Namunaviy masalalar

1-Masala

a va b musbat sonlar bo‘lsin. Isbotlang:

$$\frac{a+b}{2} \geq \sqrt{ab}$$

Yechim:

1) $(a - b)^2 \geq 0$, chunki kvadrat hech qachon manfiy bo‘lmaydi.

2) Yoyamiz: $a^2 - 2ab + b^2 \geq 0$.

3) Demak: $a^2 + b^2 \geq 2ab$.

4) $(a + b)^2 = a^2 + 2ab + b^2 \geq 4ab$.

5) Ildiz olamiz: $a + b \geq 2\sqrt{ab}$.

6) Bo‘linadi: $\frac{a+b}{2} \geq \sqrt{ab}$.

Tenglik faqat $a = b$ bo‘lganda.

2-Masala

a va b musbat sonlar bo‘lsin va $a + b = 1$. Isbotlang:

$$\frac{1}{a} + \frac{1}{b} \geq 4$$

Yechim:

1) Koshi:

$$\frac{1}{a} + \frac{1}{b} \geq \frac{(1+1)^2}{a+b}.$$

2) $a + b = 1$, shuning uchun:

$$\frac{1}{a} + \frac{1}{b} \geq 4$$

Tenglik $a = b = \frac{1}{2}$ da.

3-masala

$a, b, c > 0$ va $a + b + c = 1$ bo‘lsin. Isbotlang:

$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} \geq 9.$

Yechim:

1) Koshi: $\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)(a + b + c) \geq (1 + 1 + 1)^2 = 9.$

2) $a + b + c = 1 \rightarrow \frac{1}{a} + \frac{1}{b} + \frac{1}{c} \geq 9.$

Tenglik $a = b = c = \frac{1}{3}$ da.

4-masala

$a, b, c > 0$ bo‘lsin. Isbotlang:

$\frac{a^2}{b+c} + \frac{b^2}{c+a} + \frac{c^2}{a+b} \geq \frac{a+b+c}{2}.$

Yechim:

1) Koshi: $\frac{a^2}{b+c} + \ldots \geq (a + b + c)^2 / [(b + c) + (c + a) + (a + b)].$

2) Yig‘indi: $(b + c) + (c + a) + (a + b) = 2(a + b + c).$

3) Qisqartiramiz: $\frac{(a+b+c)^2}{[2(a+b+c)]} = \frac{a+b+c}{2}.$

Natija isbotlandi.

5-masala (Shur)

a, b, c > 0 bo‘lsin. Isbotlang:

$$a^3 + b^3 + c^3 + 3abc \geq ab(a + b) + bc(b + c) + ca(c + a).$$

Yechim:

1) Bu Shur tengsizligi.

2) Faraz qilamiz c eng kichik son.

$a = c + x,\ b = c + y,\ x \geq 0, y \geq 0.$

3) $S = a^3 + b^3 + c^3 + 3abc - [ab(a+b) + bc(b+c) + ca(c+a)]$.

4) S quyidagicha ajraladi:

$S = c(x^2 - xy + y^2) + (x - y)^2(x + y)$.

5) Ikkala qism ham ≥ 0.

6) Demak $S \geq 0$.

Tenglik faqat $a = b = c$ bo‘lganda.