MUSTAQIL ISHLASH UCHUN MASALALAR

1-masala.

ABC uchburchakda $D \in BC$ shundayki: $\frac{BD}{DC} = \frac{BA}{AC}$ $E \in CA$ va $F \in AB$ shunday tanlanganki: $BE \perp AC,\ CF \perp AB$

Isbotlang: AD, BE, CF bitta nuqtada kesishadi

Yechim (tahlil): $\frac{BD}{DC} = \frac{BA}{AC}$ — bu **bissektrisa sharti,** BE va CF — mos balandliklar

Cheva uchun: $\left(\frac{BD}{DC}\right) \cdot \left(\frac{CE}{EA}\right) \cdot \left(\frac{AF}{FB}\right) = \left(\frac{BA}{AC}\right) \cdot \left(\frac{AC}{AB}\right) \cdot 1 = 1$

Kesishadi.

2-masala.

ABC uchburchakda, D — BC ichida, E — CA davomida, F — AB ichida.

Berilgan: $BD = DC, CE = 2 \cdot EA, AF = FB$

Isbotlang: D, E, F bitta to‘g‘ri chiziqda YO‘TMAYDI

Yechim: Menelay tekshiramiz: $\left(\frac{BD}{DC}\right) \cdot \left(\frac{CE}{EA}\right) \cdot \left(\frac{AF}{FB}\right) = 1 \cdot 2 \cdot 1 = 2 \neq 1$

Demak kollinearlik YO‘Q.

3-masala.

ABC uchburchakda AD va BE kesishib P nuqtani hosil qiladi. P dan AB va AC tomonlarga tushirilgan parallel chiziqlar mos ravishda F va E nuqtalarda tutashsin.

Isbotlang: D, E, F bitta to‘g‘ri chiziqda yotadi.

Yechim: (g‘oya): $\triangle PAB$ va $\triangle PAC$ da **Menelay,** Parallellik → nisbatlar teng, Ikkala Menelayni birlashtirib: D, E, F kollinear.

4-masala.

ABC uchburchakda $D \in BC$ shundayki: $\frac{BD}{DC} = 2, E \in CA$ va $F \in AB$ shunday tanlangan: $\frac{CE}{EA} = \frac{1}{3}, \frac{FB}{FA} = 3$.

Savol: AD, BE, CF kesishadimi?

Yechim: E’tibor bering: $\frac{AF}{FB} = \frac{1}{3}$

Cheva: $2 \cdot \frac{1}{3} \cdot \frac{1}{3} = \frac{2}{9} \neq 1$ Kesishmaydi.

5-masala.

ABC uchburchakda $D \in BC, E \in CA, F \in AB$.

Berilgan: $\left(\frac{BD}{DC}\right) \cdot \left(\frac{CE}{EA}\right) \cdot \left(\frac{AF}{FB}\right) = 1$

Savol: D, E, F har doim bitta to‘g‘ri chiziqda yotadimi?

Yechim: Noto‘g‘ri xulosa qilish mumkin.

Sabab: bu formula **Cheva ham bo‘lishi mumkin,** qayerda yozilgani noma’lum

Xulosa: Har doim emas

6-masala.

ABC uchburchakda $E \in CA, F \in AB$. BE va CF kesishib P nuqtani hosil qiladi.

P dan BC ga tushirilgan chiziq D da kesadi.

Isbotlang: $\left(\frac{BD}{DC}\right) \cdot \left(\frac{CE}{EA}\right) \cdot \left(\frac{AF}{FB}\right) = 1$

Yechim: Cheva $\boldsymbol{ABC}$ **da emas.** Cheva $\triangle \boldsymbol{PBC}, \triangle \boldsymbol{PCA}, \triangle \boldsymbol{PAB}$ da.

Har uch Cheva ko‘paytiriladi → natija chiqadi.

7-masala.

ABC uchburchakda $D \in BC$ shundayki: $BD = DC$. Bir to‘g‘ri chiziq BC, CA, AB tomonlarni mos ravishda D, E, F da kesadi.

Isbotlang: $CE = FA$

Yechim: Menelay: $1 \cdot \left(\frac{CE}{EA}\right) \cdot \left(\frac{AF}{FB}\right) = 1$

Demak: $\frac{CE}{EA} = \frac{FB}{AF}$. Simmetriyadan $EA = FB$.

Natija: $CE = FA$

8-masala.

ABC uchburchakda G — og‘irlik markazi. G dan tomonlarga tushirilgan kesmalar $D \in BC, E \in CA, F \in AB$.

Isbotlang: $\left(\frac{BD}{DC}\right) \cdot \left(\frac{CE}{EA}\right) \cdot \left(\frac{AF}{FB}\right) = 1$

Yechim: Medianalar → Cheva **avtomatik**.

Bu masala: “Cheva qachon isbotsiz ishlaydi?” savoliga javob.

9-masala.

ABC uchburchakda, bir chiziq BC ni ichida, CA va AB ni davomida kesadi

Berilgan: $\frac{BD}{DC} = 3, \frac{CE}{EA} = \frac{1}{3}, \frac{AF}{FB} = 1$

Xulosa chiqaring.

Yechim: Menelay: $3 \cdot \frac{1}{3} \cdot 1 = 1$ kollinear.

10-masala.

ABC uchburchakda D, E, F nuqtalar shunday tanlanganki:

D, E, F kollinear

AD, BE, CF kesishadi

Isbotlang: Cheva va Menelay bir-birining teskari ko‘rinishi

Yechim (g‘oya)

D, E, F kollinear → Menelay

Ko‘paytma = 1

Shu ko‘paytma → Cheva sharti

Bu **nazariy yakun**.