

NAMUNAVIY MASALALAR

1-masala

Gorizontal silliq stol ustida massasi m bo'lgan kichik jism ikkala tomondan qattiqligi k bo'lgan ikkita bir xil prujinaga ulanib turibdi (devor–prujina–jism–prujina–devor shaklida). Jism muvozanat holatidan x ga siljilib qo'yib yuborildi.

- a) Qaytaruvchi kuchni x ga bog'lab yozing.
- b) Tebranish burchak chastotasi ω ni toping.
- c) Tebranish davri T ni toping.

Yechim

a) Har ikkala prujina ham siqiladi/cho'ziladi. Jism muvozanatdan x ga siljiganda har bir prujinadagi kuch modul jihatdan $k x$ bo'ladi, yo'nalish esa jismni o'rtaq tortadi.

Chap prujina: $F_1 = -k x$

O'ng prujina: $F_2 = -k x$

Natijaviy kuch:

$$F = F_1 + F_2 = -k x - k x = -2k x$$

Demak, qaytaruvchi kuch:

$$F = -2k x$$

b) Nyutonning ikkinchi qonuni:

$$m a = -2k x$$

$a = d^2x/dt^2$ ekan, demak

$$m d^2x/dt^2 = -2k x$$

$$d^2x/dt^2 + (2k/m) x = 0$$

Bu garmonik tebranish tenglamasi, burchak chastota:

$$\omega = \sqrt{\frac{2k}{m}}$$

c) Tebranish davri:

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{2k}{m}}} = 2\pi \sqrt{\frac{m}{2k}}$$

Javob:

$$F = -2kx, \quad \omega = \sqrt{\frac{2k}{m}}, T = 2\pi \sqrt{\frac{m}{2k}}$$

2-masala

Jism garmonik tebranish bajaradi. Uning tebranish davri $T = 2 \text{ s}$, amplitudasi $A = 0,1 \text{ m}$. $t = 0$ paytda jism muvozanat holatidan o'tmoqda va o'ng tomonga qarab $v_0 = 0,2 \text{ m/s}$ tezlikka ega.

- $x(t)$ tenglamasini yozing.
- Jism qachon birinchi marta chap chetki holatga yetib boradi?

Yechim

- Avval omega:

$$T = 2\pi/\omega \rightarrow \omega = 2\pi/T = 2\pi/2 = \pi \text{ rad/s}$$

Garmonik tebranish tenglamasi ko'rinishi:

$$x(t) = A \cos(\omega t + \varphi)$$

$t = 0$ da jism muvozanatdan o'tmoqda $\rightarrow x(0) = 0$.

$$x(0) = A \cos(\varphi) = 0 \rightarrow \cos \varphi = 0 \rightarrow \varphi = \pi/2 \text{ yoki } \varphi = -\pi/2$$

Tezlik:

$$v(t) = -A \omega \sin(\omega t + \varphi)$$

$t = 0$ da $v(0) = v_0 = 0,2 \text{ m/s}$. Ma'lumki jism o'ng tomonga qarab harakatlanmoqda, ya'ni $v(0) > 0$.

$$v(0) = -A \omega \sin(\varphi)$$

$A = 0,1, \omega = \pi$, demak:

$$0,2 = -0,1 \cdot \pi \cdot \sin(\varphi)$$

$$2 = -\pi \sin(\varphi)$$

$$\sin(\varphi) = -2/\pi$$

Endi φ ni tanlashimiz kerak. Oldin bilamiz $\cos(\varphi) = 0 \rightarrow \varphi = \pi/2$ yoki $-\pi/2$.

Bu nuqtalarda $\sin(\varphi)$ mos ravishda:

$$\sin(\pi/2) = 1$$

$$\sin(-\pi/2) = -1$$

Bizga $\sin(\varphi) < 0$ kerak ($-2/\pi$ manfiy), demak $\varphi = -\pi/2$ ga yaqin.

Soddalashtirib, kichik farqni hisobga olmasdan, boshlang'ich sharoitga mos model sifatida:

$x(t) \approx A \sin(\omega t)$ ko'rinishdan foydalansak ham bo'ladi, chunki $t = 0$ da $x = 0, v > 0$ uchun qulay:

$$x(t) = A \sin(\omega t)$$

Bu holda:

$$x(t) = 0,1 \sin(\pi t)$$

b) Chap chetki holat $x = -A$ ga mos keladi.

$$A \sin(\pi t) = -A \rightarrow \sin(\pi t) = -1$$

$$\sin(\pi t) = -1 \rightarrow \pi t = 3\pi/2 + 2\pi n$$

Eng kichik musbat t uchun $n = 0$:

$$\pi t = 3\pi/2 \rightarrow t = 3/2 \text{ s}$$

Javob:

$x(t) = 0,1 \sin(\pi t)$, birinchi marta chap chetki holatga $t = 1,5 \text{ s}$ da yetadi.

3-BO'LIM. ENERGIYA VA SAQLANISH QONUNI

Masala 3. Energiya bo'yicha tezlik va og'ishni topish

Massasi $m = 0,5 \text{ kg}$ bo'lgan jism prujinaga ulangan va garmonik tebranish bajarmoqda. Prujina qattiqligi $k = 200 \text{ N/m}$, tebranish amplitudasi $A = 0,05 \text{ m}$.

- a) Tizimning to'liq mexanik energiyasini toping.
 b) Jismning tezligi $v = 1 \text{ m/s}$ bo'lgan paytda og'ishi x ni toping (modul bo'yicha).
 c) Shu paytda potensial va kinetik energiyalarni toping.

Yechim

- a) To'liq energiya:

$$E = (k A^2)/2$$

$$E = 1/2 \cdot 200 \cdot (0,05)^2$$

$$0,05^2 = 0,0025$$

$$E = 100 \cdot 0,0025 = 0,25 \text{ J}$$

- b) To'liq energiya doimiy:

$$E = E_k + E_p = (m v^2)/2 + (k x^2)/2$$

$$0,25 = (0,5 \cdot 1^2)/2 + (200 x^2)/2$$

$$0,25 = 0,25 + 100 x^2$$

$$0,25 - 0,25 = 100 x^2 \rightarrow 0 = 100 x^2$$

$$x = 0$$

Demak $v = 1 \text{ m/s}$ bo'lishi aynan muvozanat holatida sodir bo'lgan.

- c) Shu paytda:

$$E_p = (k x^2)/2 = 0$$

$$E_k = E = 0,25 \text{ J}$$

Javob:

$E = 0,25 \text{ J}, v = 1 \text{ m/s}$ bo'lganda $x = 0, E_p = 0, E_k = 0,25 \text{ J}$.

4-BO'LIM. FAZA VA GRAFIKLAR

Masala 4. Grafikdan tezlik va tezlanishni topish

Tebranayotgan jismning x - t grafigidan ma'lumki:

$$\text{📖 } t = 0 \text{ da } x = 0,$$

📖 $t = 0,25 \text{ s}$ da $x = 0,1 \text{ m}$ (maksimal qiymat),

📖 $t = 0,5 \text{ s}$ da $x = 0$,

Bu tebranish davri T va amplitudasi A bo'yicha garmonik tebranishga mos keladi.

a) Tebranish davri T va burchak chastota ω ni toping.

b) $x(t)$ tenglamasini yozing.

c) $t = 0$ nuqtadagi tezlik $v(0)$ va tezlanish $a(0)$ ni toping.

Yechim

Grafikdan ko'rinib turibdi:

📖 $t = 0$ da $x = 0$

📖 $t = 0,25 \text{ s}$ da $x = A = 0,1 \text{ m}$

📖 $t = 0,5 \text{ s}$ da $x = 0$

Bu $x = 0 \rightarrow x = +A \rightarrow x = 0$ ketma-ketlik garmonik tebranishning chorak davrlari:

0 dan $+A$ ga chiqish uchun vaqt $T/4$ ga teng.

Demak:

$$T/4 = 0,25 \text{ s} \rightarrow T = 1 \text{ s}$$

$$\omega = 2\pi/T = 2\pi \text{ rad/s}$$

a) $A = 0,1 \text{ m}, T = 1 \text{ s}, \omega = 2\pi$.

b) $t = 0$ da $x = 0$ va $t = 0,25 \text{ s}$ da $x = +A$ bo'lsa, qulay shakl:

$$x(t) = A \sin(\omega t)$$

Tekshiramiz:

$t = 0$ da $x = 0$ — mos

$t = T/4 = 0,25 \text{ s}$ da:

$$x = A \sin(\omega T/4) = A \sin(2\pi \cdot 1/4) = A \sin(\pi/2) = A \text{ — mos}$$

Demak:

$$x(t) = 0,1 \sin(2\pi t)$$

c) Tezlik:

$$v(t) = dx/dt = 0,1 \cdot 2\pi \cos(2\pi t) = 0,2\pi \cos(2\pi t)$$

$t = 0$ da:

$$v(0) = 0,2\pi \cos(0) = 0,2\pi \text{ m/s} \approx 0,628 \text{ m/s}$$

Tezlanish:

$$a(t) = dv/dt = -0,1 \cdot (2\pi)^2 \sin(2\pi t)$$

$$a(0) = -0,1 \cdot (2\pi)^2 \sin(0) = 0$$

Javob:

$$\begin{aligned} T &= 1 \text{ s}, \omega = 2\pi \text{ rad/s}, \\ x(t) &= 0,1 \sin(2\pi t), \\ v(0) &\approx 0,63 \text{ m/s}, a(0) = 0. \end{aligned}$$

5-BO‘LIM. MATEMATIK MAYATNIK

Masala 5. Uzunlik o‘zgarganda davrning taqqoslanishi

Uzunligi $l_1 = 0,9 \text{ m}$ bo‘lgan matematik mayatnik davri T_1 ga teng. Mayatnik uzunligi $l_2 = 1,6 \text{ m}$ ga o‘zgartirildi.

a) T_2/T_1 nisbatni toping.

b) Agar birinchi mayatnik davri $T_1 = 1,9 \text{ s}$ bo‘lsa, T_2 ni toping ($g \approx 10 \text{ m/s}^2$ deb oling).

Yechim

Matematik mayatnik davri:

$$T = 2\pi \sqrt{\frac{l}{g}}$$

a) Demak:

$$T_2/T_1 = \sqrt{(l_2/g)} / \sqrt{(l_1/g)} = \sqrt{(l_2/l_1)}$$

$$T_2/T_1 = \sqrt{(1,6/0,9)}$$

$$1,6/0,9 \approx 1,777\dots$$

$$\sqrt{1,777\dots} \approx 1,333\dots (\text{ya'ni } \approx 4/3)$$

$$b) T_2 \approx (4/3) T_1 \approx (4/3) \cdot 1,9 \approx 7,6/3 \approx 2,53 \text{ s}$$

Javob:

$$T_2/T_1 \approx 4/3, T_2 \approx 2,5 \text{ s.}$$

6-BO‘LIM. FIZIK MAYATNIK

Masala 6. Uchidan osilgan tayoqcha tebranishi

Uzunligi $l = 0,6 \text{ m}$ bo‘lgan bir jinsli ingichka tayoqcha yuqori uchidan gorizonta o‘q atrofida osilgan (vertikal tekislikda tebranadi). Tayoqcha kichik burchak ostida tebranish bajarmoqda. $g = 10 \text{ m/s}^2$.

- Tayoqchani massa markazi osilish nuqtasidan qanchaga uzoq?
- Tayoqchani inertsia momenti I ni osilish nuqtasiga nisbatan ifodalang (m — massa).
- Tebranish davri T ni toping.

Yechim

- Bir jinsli tayoqchani massa markazi o‘rtada:

$$a = l/2 = 0,6/2 = 0,3 \text{ m}$$

- Uchiga nisbatan inertsia momenti:

$$I = (1/3) m l^2$$

- Fizik mayatnik davri:

$$T = 2\pi \sqrt{\frac{I}{m g a}}$$

O‘rniga qo‘yib:

$$T = 2\pi \sqrt{\frac{\left(\frac{1}{3}\right) m l^2}{m g \left(\frac{l}{2}\right)}}$$

m qisqaradi:

$$\begin{aligned} T &= 2\pi \sqrt{(l^2/3)/(g l/2)} = 2\pi \sqrt{(l^2/3) \cdot (2/(g l))} \\ &= 2\pi \sqrt{(2 l)/(3 g)} \end{aligned}$$

Endi $l = 0,6, g = 10$:

$$T = 2\pi \sqrt{\frac{2 \cdot 0,6}{3 \cdot 10}} = 2\pi \sqrt{\frac{1,2}{30}} = 2\pi \sqrt{0,04}$$

$$\sqrt{0,04} = 0,2$$

$$T = 2\pi \cdot 0,2 = 0,4\pi \approx 1,26 \text{ s}$$

Javob:

$$a = 0,3 \text{ m}, l = (1/3) \text{ m l}^2, T \approx 1,26 \text{ s}.$$

7-BO‘LIM. SO‘NUVCHI VA MAJBURIY TEBRANISHLAR, REZONANS

Masala 7. So‘nish va energiya kamayishi

Garmonik tebranuvchi sistemada ishqalanish sababli tebranish amplitudasi har bir davrda bir xil marta kamayib boradi. Birinchi davr oxirida amplituda $A_1 = 5 \text{ sm}$, uchinchi davr oxirida amplituda $A_3 = 3,2 \text{ sm}$ bo‘ldi.

- a) Har bir davrdan keyin amplituda necha barobar kamaymoqda (q koeffitsient)?
- b) Agar birinchi davr boshida to‘liq energiya E_1 bo‘lsa, uchinchi davr oxiridagi energiya E_3/E_1 nisbatini toping.
- c) Sönishning energiyaga ta’siriga doir fizik xulosa chiqaring.

Yechim

Amplituda har davrda q marta kamaymoqda deb olaylik:

$$\text{Ikkinchi davr oxiri: } A_2 = A_1 \cdot q$$

$$\text{Uchinchi davr oxiri: } A_3 = A_2 \cdot q = A_1 \cdot q^2$$

Berilgan: $A_1 = 5 \text{ sm}, A_3 = 3,2 \text{ sm}$:

$$A_1 \cdot q^2 = A_3 \rightarrow 5 \cdot q^2 = 3,2$$

$$q^2 = 3,2/5 = 0,64$$

$$q = \sqrt{0,64} = 0,8$$

Demak, har davrda amplituda 0,8 baravarga kamayadi.

b) Garmonik tebranishda to‘liq energiya amplitudaning kvadratiga proportsional:

$$E \sim A^2$$

$$E_3/E_1 = (A_3^2)/(A_1^2) = (A_1^2 q^4)/A_1^2 = q^4$$

$$q = 0,8 \rightarrow q^4 = (0,8)^4$$

$$0,8^2 = 0,64,$$

$$0,64^2 = 0,4096$$

$$E_3/E_1 \approx 0,41$$

c) Xulosa:

Amplituda nisbatan sekin kamayayotgan bo'lsa ham, energiya uning kvadratiga proporsional bo'lgani uchun **energiya ancha tezroq kamayadi**. Bu sönuvchi tebranishlarda juda muhim.

Javob:

$q = 0,8$; $E_3/E_1 \approx 0,41$; energiya amplitudaga qaraganda tezroq so'nadi.

8-masala. Vertikal prujina va tebranish

Massasi $m = 0,4$ kg bo'lgan jism vertikal prujinaga osilgan. Prujina qattiqligi $k = 100$ N/m. Jism muvozanat holatidan pastga $A = 6$ sm ga tortilib qo'yib yuboriladi. $g = 10$ m/s².

Topilsin:

1. Tebranish davri T
2. Maksimal tezlik $v_{\max}$
3. Jism muvozanatdan 3 sm uzoqlikda bo'lganda uning tezligi v

Yechim

1. Tebranish davri gravitatsiyaga bog'liq emas, faqat m va k ga bog'liq:

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{100}{0,4}} = \sqrt{250} \approx 15,8 \text{ rad/s}$$

$$T = 2\pi/\omega \approx 2\pi/15,8 \approx 0,40 \text{ s}$$

2. Maksimal tezlik muvozanat holatidan o'tayotganda:

$$v_{\max} = \omega A$$

$$A = 0,06 \text{ m} \rightarrow v_{\max} \approx 15,8 \cdot 0,06 \approx 0,95 \text{ m/s}$$

3. Energiya saqlanishidan foydalanamiz:

$$E = (1/2) k A^2 = (1/2) k x^2 + (1/2) m v^2$$

Bu yerda x — berilgan ondagi og‘ish (0,03 m).

$$(1/2) k A^2 - (1/2) k x^2 = (1/2) m v^2$$

$$k (A^2 - x^2) = m v^2$$

$$v = \sqrt{k \frac{A^2 - x^2}{m}}$$

$$A = 0,06 \text{ m}, x = 0,03 \text{ m}, k = 100, m = 0,4:$$

$$v \approx 15,8 \cdot \sqrt{0,06^2 - 0,03^2} \approx 0,82 \text{ m/s (aniqrog'i } \approx 0,82 \text{ m/s)}$$

Javob:

$$T \approx 0,40 \text{ s}, v_{\max} \approx 0,95 \text{ m/s}, v(3 \text{ sm da}) \approx 0,82 \text{ m/s}.$$

9-masala. Matematik mayatnik va tezliklar

Uzunligi $l = 0,9 \text{ m}$ bo‘lgan matematik mayatnik kichik burchaklarda tebranmoqda. Erkin tushish tezlanishi $g = 9,8 \text{ m/s}^2$. Mayatnik tebranish amplitudasi $\theta_0 = 10^\circ$ (radianlarda ishlaymiz).

Topilsin:

1. Tebranish davri T
2. Maximum tezlik $v_{\max}$
3. Mayatnik og‘ish burchagi $\theta = 5^\circ$ bo‘lganda uning tezligi v

Yechim

Avval burchaklarni radianlarda yozamiz:

$$\theta_0 = 10^\circ \approx 10 \cdot \pi/180 \approx 0,175 \text{ rad}$$

$$\theta = 5^\circ \approx 5 \cdot \pi/180 \approx 0,087 \text{ rad}$$

1. Tebranish davri (kichik tebranishlar uchun):

$$T = 2\pi \sqrt{\frac{l}{g}}$$

$$T \approx 2\pi \sqrt{\frac{0,9}{9,8}} \approx 2\pi \cdot 0,302 \approx 1,90 \text{ s}$$

2. Kichik tebranishda burchak tebranishi garmonik:

$$\theta(t) \approx \theta_0 \cos(\omega t + \varphi), \omega = 2\pi/T$$

$$\omega \approx 2\pi/1,90 \approx 3,30 \text{ rad/s}$$

Chiziqli tezlik: $v = l d\theta/dt$

Maksimal tezlik burchak nol bo'lganda bo'ladi:

$$v_{max} = \omega l \theta_0$$

$$v_{max} \approx 3,30 \cdot 0,9 \cdot 0,175 \approx 0,52 \text{ m/s}$$

3. Energiya saqlanishi (kichik burchaklar uchun) burchak bo'yicha ham ishlaydi:

Umumiy energiya $\sim \theta_0^2$, joriy potensial energiya $\sim \theta^2$, qolgan qismi kinetik. Shuning uchun:

$$v(\theta) = \omega l \sqrt{\theta_0^2 - \theta^2}$$

$$v \approx 3,30 \cdot 0,9 \cdot \sqrt{0,175^2 - 0,087^2}$$

$$0,175^2 \approx 0,0306$$

$$0,087^2 \approx 0,0076$$

$$\text{Farq} \approx 0,023$$

$$\sqrt{0,023} \approx 0,152$$

$$v \approx 3,30 \cdot 0,9 \cdot 0,152 \approx 0,45 \text{ m/s}$$

Javob:

$$T \approx 1,90 \text{ s}, v_{max} \approx 0,52 \text{ m/s}, v(\theta = 5^\circ) \approx 0,45 \text{ m/s}.$$

10-masala. Gorizontaal prujina, energiya va tezlanish

Massasi $m = 0,25 \text{ kg}$ bo'lgan jism gorizontaal silliq sirt ustida prujinaga ulangan. Prujina qattiqligi $k = 25 \text{ N/m}$. Jism amplitudasi $A = 0,20 \text{ m}$ bo'lgan garmonik tebranish bajarmoqda.

Topilsin:

1. Tizimning to'liq mexanik energiyasi E
2. Jismning $x = 0,10$ m og'ishda bo'lgandagi tezligi v
3. Tebranishdagi maksimal tezlanish $a_{\max}$

Yechim

1. To'liq energiya:

$$E = (1/2) k A^2$$

$$E = 1/2 \cdot 25 \cdot (0,20)^2$$

$$0,20^2 = 0,04$$

$$E = 12,5 \cdot 0,04 = 0,5 \text{ J}$$

2. Energiya saqlanishidan:

$$E = (1/2) k x^2 + (1/2) m v^2$$

$$0,5 = (1/2) \cdot 25 \cdot (0,10)^2 + (1/2) \cdot 0,25 \cdot v^2$$

$$(1/2) \cdot 25 \cdot 0,01 = 12,5 \cdot 0,01 = 0,125$$

Demak:

$$0,5 = 0,125 + 0,125 v^2$$

$$0,375 = 0,125 v^2$$

$$v^2 = 3 \rightarrow v = \sqrt{3} \approx 1,73 \text{ m/s}$$

3. Burchak chastota:

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{25}{0,25}} = \sqrt{100} = 10 \text{ rad/s}$$

Maksimal tezlanish chetki nuqtada:

$$a_{\max} = \omega^2 A = 10^2 \cdot 0,20 = 100 \cdot 0,20 = 20 \text{ m/s}^2$$

Javob:

$$E = 0,5 \text{ J}, v(x = 0,10 \text{ m}) \approx 1,73 \text{ m/s}, a_{\max} = 20 \text{ m/s}^2.$$

