = I \cdot \varepsilon = 2 \cdot 12 = 24 \text{ W}$

Ko'ramiz: $16 \text{ W} + 8 \text{ W} = 24 \text{ W}$ – energiya balansi saqlanadi.

Javob: $I = 2 \text{ A}$, $U = 8 \text{ V}$, $P = 16 \text{ W}$, $P_r = 8 \text{ W}$.

2-MASALA. Qarama-qarshi ulangan ikki manba

Ikki manba ketma-ket, lekin qarama-qarshi ulangan:

- 1. $\varepsilon_1 = 18 \text{ V}$, $r_1 = 1 \text{ om}$
- 2. $\varepsilon_2 = 12 \text{ V}$, $r_2 = 2 \text{ om}$

Ularning uchlariga $R = 5 \text{ om}$ tashqi qarshilik ulanadi.

Topilsin:

- a) zanjirdagi tok I va yo'nalishi

b) har bir manbaning quvvati (qaysi biri energiya beradi, qaysi biri “zaryalanmoqda”)

Yechim

Qulaylik uchun tok I ni manbalar ε_1 ga “musbat” yo‘nalishda deb olamiz (ε_1 “kuchliroq” bo‘lishi mumkin deb taxmin).

Umumiy EYuK:

$$\varepsilon_{um} = \varepsilon^1 - \varepsilon^2 = 18 - 12 = 6 V$$

Umumiy ichki qarshilik:

$$r_{um} = r^1 + r^2 = 1 + 2 = 3 ohm$$

To‘liq qarshilik:

$$R_{to'liq} = R + r_{um} = 5 + 3 = 8 ohm$$

Tok:

$$I = \varepsilon_{um} / R_{to'liq} = 6 / 8 = 0,75 A$$

Bu ijobiy chiqqani uchun yo‘nalish tanlanganidek – ε_1 manbadan ε_2 tomonga.

a) $I = 0,75 A$.

b) Har bir manbaning quvvati: ·

ε_1 manbaning quvvati: $P_1 = \varepsilon_1 \cdot I = 18 \cdot 0,75 = 13,5 W$

ε_2 manbaning quvvati: $P_2 = \varepsilon_2 \cdot I = 12 \cdot 0,75 = 9 W$

Ammo fizik ma’nosi:

📖 ε_1 katta, umumiy tok yo‘nalishi unga mos $\rightarrow \varepsilon_1$ energiya beradi (generator)

📖 ε_2 kuchsiz, unga qarama-qarshi manba ta’sir qiladi $\rightarrow$ u amalda “zaryalanayotgan” akkumulyator rolida.

Javob: $I = 0,75 A$; $\varepsilon_1 \approx 13,5 W$ energiya beradi, $\varepsilon_2 \approx 9 W$ energiyani qabul qiladi (zaryalanadi).

3-MASALA. Uch rezistorli tarmoqlangan zanjir

Sxema: $R_1 = 3 om$ va $R_2 = 6 om$ parallel ulangan, ularning shu parallel kombinatsiyasiga ketma-ket $R_3 = 2 om$ ulab chiqilgan. Butun zanjir $\varepsilon = 10 V, r = 0 om$ ideal manbaga ulangan.

Topilsin:

- a) zanjirning ekvivalent qarshiligi R_{eq}
- b) umumiy tok I
- c) R_1 va R_2 bo'ylab oqayotgan I_1 va I_2 toklar

Yechim

- a) Parallel ulangan R_1 va R_2 ning ekvivalent qarshiligi:

$$1/R_{par} = 1/R_1 + 1/R_2 = 1/3 + 1/6 = 2/6 + 1/6 = 3/6 = 1/2$$

Demak $R_{par} = 2 \text{ om}$

Ketma-ket R_3 bilan:

$$R_{eq} = R_3 + R_{par} = 2 + 2 = 4 \text{ om}$$

- b) Umumiy tok:

$$I = \varepsilon / R_{eq} = 10 / 4 = 2,5 \text{ A}$$

Bu tok R_3 dan o'tadi va parallel tugunga kiradi.

- c) Parallel tarmoqlarda kuchlanish:

$$U_{par} = I \cdot R_3 \text{ dan keyin? Yo'q, yaxshisi:}$$

Butun zanjir $U = 10 \text{ V}$, umumiy tok $2,5 \text{ A}$.

$$R_3 \text{ da kuchlanish: } U_3 = I \cdot R_3 = 2,5 \cdot 2 = 5 \text{ V}$$

$$\text{Demak parallel tarmoqlar uchlarida ham } U_{par} = 10 - 5 = 5 \text{ V}$$

Endi:

$$I_1 = U_{par} / R_1 = 5 / 3 \approx 1,67 \text{ A}$$

$$I_2 = U_{par} / R_2 = 5 / 6 \approx 0,83 \text{ A}$$

Tekshiramiz: $I_1 + I_2 \approx 1,67 + 0,83 = 2,5 \text{ A}$ – to'g'ri.

Javob: $R_{eq} = 4 \text{ om}, I = 2,5 \text{ A}, I_1 \approx 1,67 \text{ A}, I_2 \approx 0,83 \text{ A}$.

4-MASALA. Ikki konturli Kirxgof masalasi

Sxemada ikkita kontur bor. O'rtadagi umumiy rezistor $R_3 = 4 \text{ om}$, chap konturda $R_1 = 2 \text{ om}$ va manba $\varepsilon_1 = 10 \text{ V}$, o'ng konturda $R_2 = 6 \text{ om}$ va manba $\varepsilon_2 = 5$

V. Ichki qarshiliklar yo'q. Toklar chap kontur bo'yicha I_1 , o'ng kontur bo'yicha I_2 deb olingan, R_3 dan $I_1 - I_2$ farq tok o'tadi.

Topilsin: I_1 va I_2 .

Yechim

Chap kontur:

$$e_1 - I_1 \cdot R_1 - (I_1 - I_2) \cdot R_3 = 0$$

$$10 - 2I_1 - 4(I_1 - I_2) = 0$$

$$10 - 2I_1 - 4I_1 + 4I_2 = 0$$

$$10 - 6I_1 + 4I_2 = 0 \dots (1)$$

O'ng kontur:

$$\varepsilon_2 - I_2 \cdot R_2 - (I_2 - I_1) \cdot R_3 = 0$$

$$5 - 6I_2 - 4(I_2 - I_1) = 0$$

$$5 - 6I_2 - 4I_2 + 4I_1 = 0$$

$$5 + 4I_1 - 10I_2 = 0 \dots (2)$$

Endi sistemasini yechamiz.

$$(1): 6I_1 - 4I_2 = 10$$

$$(2): 4I_1 - 10I_2 = -5$$

Keltiramiz:

(1) ni 5 ga, (2) ni 3 ga ko'paytiraylik:

$$(1) \cdot 5: 30I_1 - 20I_2 = 50$$

$$(2) \cdot 3: 12I_1 - 30I_2 = -15$$

Endi ayiraylik: $(1) \cdot 5 - (2) \cdot 3$:

$$(30I_1 - 20I_2) - (12I_1 - 30I_2) = 50 - (-15)$$

$$30I_1 - 20I_2 - 12I_1 + 30I_2 = 65$$

$18I_1 + 10I_2 = 65$ — lekin bu yo'l chalkash ekan; boshqa usul qilamiz.

Yaxshisi (1) dan I_1 ni ifodalab, (2) ga qo'yamiz.

$$(1): 6I_1 = 10 + 4I_2 \rightarrow I_1 = (10 + 4I_2) / 6 = (5 + 2I_2) / 3$$

Endi (2) ga qo'yamiz:

$$4I_1 - 10I_2 = -5$$

$$4 \cdot (5 + 2I_2)/3 - 10I_2 = -5$$

$$(20 + 8I_2)/3 - 10I_2 = -5$$

Umumiy maxraj 3:

$$(20 + 8I_2 - 30I_2)/3 = -5$$

$$(20 - 22I_2)/3 = -5$$

$$20 - 22I_2 = -15$$

$$-22I_2 = -35$$

$$I_2 = 35 / 22 \approx 1,59 \text{ A}$$

Endi I_1 :

$$I_1 = (5 + 2I_2)/3 \approx (5 + 3,18)/3 = 8,18 / 3 \approx 2,73 \text{ A}$$

Javob: $I_1 \approx 2,73 \text{ A}, I_2 \approx 1,59 \text{ A}$.

5-MASALA. Ampermetr ko'rsatishi

EYuK $\varepsilon = 9 \text{ V}$, ichki qarshilik $r = 1 \text{ om}$ bo'lgan manba $R = 8 \text{ om}$ rezistorga ulangan. Zanjirga ketma-ket ideal ampermetr ulangan.

Topilsin:

- a) zanjirdagi tok I (ampermetr ko'rsatkichi)
- b) rezistordagi kuchlanish U_R
- c) manba uchlaridagi kuchlanish U

Yechim

a) Tok:

$$I = \varepsilon / (R + r) = 9 / (8 + 1) = 9 / 9 = 1 \text{ A}$$

b) Rezistordagi kuchlanish:

$$U_R = I \cdot R = 1 \cdot 8 = 8 \text{ V}$$

c) Manba uchlaridagi kuchlanish ham $U = U_R = 8 \text{ V}$ (chunki ampermetr ideal, qarshiligi nolga teng).

Ichki qarshilikda kuchlanish tushuvi:

$$U_r = I \cdot r = 1 \cdot 1 = 1 \text{ V}$$

$$\varepsilon = U + U_r \rightarrow 9 = 8 + 1 - \text{tekshirildi.}$$

Javob: $I = 1 \text{ A}$, $U_R = 8 \text{ V}$, $U = 8 \text{ V}$.

6-MASALA. Voltmetrning ta'siri

Ideal manba $\varepsilon = 12 \text{ V}$ ichki qarshiliksiz $R = 6 \text{ om}$ rezistorga ulangan. Rezistor uchlariga $R_v = 18 \text{ om}$ qarshilikli voltmetr ulandanda, voltmetr ko'rsatkichi U_v qancha bo'ladi?

(Voltmetr rezistor bilan parallel ulanadi.)

Yechim

Rezistor va voltmetr parallel, ularning ekvivalent qarshiligi:

$$1/R_{eq} = 1/R + 1/R_v = 1/6 + 1/18$$

$$1/6 = 3/18$$

$$1/R_{eq} = 3/18 + 1/18 = 4/18 = 2/9$$

$$\text{Demak } R_{eq} = 9/2 = 4,5 \text{ om}$$

Umumiy tok:

$$I = \varepsilon / R_{eq} = 12 / 4,5$$

$$12 / 4,5 = (24 / 9) \approx 2,67 \text{ A}$$

Parallel tarmoqlarda kuchlanish bir xil, $U_v = U_{R_{eq}} = \varepsilon = 12 \text{ V}$ bo'ladimi? Yo'q, bu yerda manba ideal va faqat R_{eq} bor, demak zanjirda faqat shu bor. Shuning uchun ularning uchlarida kuchlanish 12 V bo'ladi.

Ammo aslida bu masala "voltmetr ideal bo'lmasa, zanjir tokini o'zgartiradi" g'oyasini ko'rsatish uchun. Tok o'zgargan, lekin $U = 12 \text{ V}$ bo'lib qoladi (ichki qarshilik yo'qligi sabab).

Javob: $U_v = 12 \text{ V}$; lekin zanjir toki voltmetr ulanmasdan farq qiladi.

(Real olimpiadada ichki qarshilik ham kiritilsa, U kamayadi.)

7-MASALA. Joul–Lents bilan isitish

$R = 10 \text{ om}$ bo'lgan elektr isitkich $\varepsilon = 24 \text{ V}$, $r = 2 \text{ om}$ bo'lgan manbaga ulangan. Isitkich 1 liter suvni ($m = 1 \text{ kg}$) $t_1 = 20 \text{ }^\circ\text{C}$ dan $t_2 = 80 \text{ }^\circ\text{C}$ gacha qizdiradi. Issiqlik

yo‘qotishlarini hisobga olmaymiz. Suvning solishtirma issiqlik sig‘imi $c = 4200 \text{ J/(kg} \cdot ^\circ\text{C)}$.

Topilsin:

- a) zanjirdagi tok I
- b) isitkichdagi quvvat P
- c) suvni qizdirish uchun kerak bo‘ladigan vaqt t

Yechim

a) Tok:

$$I = \varepsilon / (R + r) = 24 / (10 + 2) = 24 / 12 = 2 \text{ A}$$

b) Isitkichdagi quvvat:

$$P = I^2 \cdot R = 2^2 \cdot 10 = 4 \cdot 10 = 40 \text{ W}$$

c) Suvga berilishi kerak bo‘lgan issiqlik:

$$\begin{aligned} Q &= m \cdot c \cdot (t_2 - t_1) \\ Q &= 1 \cdot 4200 \cdot (80 - 20) = 4200 \cdot 60 = 252000 \text{ J} \end{aligned}$$

Vaqt:

$$t = Q / P = 252000 / 40 = 6300 \text{ s}$$

$$6300 \text{ s} \approx 6300 / 60 \approx 105 \text{ min} \approx 1 \text{ soat } 45 \text{ min.}$$

Javob: $I = 2 \text{ A}, P = 40 \text{ W}, t \approx 6300 \text{ s} (1,75 \text{ soat})$.

8-MASALA. Maksimal quvvat masalasi

EYuK $\varepsilon = 18 \text{ V}$, ichki qarshilik $r = 3 \text{ om}$ bo‘lgan manbaga o‘zgaruvchan tashqi qarshilik R ulanmoqda.

Topilsin:

- a) qaysi R da tashqi zanjirda ajralayotgan quvvat maksimal bo‘ladi?
- b) bu maksimal quvvat P_{max}
- c) shu holatda tok I_{max}

Yechim

a) Maksimal quvvat sharti: $R = r$
Demak $R = 3 \text{ om}$ bo‘lganda.

b) Maksimal quvvat:

$$P_{max} = \varepsilon^2 / (4r) = 18^2 / (4 \cdot 3)$$

$$18^2 = 324$$

$$P_{max} = 324 / 12 = 27 \text{ W}$$

c) Shu holatda tok:

$$I_{max} = \varepsilon / (R + r) = 18 / (3 + 3) = 18 / 6 = 3 \text{ A}$$

Tekshirish uchun $P = I^2 R = 3^2 \cdot 3 = 9 \cdot 3 = 27 \text{ W}$ – mos keladi.

Javob: $R_{opt} = 3 \text{ om}$, $P_{max} = 27 \text{ W}$, $I_{max} = 3 \text{ A}$.

9-MASALA. Simmetrik ko‘prik va tok taqsimlanishi

Wheatstone ko‘prigi simmetrik: $R_1 = R_2 = R_3 = R_4 = R \text{ om}$. Yuqori chap va pastki chap nuqtalar orasiga $\varepsilon = 10 \text{ V}$ ideal manba ulanadi. Yuqori o‘ng va pastki o‘ng nuqtalar orasiga $R_5 = 2R$ qarshilik ulangan.

Topilsin: R_5 bo‘ylab tok I_5 .

Yechim

Simmetrik ko‘prikda: $R_1 / R_2 = R_3 / R_4 = 1$.

Bu shartda klassik natija:

Ko‘prik muvozanatda $\rightarrow$ o‘rtadagi tarmoqdan (R_5) tok OQMAYDI.

Bu yerda ham shunday: yuqori o‘ng va pastki o‘ng nuqtalarning potentsiali bir xil bo‘ladi.

Demak:

$$I_5 = 0 \text{ A.}$$

Javob: $I_5 = 0 \text{ A}$.

10-MASALA. Uch rezistorli “ko‘prik”da quvvat taqsimlanishi

Sxema:

- $R_1 = 2 \text{ om}$ va $R_2 = 4 \text{ om}$ ketma-ket ulangan.
- Shu juftlikka parallel $R_3 = 3 \text{ om}$ ulanadi.
- Zanjir $\varepsilon = 12 \text{ V}$, $r = 0 \text{ om}$ manbaga ulangan.

Topilsin:

- a) ekvivalent qarshilik R_{eq}
- b) umumiy tok I
- c) $R_1 - R_2$ tarmog'idagi tok I_{12}
- d) R_3 bo'ylab tok I_3
- e) qaysi rezistorda quvvat ko'proq ajraladi?

Yechim

- a) $R_1 + R_2$:

$$R_{12} = R_1 + R_2 = 2 + 4 = 6 \text{ om}$$

Parallel R_{12} va R_3 :

$$\frac{1}{R_{eq}} = \frac{1}{R_{12}} + \frac{1}{R_3} = \frac{1}{6} + \frac{1}{3} = \frac{1}{6} + \frac{2}{6} = \frac{3}{6} = \frac{1}{2}$$

Demak $R_{eq} = 2 \text{ om}$

- b) Umumiy tok:

$$I = \frac{\varepsilon}{R_{eq}} = \frac{12}{2} = 6 \text{ A}$$

- c) Parallel tarmoqlarda kuchlanish bir xil:

$$U_{tarmoq} = 12 \text{ V}$$

R_{12} bo'ylab tok:

$$I_{12} = \frac{U_{tarmoq}}{R_{12}} = 2 \frac{12}{6} = 2 \text{ A}$$

- d) R_3 bo'ylab tok:

$$I_3 = \frac{U_{tarmoq}}{R_3} = \frac{12}{3} = 4 \text{ A}$$

Tekshiruv: $I = I_{12} + I_3 = 2 + 4 = 6 \text{ A}$ – to'g'ri.

- e) Quvvatlar:

R_1 talab qilmaymiz, lekin R_{12} tarmoqdagi quvvat:

$$P_{12} = I_{12}^2 \cdot R_{12} = 2^2 \cdot 6 = 4 \cdot 6 = 24 \text{ W}$$

R_3 dagi quvvat:

$$P_3 = I_3^2 \cdot R_3 = 4^2 \cdot 3 = 16 \cdot 3 = 48 \text{ W}$$

Demak R_3 da quvvat ko'proq ajraladi.

Javob: $R_{eq} = 2 \text{ om}$, $I = 6 \text{ A}$, $I_{12} = 2 \text{ A}$, $I_3 = 4 \text{ A}$, eng katta quvvat R_3 da (48 W).