

FUNKSIONAL TENGLAMALAR

Mustaqil ishlash uchun masalar

1-masala

Masala.

$f : R \rightarrow R$ uchun

$$f(x + y) = f(x) + f(y)$$

va

$$f(2) = 6$$

berilgan. $f(x)$ ni toping.

Yechim.

$$x = y = 1:$$

$$f(2) = f(1) + f(1) = 2f(1) = 6$$

Demak:

$$f(1) = 3$$

Shundan:

$$f(x) = 3x$$

Javob: $f(x) = 3x$

2-masala

Masala.

$f : R \rightarrow R$ uchun

$$f(x + y) = f(x) \cdot f(y)$$

va

$$f(1) = 2$$

berilgan. $f(x)$ ni toping.

Yechim.

$$y = 0:$$

$$f(0) = 1$$

Induksiya bilan:

$$f(n) = 2^n$$

Javob: $f(x) = 2^x$

3-masala

Masala.

$f : R \rightarrow R$ uchun

$$f(3x) = 3f(x)$$

va

$$f(1) = 4$$

berilgan. $f(x)$ ni toping.

Yechim.

$f(x) = kx$ deb olamiz.

$$k \cdot 3x = 3kx$$

Mos keladi.

$$f(1) = 4 \rightarrow k = 4.$$

Javob: $f(x) = 4x$

4-masala

Masala.

$$f(x + y) = f(x) + f(y) + 2xy$$

$$va$$

$$f(0) = 0$$

berilgan. $f(x)$ ni toping.

Yechim.

$$f(x) = ax^2 \text{ deb olamiz.}$$

Chap:

$$a(x + y)^2$$

O'ng:

$$ax^2 + ay^2 + 2xy$$

Taqqoslaymiz:

$$2a = 2 \rightarrow a = 1$$

$$\textbf{Javob: } f(x) = x^2$$

5-masala

Masala.

$$f(x + 1) = f(x) + 5$$

Isbotlang: f funksiya injektiv.

Yechim.

Agar:

$$f(a) = f(b)$$

bo'lsa, u holda:

$$f(b + n) = f(b) + 5n$$

bo'lgani uchun:

$$f(a) = f(b + (a - b)) = f(b) + 5(a - b)$$

Bu faqat:

$$a - b = 0$$

da mumkin.

Xulosa: f injektiv.

6-masala

Masala.

$$f(x) = 4x - 7$$

Isbotlang: $f : R \rightarrow R$ sur'yektiv.

Yechim.

Ixtiyoriy $y \in R$ uchun:

$$\begin{aligned} 4x - 7 &= y \\ x &= \frac{y + 7}{4} \end{aligned}$$

Har doim haqiqiy.

Xulosa: f sur'yektiv.

7-masala

Masala.

$$f(x + y) = f(x) + f(y) + 1$$

Konstant yechimni toping.

Yechim.

$f(x) = C$ desak:

$$C = C + C + 1$$

Bu:

$$C = -1$$

da bajariladi.

Javob: $f(x) = -1$

8-masala

Masala.

$$f(x - y) = f(x) + f(y)$$

Toping f .

Yechim.

$$x = y:$$

$$f(0) = f(x) + f(x)$$

Demak:

$$f(x) = 0$$

Javob: $f(x) = 0$

9-masala

Masala.

$$f(x + y) + f(x - y) = 2f(x) + 2y^2$$

Toping $f(x)$.

Yechim.

$$f(x) = ax^2 \text{ deb olamiz.}$$

Chap:

$$a(x + y)^2 + a(x - y)^2 = 2ax^2 + 2ay^2$$

O'ng:

$$2ax^2 + 2y^2$$

Demak:

$$a = 1$$

Javob: $f(x) = x^2$

10-masala

Masala.

$$f(x + y) = f(x) + f(y)$$

Va f uzluksiz, $f(1) = 5$

Toping $f(x)$.

Yechim.

Uzluksizlik sabab:

$$f(x) = kx$$

$$f(1) = 5 \rightarrow k = 5.$$

Javob: $f(x) = 5x$

11-masala.

$f : R \rightarrow R$ funksiya uchun:

$$\begin{aligned} f(x + 1) - f(x) &= 2x + 3 \\ f(0) &= 1 \end{aligned}$$

berilgan. $f(x)$ ni toping.

Yechim:

E'tibor beramiz:

$$\begin{aligned}(x + 1)^2 - x^2 &= 2x + 1 \\ (x + 1)^2 + 2 - (x^2 + 2) &= 2x + 3\end{aligned}$$

demak, $2x + 3$ ifodasi kvadrat bilan bog'liq.

$f(x)$ ni kvadrat ko'rinishda taxmin qilamiz:

$$f(x) = ax^2 + bx + c$$

Hisoblaymiz:

$$\begin{aligned}f(x + 1) &= a(x + 1)^2 + b(x + 1) + c \\ &= a(x^2 + 2x + 1) + bx + b + c \\ &= ax^2 + 2ax + a + bx + b + c\end{aligned}$$

Demak:

$$\begin{aligned}&f(x + 1) - f(x) \\ &= (ax^2 + 2ax + a + bx + b + c) - (ax^2 + bx + c) \\ &= 2ax + a + b\end{aligned}$$

Bu teng bo'lishi kerak:

$$2ax + a + b = 2x + 3$$

Koeffitsientlarni taqqoslaymiz:

x ning oldidagi koeffitsient:

$$2a = 2 \rightarrow a = 1$$

erkin had:

$$\begin{aligned}a + b &= 3 \\ 1 + b &= 3 \rightarrow b = 2\end{aligned}$$

Endi $f(0)$ dan c ni topamiz:

$$f(0) = a \cdot 0^2 + b \cdot 0 + c = c = 1$$

Shunday qilib: $f(x) = x^2 + 2x + 1$

Javob:

$$f(x) = x^2 + 2x + 1$$

12-masala. (Aralash qo'shuvchi + kvadrat)

$f : R \rightarrow R$ uchun:

$$\begin{aligned} f(x + y) &= f(x) + f(y) + 4xy \\ f(0) &= 0 \end{aligned}$$

berilgan. $f(x)$ ni toping.

Yechim:

Taxmin qilamiz:

$$f(x) = a x^2$$

(Chunki qo'shimcha had $4xy$ kvadrat bilan bog'liq.)

Chap:

$$\begin{aligned} f(x + y) &= a (x + y)^2 = a (x^2 + 2xy + y^2) \\ &= a x^2 + 2a xy + a y^2 \end{aligned}$$

O'ng:

$$f(x) + f(y) + 4xy = a x^2 + a y^2 + 4xy$$

Taqqoslaymiz:

- x^2 va y^2 uchun:
mos tushadi.
- xy uchun:
chapda: $2a xy$
o'ngda: $4 xy$

Demak:

$$2a = 4 \rightarrow a = 2$$

Shunday qilib:

$$f(x) = 2x^2$$

$f(0) = 0$ shart ham bajariladi.

Javob:

$$f(x) = 2x^2$$

13-masala.

$f : R \rightarrow R$ uchun:

$$\begin{aligned} f(x + y) &= f(x) + f(y) + x * y \\ f(0) &= 0 \end{aligned}$$

berilgan. $f(x)$ ni toping.

Yechim.

Taxmin qilamiz:

$$f(x) = a x^2$$

Chap:

$$f(x + y) = a(x + y)^2 = a(x^2 + 2xy + y^2)$$

O'ng:

$$f(x) + f(y) + x \cdot y = a x^2 + a y^2 + x \cdot y$$

Taqqoslaymiz:

$$a x^2 + 2a x y + a y^2 = a x^2 + a y^2 + x y$$

Bu yerda:

$$2a x y = x y \rightarrow 2a = 1 \rightarrow a = \frac{1}{2}$$

Demak:

$$f(x) = \frac{1}{2} x^2$$

Javob:

$$f(x) = \frac{x^2}{2}$$

14-masala.

$f : R \rightarrow R$ funksiya uchun:

$$\begin{aligned} f(x - y) &= f(x) + f(y) - 4xy \\ f(0) &= 0 \end{aligned}$$

berilgan. $f(x)$ ni toping.

Yechim:

Kvadrat tanlaymiz:

$$f(x) = a x^2$$

Chap:

$$f(x - y) = a(x - y)^2 = a(x^2 - 2xy + y^2)$$

O'ng:

$$f(x) + f(y) - 4xy = a x^2 + a y^2 - 4xy$$

Taqqoslaymiz:

$$a x^2 - 2a x y + a y^2 = a x^2 + a y^2 - 4xy$$

xy koeffitsientlari:

$$-2a = -4 \rightarrow a = 2$$

Demak:

$$f(x) = 2x^2$$

$f(0) = 0$ ham mos keladi.

Javob:

$$f(x) = 2x^2$$

15-masala.

$f : R \rightarrow R$ uchun:

$$\begin{aligned} f(2x) &= 2f(x) + 3x^2 \\ f(0) &= 0 \end{aligned}$$

berilgan. $f(x)$ ni toping.

Yechim:

Taxmin qilamiz:

$$f(x) = a x^2 + b x$$

Chap:

$$f(2x) = a(2x)^2 + b(2x) = 4a x^2 + 2b x$$

O'ng:

$$\begin{aligned} 2f(x) + 3x^2 &= 2(ax^2 + bx) + 3x^2 \\ &= (2a x^2 + 2b x + 3x^2) \\ &= (2a + 3)x^2 + 2b x \end{aligned}$$

Taqqoslaymiz:

$$4a x^2 + 2b x = (2a + 3)x^2 + 2b x$$

Demak:

$$4a = 2a + 3 \rightarrow 2a = 3 \rightarrow a = \frac{3}{2}$$

b haqida cheklov yo'q (chunki $2b x$ ikkala tomonda ham bir xil).

Endi $f(0)$ dan b ni topamiz:

$$f(0) = a \cdot 0 + b \cdot 0 = 0 \quad (\text{bu har qanday } b \text{ uchun rost})$$

Demak b ixtiyoriy bo'ladi. Lekin ko'pincha eng sodda yechim sifatida $b = 0$ tanlanadi.

Standart yechim:

$$f(x) = \frac{3}{2}x^2$$