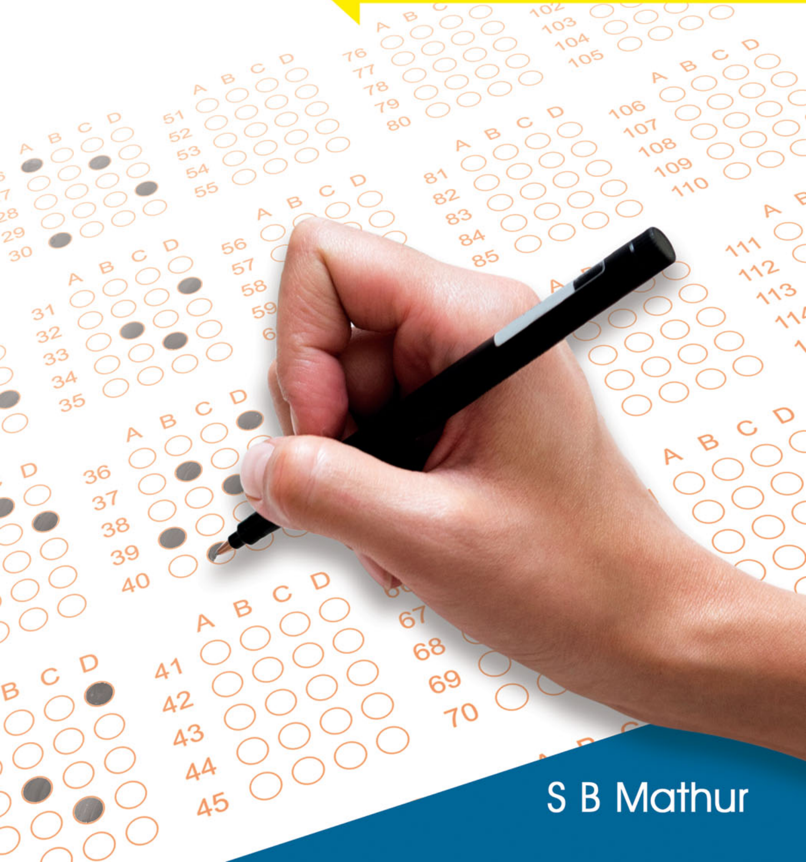
BHARATI BHAWAN (P & D)

Solved Problems in Physics

for IIT JEE Advanced



S B Mathur

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S B Mathur, PhD

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PREFACE

The IIT JEE (Advanced) entrance examination is unique in terms of the various types of questions asked to determine a student's depth of understanding of a subject. Keeping this in mind, this book has been designed to provide the students with sufficient questions to let them take the test confidently. The questions are given in the following six chapters, each based on a specific pattern:

1. Objective Questions: Type 1 (with only *one* option correct),
2. Objective Questions: Type 2 (with *one* or *more* options correct),
3. Integer-Answer-Type Questions,
4. Matrix-Matching-Type Questions,
5. Linked-Comprehension-Type Questions, and
6. Assertion–Reason-Type Questions.

Solutions to all the questions are given separately in detail at the end of each chapter to assist self-assessment.

Finally, I would like to thank my family members—Uma Mathur, Nitin, Manjari, and little Aishanya—for their constant encouragement and support.

Suggestions from teachers and students for the improvement of the book will be highly appreciated.

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❑

1

Multiple-Choice Questions: Type 1

*In this chapter, each question has **four** options (a, b, c and d), out of which **only one** option is correct.*

1.1 General Physics

1. A vernier callipers has 1-mm marks on the main scale. It has 20 equal divisions on the vernier scale which match with 16 main-scale divisions. For this vernier callipers, the least count is

(a) 0.02 mm (b) 0.05 mm
(c) 0.1 mm (d) 0.2 mm

2. A quantity X is given by $\varepsilon_0 L \dfrac{\Delta V}{\Delta t}$, where ε_0 is the permittivity of vacuum, L is the length, ΔV is the potential difference and Δt is the time interval. The dimensional formula for X is the same as that for

(a) resistance (b) charge
(c) voltage (d) current

3. The density of a solid ball is to be determined in an experiment. The diameter of the ball is measured with a screw gauge whose pitch is 0.5 mm, with 50 divisions on the circular scale. The reading on the main scale is 2.5 mm and that on the circular scale is 20 divisions. If the measured mass of this ball has a relative error of 2%, the relative percentage error in the density is

(a) 0.9 (b) 2.4
(c) 3.1 (d) 4.2

4. In the determination of Young modulus $\left(Y = \dfrac{4MgL}{\pi d^2 l}\right)$ by using Searle's method, a wire of length $L = 2$ m and diameter $d = 0.5$ mm is used. For a load $M = 2.5$ kg, an extension $l = 0.25$ mm in the length of the wire is observed. Quantities d and l are measured using a screw gauge and a micrometer respectively. They have equal pitches of 0.5 mm. The number of divisions on each of their circular scales is 100. The maximum probable error in the measurement of Y arises due to the error in l and d both.

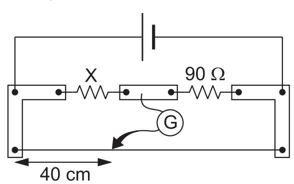
(a) The error in the measurement of d and l are equal.

(b) The error in the measurement of d is twice the error in the measurement of l.

(c) The error in the measurement of l is twice the error in the measurement of d.

(d) The error in the measurement of d is four times the error in the measurement of l.

5. During an experiment with a metre bridge, the galvanometer shows a null point when the jockey is pressed at 40.0 cm using a standard resistance (R) of 90 Ω, as shown in the figure. The least count of the scale used in the metre bridge is 1 mm. The unknown resistance (X) is



(a) (60 ± 0.15) Ω (b) (135 ± 0.56) Ω

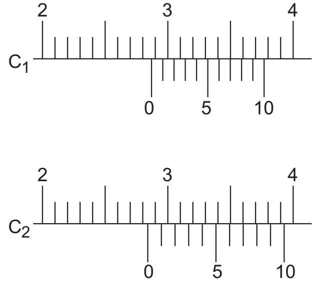
(c) (60 ± 0.25) Ω (d) (135 ± 0.23) Ω

6. The diameter of a cylinder is measured using a vernier callipers with no zero errors. It is found that the zero of the vernier scale lies between 5.10 cm and 5.15 cm of the main scale. The vernier scale has 50 divisions, equivalent to 2.45 cm. The 24th division of the vernier scale exactly coincides with one of the main-scale divisions. The diameter of the cylinder is

(a) 5.112 cm (b) 5.124 cm

(c) 5.136 cm (d) 5.148 cm

7. There are two vernier callipers, both of which have 1 cm divided into 10 equal divisions on the main scale. The vernier scale of one of the callipers (C_1) has 10 equal divisions, corresponding to 9 main-scale divisions. The vernier scale of the other calliper (C_2) has 10 equal divisions that correspond to 11 main-scale divisions. The readings of the two callipers are shown in the figure. The measured values by the callipers C_1 and C_2 are respectively



(a) 2.87 cm and 2.86 cm (b) 2.87 cm and 2.87 cm

(c) 2.87 cm and 2.83 cm (d) 2.85 cm and 2.82 cm

8. Consider an expanding sphere of instantaneous radius R whose mass (M) remains constant. The expansion is such that the instantaneous density (ρ) remains uniform throughout the volume. The rate of fractional change in density $\left(\frac{1}{\rho}\frac{d\rho}{dt}\right)$ is also constant. The velocity (v) of any point on the surface of the expanding sphere is proportional to

(a) $R^{2/3}$ (b) R

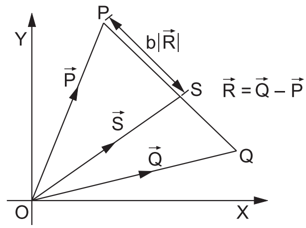
(c) R^3 (d) $\frac{1}{R}$

9. A person measures the depth of a well by measuring the time interval between dropping a stone and receiving the sound of impact of the stone with the bottom of the well. The error in his measurement of time is $\delta T = 0.01$ second, and he measures the depth of the well (L) to be 20 metres. Take the acceleration due to gravity (g) as 10 m s^{-2} and the velocity of sound as 300 m s^{-1}. Then the fractional error $\left(\frac{\delta L}{L}\right)$ in the measurement is close to

(a) 0.2% (b) 5%

(c) 1% (d) 3%

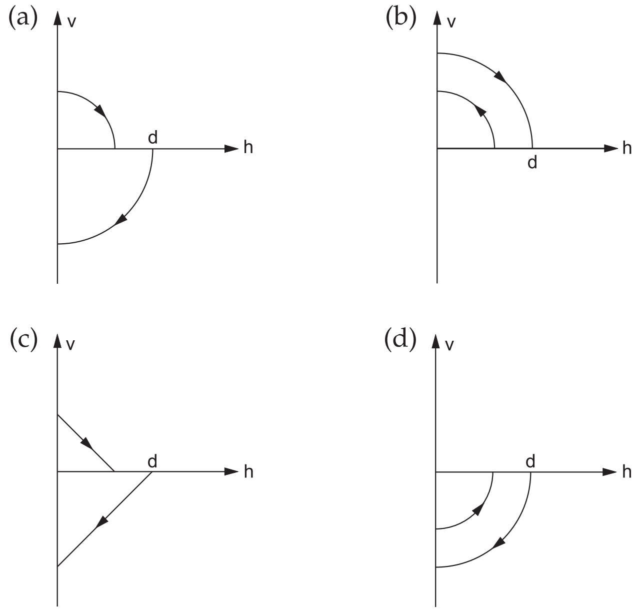
10. Three vectors $\vec{P}, \vec{Q}$ and $\vec{R}$ are shown in the figure. Let S be a point on the vector $\vec{R}$. The distance between the points P and S is $b|\vec{R}|$. The general relation among $\vec{P}, \vec{Q}$ and $\vec{S}$ is



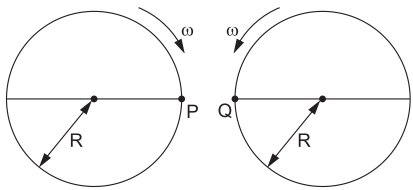
(a) $\vec{S} = (1-b)\vec{P} + b^2\vec{Q}$ (b) $\vec{S} = (1-b^2)\vec{P} + b\vec{Q}$

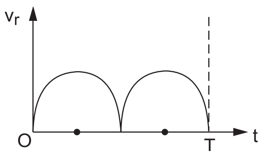
(c) $\vec{S} = (1-b)\vec{P} + b\vec{Q}$ (d) $\vec{S} = (b-1)\vec{P} + b\vec{Q}$

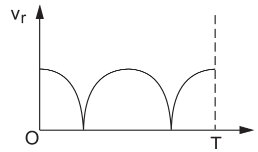
11. A ball is dropped vertically from a height d above the ground. It hits the ground and bounces up vertically to a height $d/2$. Neglecting the subsequent motion and the air resistance, how its velocity (v) varies with the height (h) above the ground?

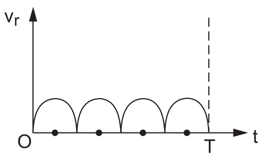


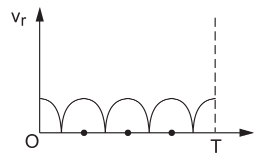
12. Two identical discs of the same radius R are rotating about their axes in opposite directions at the same angular speed ω. The discs are in the same horizontal plane. At the time $t = 0$, the points P and Q are facing each other, as shown in the figure. The relative speed between the two points P and Q is v_r. In each period of rotation (T) of the discs, v_r as a function of time is best represented by



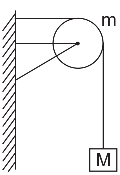
(a) 

(b) 

(c) 

(d) 

13. A string of negligible mass going over a clamped pulley of mass m supports a block of mass M, as shown in the figure. The force on the pulley by the clamp is given by



(a) $\sqrt{2}Mg$

(b) $\sqrt{2}mg$

(c) $\sqrt{(M+m)^2+m^2} \cdot g$

(d) $\sqrt{(M+m)^2+M^2} \cdot g$

14. Two particles of masses m_1 and m_2 in projectile motion have the velocities $\vec{v_1}$ and $\vec{v_2}$ respectively at the time $t = 0$. They collide at $t = t_0$. Their velocities become $\vec{v_1'}$ and $\vec{v_2'}$ at $t = 2t_0$ while still moving in air. The value of $\left|\left(m_1\vec{v_1'} + m_2\vec{v_2'}\right) - \left(m_1\vec{v_1} + m_2\vec{v_2}\right)\right|$ is

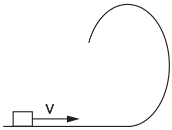
(a) zero

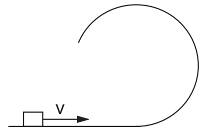
(b) $(m_1 + m_2)gt_0$

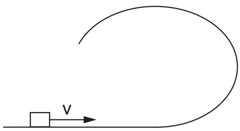
(c) $2(m_1 + m_2)gt_0$

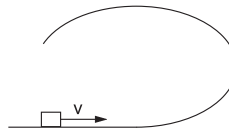
(d) $\frac{1}{2}(m_1 + m_2)gt_0$

15. A small block is shot into each of the four tracks shown below. Each of the tracks rises to the same height. The speed with which the block enters the track is the same in all cases. At the highest point of the track, the normal reaction is the maximum in the case

(a) 

(b) 

(c) 

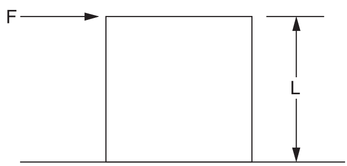
(d) 

16. A long horizontal rod has a bead which can slide along its length and is initially placed at a distance L from one end A of the rod. The rod is set in angular motion about A with a constant angular acceleration α. If the coefficient of friction between the rod and the bead is μ and the gravity is neglected, the time after which the bead starts slipping is

(a) $\sqrt{\dfrac{\mu}{\alpha}}$ (b) $\dfrac{\mu}{\sqrt{\alpha}}$

(c) $\dfrac{1}{\sqrt{\mu\alpha}}$ (d) infinitesimal

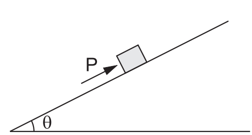
17. A cubical block of side length L rests on a rough horizontal surface having the coefficient of friction μ. A horizontal force F is applied on the block, as shown. If the coefficient of friction is sufficiently high so that the block does not slide before toppling, the minimum force required to topple the block is



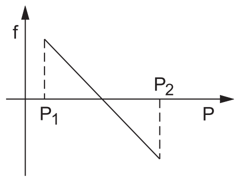
(a) infinitesimal (b) $\dfrac{mg}{4}$

(c) $\dfrac{mg}{2}$ (d) $mg(1 - \mu)$

18. A block of mass m is on an inclined plane of angle θ. The coefficient of friction between the block and the plane is μ, where $\tan\theta > \mu$. The block is held stationary by applying a force P parallel to the plane. The direction of force pointing up the plane is taken



to be positive. As P is varied from $P_1 = mg(\sin\theta - \mu\cos\theta)$ to $P_2 = mg(\sin\theta + \mu\cos\theta)$, the frictional force f versus P graph will look like

(a)


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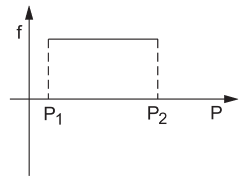

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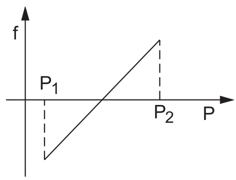

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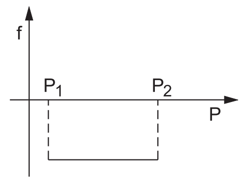
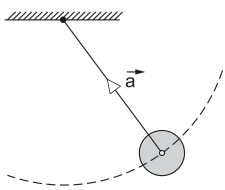


19. A uniform wooden stick of mass 1.6 kg and length l rests in an inclined manner on a smooth vertical wall of height h ($< l$) such that a small portion of the stick extends beyond the wall. The reaction force of the wall on the stick is perpendicular to the stick. The stick makes an angle of 30° with the wall, and the bottom of the stick is on a rough floor. The reaction of the wall on the stick is equal in magnitude to the reaction of the floor on the stick. The ratio h/l and the frictional force f at the bottom of the stick are respectively (when $g = 10\ \text{m s}^{-2}$)

(a) $\dfrac{\sqrt{3}}{16}$ and $\dfrac{16\sqrt{3}}{3}$ N (b) $\dfrac{3}{16}$ and $\dfrac{16\sqrt{3}}{3}$ N

(c) $\dfrac{3\sqrt{3}}{16}$ and $\dfrac{8\sqrt{3}}{3}$ N (d) $\dfrac{3\sqrt{3}}{16}$ and $\dfrac{16\sqrt{3}}{3}$ N

20. A simple pendulum is oscillating without damping. When the displacement of the bob is less than the maximum, its acceleration vector $\vec{a}$ is correctly shown in the figure

(a)


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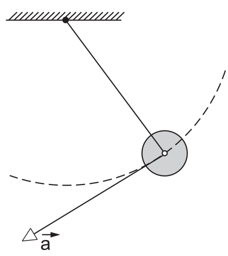
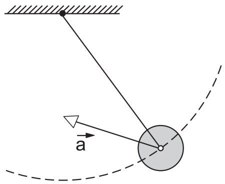


(c)



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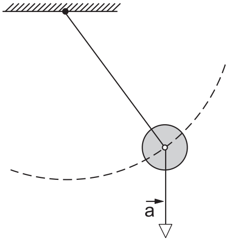
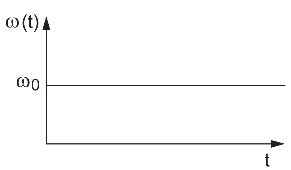


21. A cylinder rolls up an inclined plane, reaches some height, and then rolls down (without slipping throughout these motions). The directions of the frictional force acting on the cylinder are

(a) up the incline while ascending and down the incline while descending
(b) up the incline while ascending as well as descending
(c) down the incline while ascending and up the incline while descending
(d) down the incline while ascending as well as descending

22. A circular platform is free to rotate in a horizontal plane about a vertical axis passing through its centre. A tortoise is sitting at the edge of the platform. Now, the platform is given an angular velocity ω_0. When the tortoise moves along a chord of the platform with a constant velocity (with respect to the platform), the angular velocity of the platform $\omega(t)$ will vary with time t as

(a)



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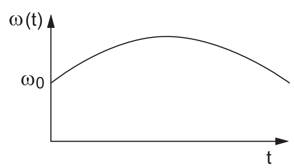

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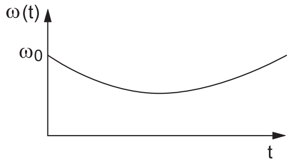
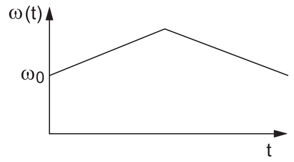


(d)



23. A ball of mass $m = 0.5$ kg is attached to the end of a string of length $L = 0.5$ m. The ball is rotated on a horizontal circular path about the vertical axis. The maximum tension that the string can bear is

324 N. The maximum possible value of the angular velocity of the ball is

(a) 9 rad s^{-1} (b) 18 rad s^{-1}

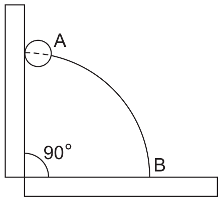
(c) 27 rad s^{-1} (d) 36 rad s^{-1}

24. The work done on a particle of mass m by a force $K\left\{\frac{x}{(x^2+y^2)^{3/2}}\hat{i}+\frac{y}{(x^2+y^2)^{3/2}}\hat{j}\right\}$ when the particle is taken from the point $(a, 0)$ to another point $(0, a)$ along a circular path of radius a about the origin in the xy-plane is

(a) $\frac{2\pi K}{a}$ (b) $\frac{\pi K}{a}$

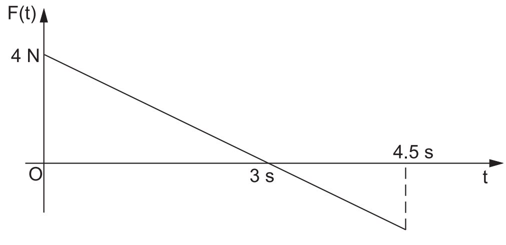
(c) $\frac{\pi K}{2a}$ (d) 0

25. A wire which passes through the hole in a small bead is bent in the form of a quarter of a circle. The wire is fixed vertically on the ground, as shown in the figure. The bead is released from near the top of this wire and it slides along the wire without friction. As the bead moves from A to B, the force it applies on the wire is



(a) always radially outwards
(b) always radially inwards
(c) initially radially outwards and later radially inwards
(d) initially radially inwards and later radially outwards

26. A block of mass 2 kg is free to move along the x-axis. It is at rest and from $t = 0$ onwards it is subjected to a time-dependent force $F(t)$ in the x-direction. The force $F(t)$ varies with t as shown in the figure above. The kinetic energy of the block after 4.5 seconds is

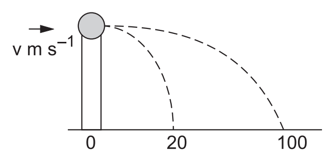


(a) 4.50 J (b) 7.50 J
(c) 5.06 J (d) 14.06 J

27. A moving point mass of 1 kg collides elastically with a stationary point mass of 5 kg. After their collision, the 1-kg mass reverses its direction and moves with a speed of 2 m s^{-1}. Which of the following statement is correct for the system of these two masses?

(a) The total momentum of the system is 4 kg m s^{-1}.
(b) The momentum of the 5-kg mass after collision is 4 kg m s^{-1}.
(c) The kinetic energy of the centre of mass is 0.75 J.
(d) The total kinetic energy of the system is 4 J.

28. A ball of mass 0.2 kg rests on a vertical post of height 5 m. A bullet of mass 0.01 kg, travelling with a velocity v m s^{-1} in a horizontal direction, hits the centre of the ball. After the collision, the ball and the bullet travel independently. The ball hits the ground at a distance of 20 m and the bullet, at a distance of 100 m from the foot of the post. The velocity v of this bullet is



(a) 250 m s^{-1} (b) $250\sqrt{2}$ m s^{-1}
(c) 400 m s^{-1} (d) 500 m s^{-1}

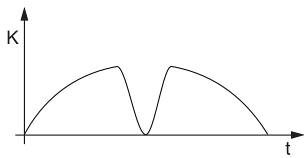
29. A particle of mass m is projected from the ground with an initial speed u_0 at an angle α with the horizontal. At the highest point of its trajectory, it makes a perfectly inelastic collision with another identical particle, which was thrown vertically upward from the ground with the same initial speed (u_0). The angle that the composite system makes with the horizontal immediately after the collision is

(a) $\dfrac{\pi}{4}$ (b) $\dfrac{\pi}{4}+\alpha$

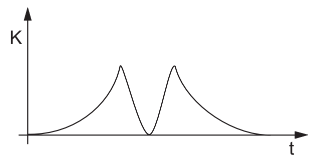
(c) $\dfrac{\pi}{4}-\alpha$ (d) $\dfrac{\pi}{2}$

30. A tennis ball is dropped on a smooth horizontal surface. It bounces back to its original position after hitting the surface. The force on the ball during the collision is proportional to the length of compression of the ball. Which one of the following sketches describes the variation of its kinetic energy K with the time t most appreciably? (The figures are only illustrative and not drawn to the scale.)

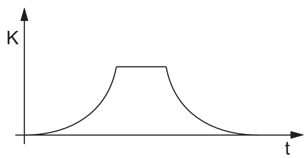
(a)



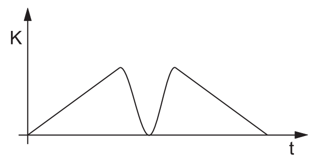
(b)



(c)



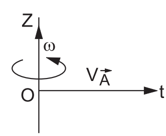
(d)



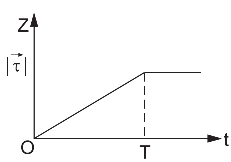
31. Two blocks of masses 10 kg and 4 kg are connected by a spring of negligible mass and placed on a frictionless horizontal surface. An impulse gives a velocity of 14 m s^{-1} to the heavier block in the direction of the lighter block. The velocity of the centre of mass is

(a) 30 m s^{-1} (b) 20 m s^{-1}
(c) 10 m s^{-1} (d) 5 m s^{-1}

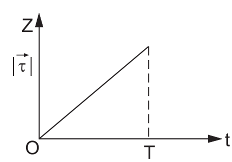
32. A thin uniform rod, pivoted at O, is rotating in the horizontal plane with a constant angular speed ω, as shown in the figure. At $t = 0$, a small insect starts from O and moves with a constant speed v relative to the rod towards the other end. It reaches the end of the rod at $t = T$ and stops. The magnitude of the torque ($|\vec{\tau}|$) on the system about O as a function of time (t) is best represented by



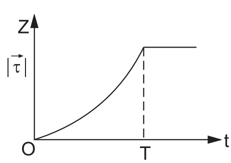
(a)



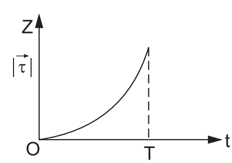
(b)



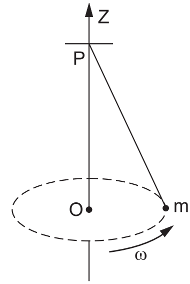
(c)



(d)



33. A small mass m is attached to a massless string whose other end is fixed at P, as shown in the figure. The mass is undergoing a circular motion in the xy-plane with the centre at O and a constant angular speed ω. If the angular momentum of the system calculated about O and P are respectively denoted by $\overrightarrow{L_O}$ and $\overrightarrow{L_P}$ then



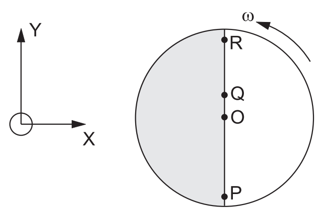
(a) $\overrightarrow{L_O}$ and $\overrightarrow{L_P}$ do not vary with time

(b) $\overrightarrow{L_O}$ varies with time, while $\overrightarrow{L_P}$ remains constant

(c) $\overrightarrow{L_O}$ remains constant, while $\overrightarrow{L_P}$ varies with time

(d) $\overrightarrow{L_O}$ and $\overrightarrow{L_P}$ both vary with time

34. Consider a disc rotating in the horizontal plane with a constant angular velocity ω about its centre O. The disc has a shaded region on one side of the diameter and an unshaded region on the other side, as shown in the figure. When the disc is in the orientation as shown, two pebbles P and Q are simultaneously projected at an angle towards R. The velocity of projection is in the yz-plane and is the same for both the pebbles with respect to this disc. Assume that (i) they land back on the disc before the disc has completed 1/8 rotation, (ii) their range is less than half the disc radius and (iii) ω remains constant throughout. Then,



(a) P lands in the shaded region and Q, in the unshaded region

(b) P lands in the unshaded region and Q, in the shaded region

(c) both P and Q land in the unshaded region

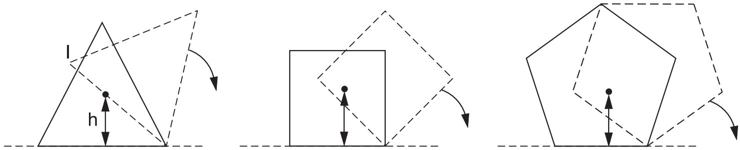
(d) both P and Q land in the shaded region

35. One end of a thick horizontal copper wire of length $2L$ and radius $2R$ is welded to an end of another thin horizontal copper wire of length L and radius R. When the arrangement is stretched by applying forces at two ends, the ratio of the elongation in the thin wire to that in the thick wire is

(a) 0.25 (b) 0.50

(c) 2.00 (d) 4.00

36. Consider three regular polygons with the number of sides n = 3, 4 and 5 as shown in the figures.



The centre of mass of all the polygons is at the height h from the ground. They roll on a horizontal surface about the leading vertex without slipping and sliding as depicted. The maximum increase in the height of the locus of the centre of mass for each polygon is Δ. Then Δ depends on n and h as

(a) $\Delta = h\sin\left(\frac{2\pi}{n}\right)$ (b) $\Delta = h\sin^2\left(\frac{\pi}{n}\right)$

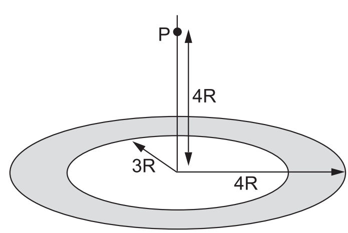
(c) $\Delta = h\tan^2\left(\frac{\pi}{2n}\right)$ (d) $\Delta = h\left[\frac{1}{\cos\left(\frac{\pi}{n}\right)} - 1\right]$

37. A geostationary satellite orbits around the earth in a circular orbit of radius 36000 km. Then, the time period of a spy satellite orbitting a few hundred kilometres above the earth's surface (R_e = 6400 km) will approximately be

(a) $\frac{1}{2}$ h (b) 1 h

(c) 2 h (d) 4 h

38. A thin uniform annular disc (see figure) of mass M has the outer radius $4R$ and the inner radius $3R$. The work required to take a unit mass from the point P on its axis to infinity is



(a) $\frac{2GM}{7R}(4\sqrt{2}-5)$ (b) $-\frac{2GM}{7R}(4\sqrt{2}-5)$

(c) $\frac{GM}{4R}$ (d) $\frac{2GM}{5R}(\sqrt{2}-1)$

39. A satellite is moving with a constant speed v in a circular orbit around the earth. An object of mass m is ejected from the satellite such that it just escapes from the gravitational pull of the earth. At the time of its ejection, the kinetic energy of this object is

(a) $\frac{1}{2}mv^2$ (b) mv^2

(c) $\frac{3}{2}mv^2$ (d) $2mv^2$

40. A planet of radius $R = \frac{1}{10}$ (radius of the earth) has the same mass density as the earth. Scientists dig a well of depth $\frac{R}{5}$ on it and lower a wire of the same length and of linear mass density 10^{-3} kg m^{-1} into it. If the wire is not touching anywhere, the force applied at the top of the wire by a person holding it in place is (taking the radius of the earth as 6×10^6 m and the acceleration due to gravity of the earth as 10 m s^{-2})

(a) 96 N (b) 108 N

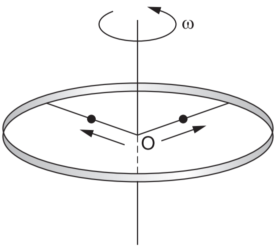
(c) 120 N (d) 150 N

41. A rocket is launched normal to the surface of the earth, away from the sun, along the line joining the sun and the earth. The sun is 3×10^5 times heavier than the earth and is at a distance 2.5×10^4 times larger than the radius of the earth. The escape velocity from the earth's gravitational field is $v_e = 11.2$ km s^{-1}. The minimum initial velocity required for the rocket to be able to leave the sun–earth system is close to

(a) 60 km s^{-1} (b) 40 km s^{-1}

(c) 70 km s^{-1} (d) 20 km s^{-1}

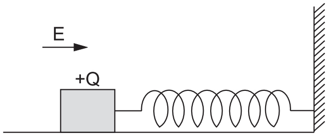
42. A ring of mass M and radius R is rotating with the angular speed ω about a fixed vertical axis passing through its centre O with two point masses each of mass $M/8$ at rest at O. These masses can move radially outwards along the two massless rods fixed on the ring as shown in the figure. At some instant the angular speed of the system is $\frac{8}{9}\omega$ and one of the masses is at a distance of $3R/5$ from O. At this instant, the distance of the other point mass from O is



(a) $\frac{2}{3}R$ (b) $\frac{3}{5}R$

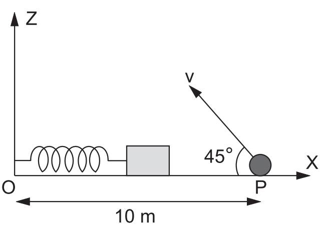
(c) $\frac{1}{3}R$ (d) $\frac{4}{5}R$

43. A wooden block performs SHM on a frictionless surface with a frequency of ν_0. The block carries a charge $+Q$ on its surface. If a uniform electric field $\vec{E}$ is switched on as shown then the SHM of the block will be of



(a) the same frequency and with a shifted mean position
(b) the same frequency and with the same mean position
(c) a changed frequency and with a shifted mean position
(d) a changed frequency and with the same mean position

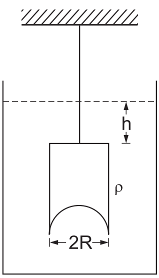
44. A small block is connected to one end of a massless spring of unstretched length 4.9 m. The other end of the spring (see the figure) is fixed. The system lies on a horizontal frictionless surface. The block is stretched by 0.2 m and then released. It then executes SHM with an angular frequency of $\omega = (\pi/3)$ rad s^{-1}. Simultaneously at $t = 0$, a small pebble is projected with a speed v from the point P at an angle of 45° as shown in the figure. The point P is at a horizontal distance of 10 cm from O. If this pebble hits the block at $t = 1$ s, the value of v is



(a) $\sqrt{50}$ m s^{-1} (b) $\sqrt{51}$ m s^{-1}

(c) $\sqrt{52}$ m s^{-1} (d) $\sqrt{53}$ m s^{-1}

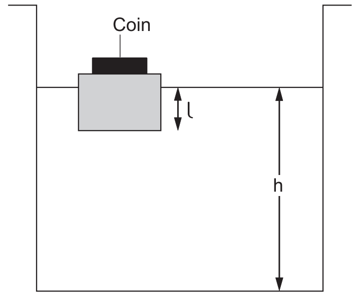
45. A hemispherical portion of radius R is removed from the bottom of a cylinder of radius R. The volume of the remaining cylinder is V and its mass is M. It is suspended by a string in a liquid of density ρ. It stays vertical inside the liquid. The upper surface of the cylinder is at a depth h below the liquid surface. The force on the bottom of the cylinder by the liquid is



(a) Mg (b) $Mg - V\rho g$

(c) $Mg + \pi R^2 h\rho g$ (d) $\rho g(V + \pi R^2 h)$

46. A wooden block, with a coin placed on its top, floats in water, as shown in the figure. The distance l and h are shown. After some time the coin falls into the water. Then,

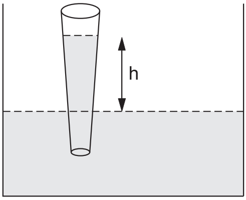


(a) l decreases and h increases
(b) l increases and h decreases
(c) both l and h increase
(d) both l and h decrease

47. A thin uniform cylindrical shell, closed at both ends, is partially filled with water. It is floating vertically in water in the half-submerged state. If ρ_c is the relative density of the material of the shell with respect to water then the correct statement is that the shell is

(a) more than half-filled if ρ_c is less than 0.5
(b) more than half-filled if ρ_c is more than 1
(c) half-filled if ρ_c is more than 0.5
(d) less than half-filled if ρ_c is less than 0.5

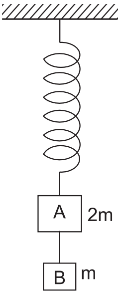
48. A glass capillary tube is of the shape of a truncated cone with an apex angle α so that its two ends have cross sections of different radii. When dipped in water vertically, water rises in it to a height h, where the radius of its cross section is b. If the surface tension of water is s, its density is ρ and its contact angle with glass is θ, the value of h will be



(a) $\dfrac{2s}{b\rho g}\cos(\theta-\alpha)$ (b) $\dfrac{2s}{b\rho g}\cos(\theta+\alpha)$

(c) $\dfrac{2s}{b\rho g}\cos\left(\theta-\dfrac{\alpha}{2}\right)$ (d) $\dfrac{2s}{b\rho g}\cos\left(\theta+\dfrac{\alpha}{2}\right)$

49. Two blocks A and B of masses $2m$ and m respectively are connected by a massless and inextensible string. The whole system is suspended by a massless spring, as shown in the figure. The magnitude of acceleration of A and B immediately after the string is cut are respectively



(a) g and $\frac{g}{2}$ (b) $\frac{g}{2}$ and g

(c) g and g (d) $\frac{g}{2}$ and $\frac{g}{2}$

50. A system of binary stars of masses m_A and m_B are moving in circular orbits of radii r_A and r_B respectively. If T_A and T_B are the time periods of the stars A and B respectively,

(a) $\frac{T_A}{T_B} = \left(\frac{r_A}{r_B}\right)^{3/2}$ (b) $T_A > T_B$ if $r_A > r_B$

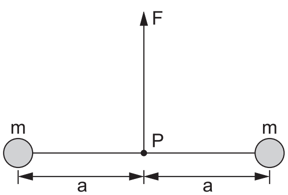
(c) $T_A > T_B$ if $m_A > m_B$ (d) $T_A = T_B$

51. A solid sphere of mass M, radius R and having moment of inertia as I is recast into a disc of thickness t, whose moment of inertia about an axis passing through its edge and perpendicular to its plane remains I. Then the radius of the disc will be

(a) $\frac{2R}{\sqrt{15}}$ (b) $R\sqrt{\frac{2}{15}}$

(c) $\frac{4R}{\sqrt{15}}$ (d) $\frac{R}{4}$

52. Two particles each of mass m are tied at the ends of a light string of length $2a$. The whole system is kept on a smooth horizontal surface with the string held tight so that each particle remains at a distance a from the centre P (as shown in the figure). Now the midpoint of the string is pulled vertically upwards with a small but constant force F. As a result, the particles move towards each other on the surface. The magnitude of the acceleration when the seperation between them becomes $2x$ is



(a) $\dfrac{F}{2m}\cdot\dfrac{a}{\sqrt{a^2-x^2}}$ (b) $\dfrac{F}{2m}\cdot\dfrac{x}{\sqrt{a^2-x^2}}$

(c) $\dfrac{F}{2m}\cdot\dfrac{x}{a}$ (d) $\dfrac{F}{2m}\cdot\dfrac{\sqrt{a^2-x^2}}{x}$

53. A small object of uniform density rolls up a curved surface with an initial velocity v. It reaches up to a maximum height of $3v^2/4g$ with respect to the initial position. The object is a

(a) ring (b) solid sphere

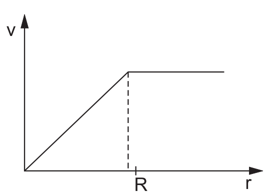
(c) hollow sphere (d) disc

54. A spherically symmetric gravitational system of particles has a mass density

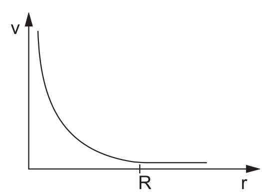
$$\rho = \begin{cases} \rho_0 \text{ for } r < R \\ 0 \text{ for } r > R \end{cases},$$

where ρ_0 is a constant. A test mass can undergo a circular motion under the influence of the gravitational field of particles. Its speed v as a function of the distance r (where $0 < r < \infty$) from the centre of the system is represented by

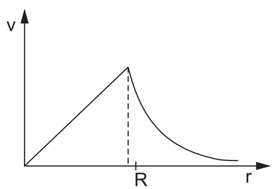
(a)



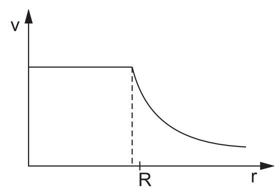
(b)



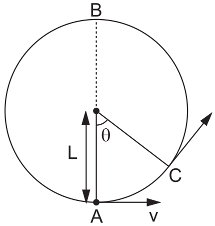
(c)



(d)



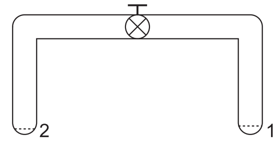
55. A bob of mass m is suspended by a massless string of length L. The horizontal velocity v at the position A is just sufficient to make it reach the point B. The angle θ at which the speed of the bob is the half of that at A satisfies



(a) $\theta = \frac{\pi}{4}$ (b) $\frac{\pi}{4} < \theta < \frac{\pi}{2}$

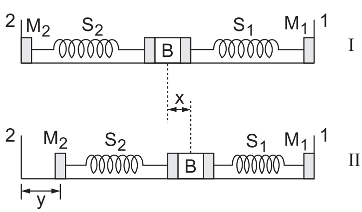
(c) $\frac{\pi}{2} < \theta < \frac{3\pi}{4}$ (d) $\frac{3\pi}{4} < \theta < \pi$

56. A glass tube of uniform internal radius (r) has a valve separating the two identical ends. Initially, the valve is in a tightly closed position. The end 1 has a hemispherical soap bubble of radius r. The end 2 has a subhemispherical soap bubble, as shown in the figure. Just after opening the valve,



(a) air from the end 1 flows towards the end 2: no change occurs in the volume of the soap bubbles
(b) air from the end 1 flows towards the end 2: the volume of the soap bubble at the end 1 decreases
(c) no change occurs
(d) air from the end 2 flows towards the end 1: the volume of the soap bubble at the end 1 increases

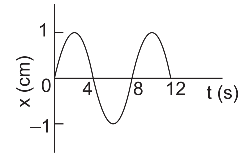
57. A block B is attached to two unstretched springs S_1 and S_2 with spring constants k and $4k$ respectively (see the figure I). The other ends are attached to the identical supports M_1 and M_2 which are not fixed to the walls. The springs and supports have negligible masses. There is no friction anywhere. The block B is displaced towards the wall 1 by a small distance x (see the figure II) and then released. The block returns and moves a maximum distance y towards the wall 2. The displacements x and y are measured with respect to the equilibrium position of the block B. The ratio $\frac{y}{x}$ is



(a) 4 (b) 2

(c) $\frac{1}{2}$ (d) $\frac{1}{4}$

58. The x–t graph of a particle undergoing a simple harmonic motion is shown here. The acceleration of the particle at $t = \frac{4}{3}$ s is



(a) $\frac{\sqrt{3}\pi^2}{32}$ cm s^{-2} (b) $\frac{-\pi^2}{32}$ cm s^{-2}

(c) $\frac{\pi^2}{32}$ cm s^{-2} (d) $-\frac{\sqrt{3}\pi^2}{32}$ cm s^{-2}

59. A block of base 10 cm × 10 cm and height 15 cm is kept on an inclined plane. The coefficient of friction between them is $\sqrt{3}$. The inclination θ of this inclined plane from the horizontal plane is gradually increased from 0°. Then,

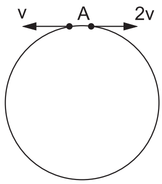
(a) the block will start sliding down the plane at $\theta = 30°$

(b) the block will remain at rest on the plane up to a certain θ and then it will topple

(c) the block will start sliding down the plane at $\theta = 60°$ and continue to do so at higher angles

(d) the block will start sliding down the plane at $\theta = 60°$ and, on further increasing θ, it will topple at a certain θ

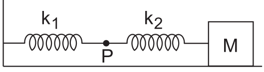
60. Two small particles of equal masses start moving in opposite directions from a point A in a horizontal circular orbit. Their tangential velocities are v and $2v$ respectively, as shown in the figure. Between collisions, the particles move with constant speeds. After making how many elastic collisions, other than that at A, will these two particles again reach the point A?



(a) Four (b) Three

(c) Two (d) One

61. A block M shown in the figure oscillates in a simple harmonic motion with amplitude A. The amplitude of the point P is



(a) $\frac{k_1 A}{k_2}$ (b) $\frac{k_2 A}{k_1}$

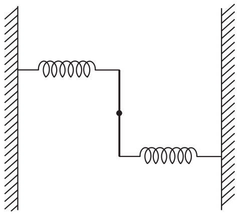
(c) $\dfrac{k_1 A}{k_1 + k_2}$ (d) $\dfrac{k_2 A}{k_1 + k_2}$

62. A piece of wire is bent in the shape of the parabola $y = kx^2$ (the y-axis being vertical), with a bead of mass m on it. The bead can slide on the wire without friction. It stays at the lowest point of the parabola when the wire is at rest. The wire is now accelerated parallel to the x-axis with a constant acceleration a. The distance of the new equilibrium position of the bead, where the bead can stay at rest with respect to the wire, from the y-axis is

(a) $\dfrac{a}{gk}$ (b) $\dfrac{a}{2gk}$

(c) $\dfrac{2a}{gk}$ (d) $\dfrac{a}{4gk}$

63. A uniform rod of length L and mass M is pivoted at the centre. Its two ends are attached to two springs of equal spring constants (k). The springs are fixed to rigid supports, as shown in the figure, and the rod is free to oscillate in the horizontal plane. The rod is gently pushed through a small angle θ in one direction and then released. The frequency of the oscillation is



(a) $\dfrac{1}{2\pi}\sqrt{\dfrac{2k}{M}}$ (b) $\dfrac{1}{2\pi}\sqrt{\dfrac{k}{M}}$

(c) $\dfrac{1}{2\pi}\sqrt{\dfrac{6k}{M}}$ (d) $\dfrac{1}{2\pi}\sqrt{\dfrac{24k}{M}}$

1.2 Heat and Thermodynamics

1. When a block of iron floats in mercury at 0 °C, a fraction k_1 of its volume is submerged; while at the temperature 60 °C, a fraction k_2 is seen to be submerged. If the coefficient of volume expansion for iron is γ_{Fe} and that for mercury is γ_{Hg}, the ratio k_1/k_2 can be expressed as

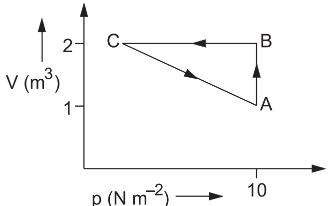
(a) $\dfrac{1 + (60\ °\mathrm{C})\gamma_{Fe}}{1 + (60\ °\mathrm{C})\gamma_{Hg}}$ (b) $\dfrac{1 - (60\ °\mathrm{C})\gamma_{Fe}}{1 + (60\ °\mathrm{C})\gamma_{Hg}}$

(c) $\dfrac{1+(60\,°C)\gamma_{Fe}}{1-(60\,°C)\gamma_{Hg}}$ (d) $\dfrac{1+(60\,°C)\gamma_{Hg}}{1+(60\,°C)\gamma_{Fe}}$

2. Starting with the same initial conditions, an ideal gas expands from the volume V_1 to V_2 in three different ways. The work done by the gas is W_1 if the process is purely isothermal, W_2 if purely isobaric, and W_3 if purely adiabatic. Then,

(a) $W_2 > W_1 > W_3$ (b) $W_2 > W_3 > W_1$
(c) $W_1 > W_2 > W_3$ (d) $W_1 > W_3 > W_2$

3. An ideal gas is taken through the cycle $A \to B \to C \to A$, as shown in the figure. If the net heat supplied to the gas in the cycle is 5 J, the work done by the gas in the process $C \to A$ is



(a) –5 J (b) –10 J
(c) –15 J (d) –20 J

4. 5.6 L of helium gas at STP is adiabatically compressed to 0.7 L. Taking the initial temperature to be T_1, the work done in the process is

(a) $\dfrac{9}{8}RT_1$ (b) $\dfrac{3}{2}RT_1$

(c) $\dfrac{15}{8}RT_1$ (d) $\dfrac{9}{2}RT_1$

5. A mixture of 2 mol of helium gas (atomic mass = 4 amu) and 1 mol of argon gas (atomic mass = 40 amu) is kept at 300 K in a container. The ratio of the rms speeds $\left(\dfrac{v_{rms(He)}}{v_{rms(Ar)}}\right)$ is

(a) 0.32 (b) 0.45
(c) 2.24 (d) 3.16

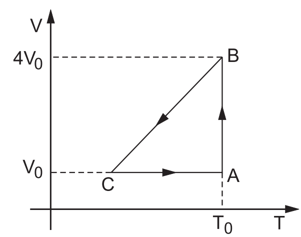
6. Two moles of ideal helium gas are in a rubber balloon at 30 °C. The balloon is fully expandable and can be assumed to require no energy for its expansion. The temperature of the gas in the balloon is slowly changed to 35 °C. The amount of heat required in raising the temperature is nearly (take $R = 8.31$ J mol^{-1} K^{-1})

(a) 62 J (b) 104 J (c) 124 J (d) 208 J

7. A real gas behaves like an ideal gas if its

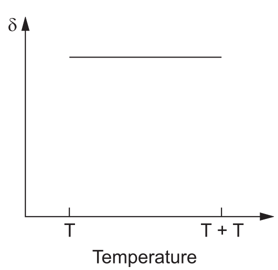
(a) pressure and temperature are both high
(b) pressure and temperature are both low
(c) pressure is high and temperature is low
(d) pressure is low and temperature is high

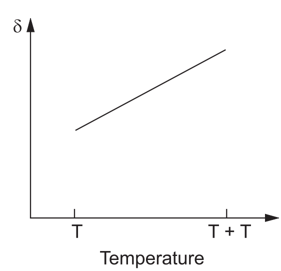
8. One mole of an ideal gas in the initial state A undergoes a cyclic process ABCA, as shown in the figure. Its pressure at A is p_0. Choose the correct statement from the following.

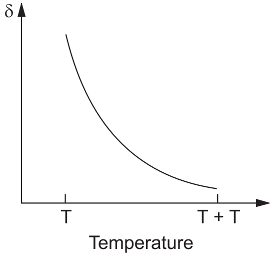


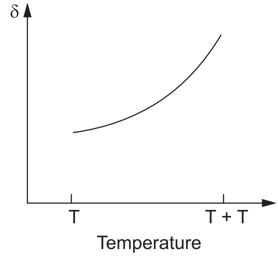
(a) The internal energies at A and B are different.
(b) The work done by the gas in process AB is $p_0V_0 \ln 4$.
(c) The pressure at C is $\dfrac{p_0}{4}$.
(d) The temperature at C is $\dfrac{T_0}{4}$.

9. An ideal gas is initially at a thermodynamic temperature T and has a volume V. Its volume is increased by ΔV due to an increase in temperature ΔT, the pressure remaining constant. Which of the following graphs shows how the quantity $\delta = \Delta V/(V\Delta T)$ varies with temperature?

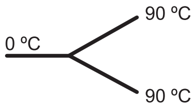
(a) 

(b) 

(c) 

(d) 

10. Three rods made of the same material and having the same cross section have been joined as shown in the figure. Each rod is of the same length. The left and right ends are kept at 0 °C and 90 °C respectively. The temperature of the junction of the three rods will be



(a) 45 °C
(b) 60 °C
(c) 30 °C
(d) 20 °C

11. An ideal black body at the room temperature is thrown into a furnace. It is observed that

(a) initially it is the darkest body and at later times the brightest
(b) it is the darkest body at all times
(c) it cannot be distinguished at all times
(d) initially it is the darkest body and at later times it cannot be distinguished

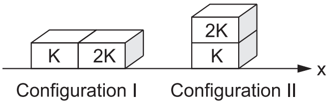
12. Three very large plates of same area are kept parallel and close to each other. They are considered ideal black surfaces and have very high thermal conductivity. The first and the third plates are maintained at the temperatures $2T$ and $3T$ respectively. The temperature of the middle (i.e., the second) plate under the steady-state condition is

(a) $\left(\frac{65}{2}\right)^{\frac{1}{4}} T$
(b) $\left(\frac{97}{4}\right)^{\frac{1}{4}} T$
(c) $\left(\frac{97}{2}\right)^{\frac{1}{4}} T$
(d) $(97)^{\frac{1}{4}} T$

13. Two nonreactive monatomic ideal gases have their atomic masses in the ratio 2 : 3. The ratio of their partial pressures when enclosed in a vessel kept at a constant temperature is 4 : 3. The ratio of their densities is

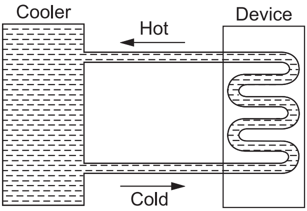
(a) 1 : 4 (b) 1 : 2
(c) 6 : 9 (d) 8 : 9

14. Two rectangular blocks having identical dimensions can be arranged either in Configuration I or in Configuration II, as shown in the figure. One of the blocks has the thermal conductivity K and the other, $2K$. The temperature difference between the ends along the x-axis is the same in both the configurations. It takes 9 s to transport a certain amount of heat from the hot end to the cold end in Configuration I. The time required in transporting the same amount of heat in Configuration II is



(a) 2.0 s (b) 3.0 s
(c) 4.5 s (d) 6.0 s

15. A water cooler of storage capacity 120 litres can cool water at a constant rate of P. In a closed circulation system (as shown schematically in this figure), the water from the cooler is used to cool an external device that generates constantly 3 kW of power (thermal load). The temperature of the water fed into the device cannot exceed 30 °C and the entire 120 litres of water is initialy cooled to 10 °C. The entire system is thermally insulated. The minimum value of P for which the device can be operated for 3 hours (taking the specific heat capacity of water = 4.2 kJ kg^{-1} K^{-1} and the density of water = 1000 kg m^{-3}) is



(a) 1600 W (b) 2067 W
(c) 2533 W (d) 3933 W

16. A gas is enclosed in a cylinder with a movable frictionless piston. Its initial thermodynamic state at the pressure $p_i = 10^5$ Pa and the volume $V_i = 10^{-3}$ m^3 changes to a final state at $p_f = \left(\frac{1}{32}\right) \times 10^5$ Pa and $V_f = 8 \times 10^{-3}$ m^3 in an adiabatic quasi-static process, such that p^3V^5 = constant. Consider another thermodynamic process that brings the system from the same initial state to the same final state in two steps: an isobaric expansion at p_i followed by an isochoric process at V_f. The amount of the heat supplied to the system in the two-step process is approximately

(a) 112 J (b) 294 J
(c) 588 J (d) 813 J

17. Parallel rays of light of the intensity $I = 912$ W m^{-2} are incident on a spherical black body kept in the surroundings of temperature 300 K. Take the Stefan–Boltzmann constant $\sigma = 5.7 \times 10^{-8}$ W m^{-2} K^{-4}, and assume that the energy exchange with the surroundings is only through radiations. The final steady-state temperature of the black body is close to

(a) 330 K (b) 660 K
(c) 990 K (d) 1550 K

18. The ends Q and R of two thin wires PQ and RS are soldered (joined) together. Initially each of the wires has a length of 1 m at 10 °C. Now the end P is maintained at 10 °C, while the end S is heated and maintained at 400 °C. The system is thermally insulated from its surroundings. If the thermal conductivity of the wire PQ is twice that of the wire RS, and the coefficient of linear thermal expansion for PQ is 1.2×10^{-5} K^{-1}, the change in the length of the wire PQ is

(a) 0.78 mm (b) 0.90 mm
(c) 1.56 mm (d) 2.34 mm

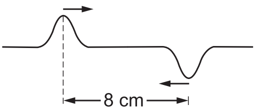
19. An ideal gas is expanding such that pT^2 = constant. The coefficient of volume expansion for the gas is

(a) $\frac{1}{T}$ (b) $\frac{2}{T}$

(c) $\frac{3}{T}$ (d) $\frac{4}{T}$

1.3 Sound Waves

1. Two pulses in a stretched string whose centres are initially 8 cm apart are moving towards each other, as shown in the figure. The speed of each pulse is 2 cm s^{-1}. After 2 s, the total energy of the pulses will be



(a) zero
(b) purely kinetic
(c) purely potential
(d) partly kinetic and partly potential

2. A point mass is subjected to two simultaneous sinusoidal displacements in the x-directions: $x_1(t) = A \sin \omega t$ and $x_2(t) = A \sin\left(\omega t + 2\pi/3\right)$. Adding a third sinusoidal displacement $x(t) = B \sin(\omega t + \phi)$ brings the mass to a complete rest. The values of B and ϕ are respectively

(a) $\sqrt{2}A$ and $\frac{3\pi}{4}$
(b) A and $\frac{4\pi}{3}$
(c) $\sqrt{3}A$ and $\frac{5\pi}{6}$
(d) A and $\frac{\pi}{3}$

3. The ends of a stretched wire of length L are fixed at $x = 0$ and $x = L$. In one experiment, the displacement of the wire is $y_1 = A \sin(\pi x/L) \sin \omega t$ and the energy is E_1, and in another experiment the displacement is $y_2 = A \sin(2\pi x/L) \sin 2\omega t$ and the energy is E_2. Then

(a) $E_2 = E_1$
(b) $E_2 = 2E_1$
(c) $E_2 = 4E_1$
(d) $E_2 = 16E_1$

4. Two vibrating strings made of the same material but of lengths L and $2L$ have the radii $2r$ and r respectively. They are stretched under the same tension. Both the strings vibrate in their fundamental modes—the one of length L with frequency ν_1 and the other with frequency ν_2. The ratio ν_1/ν_2 is equal to

(a) 2
(b) 4
(c) 8
(d) 1

5. A sonometer wire resonates with a given tuning fork, forming standing waves with five antinodes between the two bridges when a mass of 9 kg is suspended from the wire. When this mass is replaced by a mass M, the wire resonates with the same tuning fork, forming three antinodes for the same positions of the bridges. The value of M is

(a) 25 kg (b) 5 kg

(c) 12.5 kg (d) $\frac{1}{25}$ kg

6. A student is performing an experiment with a resonance column. The diameter of the column tube is 4 cm. The frequency of the tuning fork is 512 Hz. The air temperature is 38 °C in which the speed of sound is 336 m s^{-1}. The zero of the metre scale coincides with the top end of the resonance column tube. When the first resonance occurs, the reading of the water level in the column is

(a) 14.0 cm (b) 15.2 cm

(c) 16.4 cm (d) 17.6 cm

7. A hollow pipe of length 0.8 m is closed at one end. At its open end, a 0.5-m-long uniform string is vibrating in its second harmonic and it resonates with the fundamental frequency of the pipe. If the tension in the wire is 50 N and the speed of sound is 320 m s^{-1}, the mass of the string is

(a) 5 grams (b) 10 grams

(c) 20 grams (d) 40 grams

8. A train moves towards a stationary observer with a speed of 34 ms^{-1}. The train sounds a whistle and its frequency registered by the observer is f_1. If the train's speed is reduced to 17 ms^{-1}, the frequency registered is f_2. If the speed of sound is 340 ms^{-1} then the ratio f_1/f_2 is

(a) $\frac{18}{19}$ (b) $\frac{1}{2}$

(c) 2 (d) $\frac{19}{18}$

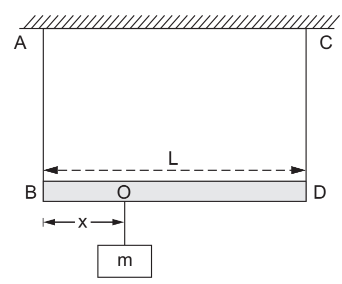
9. A siren placed at a railway platform is emitting sound waves of 5 kHz frequency. A passenger sitting in a moving train A records a frequency of 5.5 kHz while the train approaches the siren. During his return journey by a different train B, he records a frequency of 6.0 kHz while approaching the same siren. The ratio of the velocity of the train B to that of the train A is

(a) 242 : 252 (b) 2 : 1
(c) 5 : 1 (d) 11 : 6

10. A police car with a siren of frequency of 8 kHz is moving with a uniform velocity of 36 km h^{-1} towards a tall building which reflects the sound waves. The speed of sound in air is 320 m s^{-1}. The frequency of the siren heard by the car driver is

(a) 8.50 kHz (b) 8.25 kHz
(c) 7.75 kHz (d) 7.50 kHz

11. A massless rod is suspended by two identical strings, AB and CD, of equal lengths. A block of mass m is suspended from the point O such that BO = x, as shown in the figure. Further, it is observed that the frequency of the first harmonic (fundamental frequency) in AB is equal to the 2nd harmonic in CD. Then the length of BO is



(a) $\frac{L}{5}$ (b) $\frac{4L}{5}$

(c) $\frac{3L}{4}$ (d) $\frac{L}{4}$

12. In the experiment to determine the speed of sound using a resonance column,

(a) the prongs of the tuning fork are kept in a vertical plane
(b) the prongs of the tuning fork are kept in a horizontal plane
(c) in one of the two resonances observed, the length of the

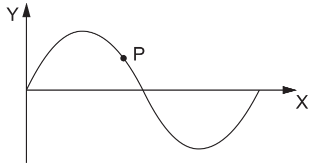
resonating air column is close to the wavelength of sound in air

(d) in one of the two resonances observed, the length of the resonating air column is close to half the wavelength of sound in air

13. A vibrating string of length l under the tension T resonates with a mode corresponding to the first overtone (third harmonic) of an air column of length 75 cm inside a tube closed at one end. The string also generates 4 beats per second when excited with a tuning fork of frequency n. When the tension of the string is slightly increased, the beat frequency reduces to 2 beats per second. Assuming the speed of sound in air to be 340 m s^{-1}, the frequency n of the tuning fork is

(a) 344 Hz (b) 336 Hz
(c) 117.3 Hz (d) 109.3 Hz

14. A transverse sinusoidal wave moves along a string in the positive x-direction at a speed of 10 cm s^{-1}. The wavelength of the wave is 0.5 m and its amplitude is 10 cm. At a particular time t, the snapshot of the wave is shown in the figure. The velocity of the point P when its displacement is 5 cm is

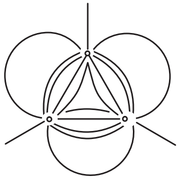


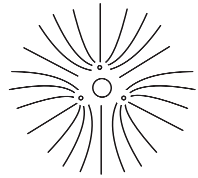
(a) $\frac{\sqrt{3}\pi}{50}\hat{j}$ m s^{-1} (b) $-\frac{\sqrt{3}\pi}{50}\hat{j}$ m s^{-1}

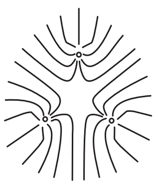
(c) $\frac{\sqrt{3}\pi}{50}\hat{i}$ m s^{-1} (d) $-\frac{\sqrt{3}\pi}{50}\hat{i}$ m s^{-1}

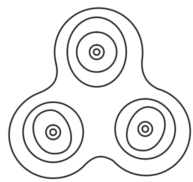
1.4 Electrostatics

1. Three positive charges each having the value q are placed at the vertices of an equilateral triangle. The resulting lines of force should be sketched as

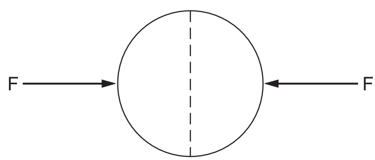
(a) 

(b) 

(c) 

(d) 

2. A uniformly charged thin spherical shell of radius R carries a uniform surface charge density of σ per unit area. It is made of two hemispherical shells, held together by pressing them with a force F (see the figure). F is proportional to



(a) $\frac{1}{\varepsilon_0}\sigma^2 R^2$ (b) $\frac{1}{\varepsilon_0}\sigma^2 R$

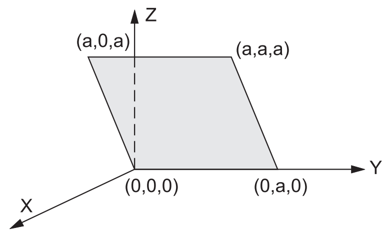
(c) $\frac{1}{\varepsilon_0}\frac{\sigma^2}{R}$ (d) $\frac{1}{\varepsilon_0}\frac{\sigma^2}{R^2}$

3. A tiny spherical oil drop carrying a net charge q is balanced in still air with a vertical uniform electric field of strength $\frac{81\pi}{7} \times 10^5$ V m^{-1}. When the field is switched off, the drop is observed to fall with a terminal velocity of 2×10^{-3} m s^{-1}. Given g = 9.8 m s^{-2}, viscosity of the air = 1.8×10^{-5} N s m^{-2} and density of oil = 900 kg m^{-3}, the magnitude of q is

(a) 1.6×10^{-19} C (b) 3.2×10^{-19} C

(c) 4.8×10^{-19} C (d) 8.0×10^{-19} C

4. Consider the electric field $\vec{E} = E_0\hat{x}$, where E_0 is a constant. The flux through the shaded area shown in the figure due to this field is



(a) $2E_0a^2$ (b) $\sqrt{2}E_0a^2$

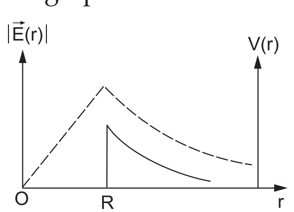
(c) E_0a^2 (d) $\dfrac{E_0a^2}{\sqrt{2}}$

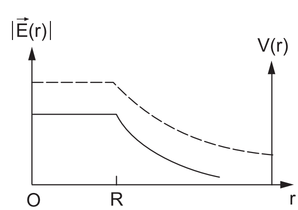
5. Two large vertical and parallel metal plates having a separation of 1 cm are connected to a d.c. voltage source of potential difference X. A proton is released from the rest, midway between the two plates. It is found to move at 45° to the vertical just after the release. Then X is nearly

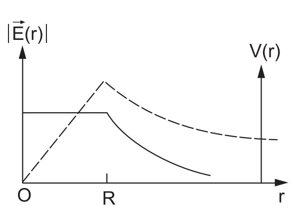
(a) 1×10^{-5} V (b) 1×10^{-7} V

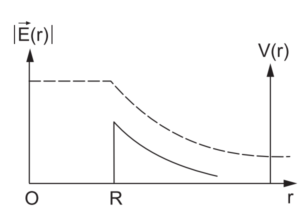
(c) 1×10^{-9} V (d) 1×10^{-10} V

6. Consider a thin spherical shell of radius R with its centre at the origin, carrying a uniform positive surface charge density. The variation of the magnitude of the electric field $|\vec{E}(r)|$ and the electric potential $V(r)$ with the distance r from the centre is best represented by which graph?

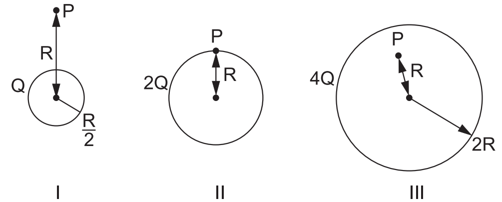
(a)


(b)


(c)


(d)


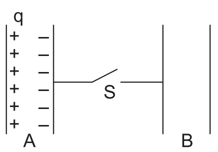
7. The charges Q, $2Q$ and $4Q$ are uniformly distributed in three dielectric solid spheres of radii $R/2$, R and $2R$ respectively as shown in the figure. If the magnitudes of electric field at the point P at a distance R from the centre of the spheres I, II and III are E_1, E_2 and E_3 respectively then



(a) $E_1 > E_2 > E_3$ (b) $E_3 > E_1 > E_2$

(c) $E_2 > E_1 > E_3$ (d) $E_3 > E_2 > E_1$

8. Consider the situation shown in the figure. The capacitor A has a charge q on it, whereas B is uncharged. The charge appearing on the capacitor B a long time after the switch is closed is



(a) zero (b) $q/2$

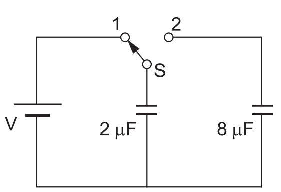
(c) q (d) $2q$

9. Two identical capacitors have the same capacitance C. One of them is charged to a potential V_1 and the other to V_2. The negative ends of the capacitors are connected. When the positive ends are also connected, the decrease in energy of the combined system is

(a) $\frac{1}{4}C(V_1^2 - V_2^2)$ (b) $\frac{1}{4}C(V_1^2 + V_2^2)$

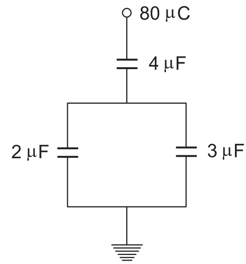
(c) $\frac{1}{4}C(V_1 - V_2)^2$ (d) $\frac{1}{4}C(V_1 + V_2)^2$

10. A 2-μF capacitor is charged as shown in the figure. The percentage of its stored energy dissipated after the switch S is turned to the position 2 is



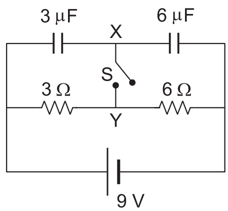
(a) 0% (b) 20%
(c) 75% (d) 80%

11. In the given circuit, a charge of 80 μC is given to the upper plate of the 4-μF capacitor. Then in the steady state, the charge on the upper plate of the 3-μF capacitor is



(a) +32 μC (b) +40 μC
(c) +48 μC (d) +80 μC

12. A circuit is connected as shown in the figure, with the switch S open. When the switch is closed, the total amount of charge that flows from Y to X is



(a) zero (b) 54 μC
(c) 27 μC (d) 81 μC

13. A long, hollow conducting cylinder is kept coaxially inside another long, hollow conducting cylinder of larger radius. Both the cylinders are initially electrically neutral. Choose the correct statement given below.

(a) A potential difference appears between the two cylinders when a charge density is given to the inner cylinder.
(b) A potential difference appears between the two cylinders when a charge density is given to the outer cylinder.
(c) No potential difference appears between the two cylinders when a uniform line charge is kept along the axis of the cylinders.
(d) No potential difference appears between the two cylinders when the same charge density is given to both the cylinders.

14. Consider a neutral conducting sphere. A positive point charge is placed outside the sphere. The net charge on the sphere is then

(a) negative and distributed uniformly over the surface of the sphere

(b) negative and appears only at the point on the sphere closest to the point charge

(c) negative and distributed nonuniformly over the entire surface of the sphere

(d) zero

15. A positive and a negative point charges of equal magnitudes are kept at $\left(0, 0, \frac{a}{2}\right)$ and $\left(0, 0, \frac{-a}{2}\right)$ respectively. The work done by the electric field when another positive point charge is moved from $(-a, 0, 0)$ to $(0, a, 0)$ is

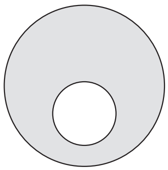
(a) positive

(b) negative

(c) zero

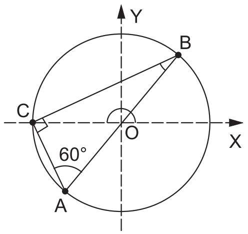
(d) dependent on the path connecting the initial and final positions

16. A spherical portion has been removed from a solid sphere having a charge distributed uniformly in its volume, as shown in the figure. The electric field inside the emptied space is



(a) zero everywhere
(b) nonzero and uniform
(c) nonuniform
(d) zero only at its centre

17. Consider a system of three charges $q/3$, $q/3$ and $-2q/3$ at the points A, B and C respectively, as shown in the figure. Take O as the centre of the circle of radius R, and $\angle CAB = 60°$. Which of the following option is correct?



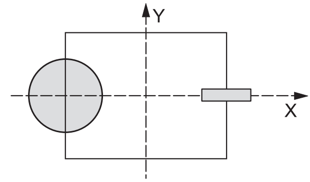
(a) The electric field at the point O is $q/8\pi\varepsilon_0 R^2$ directed along the negative x-axis.

(b) The potential energy of the system is zero.

(c) The magnitude of the force between the charges at C and B is $q^2/54\pi\varepsilon_0 R^2$.

(d) The potential at the point O is $q/12\pi\varepsilon_0 R$.

18. A disc of radius $a/4$ having a uniformly distributed charge of 6 C is placed in the xy-plane with its centre at $(-a/2, 0, 0)$. A rod of length a carrying a uniformly distributed charge of 8 C is placed on the x-axis from $x = a/4$ to $5a/4$. Two point charges –7 C and 3 C are placed at $(a/4, -a/4, 0)$ and $(-3a/4, 3a/4, 0)$ respectively. Consider a cubical surface formed by six surfaces $x = \pm a/2, y = \pm a/2, z = \pm a/2$. The electric flux through this cubical surface is



(a) $\dfrac{-2\,\text{C}}{\varepsilon_0}$ (b) $\dfrac{2\,\text{C}}{\varepsilon_0}$

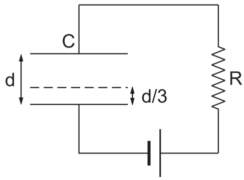
(c) $\dfrac{10\,\text{C}}{\varepsilon_0}$ (d) $\dfrac{12\,\text{C}}{\varepsilon_0}$

19. Three concentric metallic spherical shells of radii R, $2R$ and $3R$ are given the charges Q_1, Q_2 and Q_3 respectively. It is found that the surface charge densities on the outer surfaces of the shells are equal. Then the ratio of the charges given to the shells, i.e., $Q_1 : Q_2 : Q_3$, is equal to

(a) $1:2:3$ (b) $1:3:5$

(c) $1:4:9$ (d) $1:8:18$

20. A parallel-plate capacitor of capacitance C with plates of unit areas and separated by a distance of d is filled with a liquid of dielectric constant $K = 2$. The level of liquid is $d/3$ initially. Suppose that the liquid level decreases at a constant speed of V, the time constant as a function of time t is

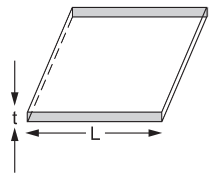


(a) $\dfrac{6\varepsilon_0 R}{5d+3Vt}$ (b) $\dfrac{(15d+9Vt)\,\varepsilon_0 R}{2d^2-3dVt-9V^2t^2}$

(c) $\dfrac{6\varepsilon_0 R}{5d-3Vt}$ (d) $\dfrac{(15d-9Vt)\,\varepsilon_0 R}{2d^2+3dVt-9V^2t^2}$

1.5 Current Electricity and Magnetism

1. Consider a thin square sheet of side L and thickness t which is made of a material of resistivity ρ. The resistance between the two opposite faces shown by the shaded areas in the adjoining figure is

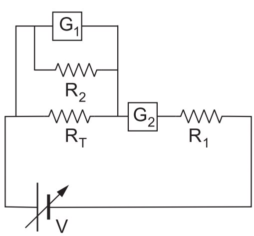


(a) directly proportional to L (b) directly proportional to t

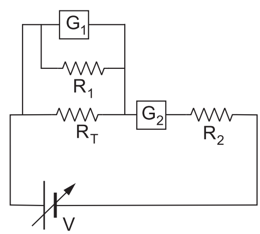
(c) independent of L (d) independent of t

2. To verify Ohm's law, a student is provided with a test resistor R_T, a high resistance R_1, a small resistance R_2, two identical galvanometers G_1 and G_2, and a variable voltage source V. The correct circuit to carry out the experiment is

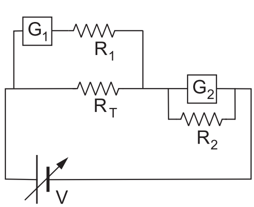
(a)



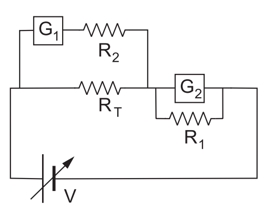
(b)



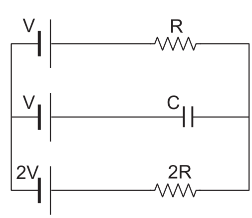
(c)



(d)



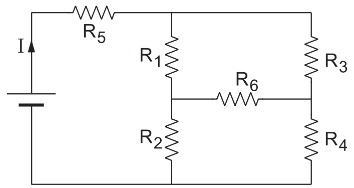
3. In the given circuit with a steady current, the potential drop across the capacitor must be



(a) V (b) $\frac{V}{2}$

(c) $\frac{V}{3}$ (d) $\frac{2V}{3}$

4. In the given circuit it is observed that the current I is independent of the value of the resistance R_6. Then the resistance values must satisfy



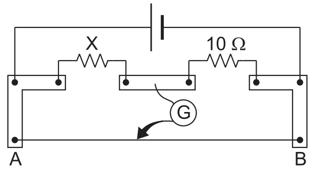
(a) $R_1R_2R_5 = R_3R_4R_6$

(b) $\dfrac{1}{R_5} + \dfrac{1}{R_6} = \dfrac{1}{R_1 + R_2} + \dfrac{1}{R_3 + R_4}$

(c) $R_1R_4 = R_2R_3$

(d) $R_1R_3 = R_2R_4 = R_5R_6$

5. A meter bridge is set up, as shown, to determine an unknown resistance X using a standard 10-Ω resistor. The galvanometer G shows a null deflection when the tapping key is at the 52-cm mark. The end corrections are 1 cm and 2 cm respectively for the ends A and B. The determined value of X is

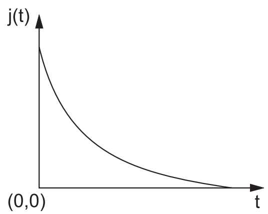


(a) 10.2 Ω (b) 10.6 Ω

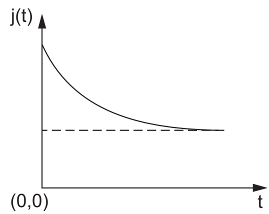
(c) 10.8 Ω (d) 11.1 Ω

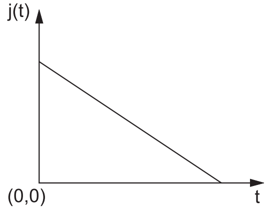
6. An infinite line charge of uniform electric charge density λ lies along the axis of an electrically conducting infinite cylindrical shell of radius R. At the time $t = 0$, the space inside the cylinder is filled with a material of permittivity ε and electrical conductivity σ. The electrical conduction in the material follows Ohm's law. Which of the following graphs best describes the subsequent variation of the magnitude of the current density $j(t)$ at any point in the material?

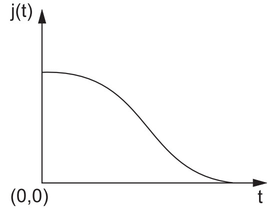
(a)



(b)



(c) 

(d) 

7. Incandescent bulbs are designed keeping in mind that the resistance of their filament increases with an increase in temperature. If at the room temperature, 100-W, 60-W and 40-W bulbs have filament resistances R_{100}, R_{60} and R_{40} respectively, the relation between these resistances is

(a) $\dfrac{1}{R_{100}} = \dfrac{1}{R_{40}} + \dfrac{1}{R_{60}}$ (b) $R_{100} = R_{40} + R_{60}$

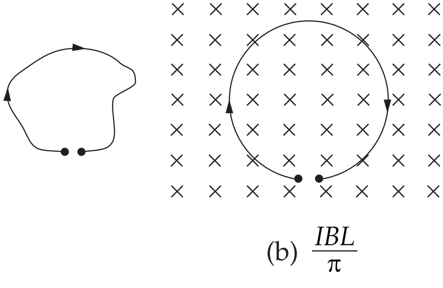
(c) $R_{100} > R_{60} > R_{40}$ (d) $\dfrac{1}{R_{100}} > \dfrac{1}{R_{60}} > \dfrac{1}{R_{40}}$

8. A particle of charge q and mass m moves in a circular orbit of radius r with the angular speed ω. The ratio of the magnitude of its magnetic moment to that of its angular momentum depends on

(a) ω and q (b) ω, q and m

(c) q and m (d) ω and m

9. A thin flexible wire of length L is connected to two adjacent fixed points and carries a current I in the clockwise direction, as shown in the figure. When the system is put in a uniform magnetic field of strength B going into the plane of the paper, the wire takes the shape of a circle. The tension in the wire is



(a) IBL (b) $\dfrac{IBL}{\pi}$

(c) $\dfrac{IBL}{2\pi}$ (d) $\dfrac{IBL}{4\pi}$

10. A particle of mass m and charge q moves with a constant velocity v along the positive x-direction. It enters a region containing a uniform magnetic field B directed along the negative z-direction, extending from $x = a$ to $x = b$. The minimum value of v required so that the particle can just enter the region $x > b$ is

(a) $\dfrac{qbB}{m}$ (b) $\dfrac{q(b-a)B}{m}$

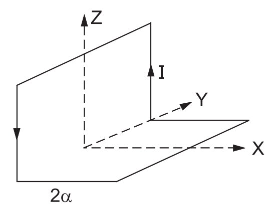
(c) $\dfrac{qaB}{m}$ (d) $\dfrac{q(b+a)B}{2m}$

11. A long straight wire along the z-axis carries a current I in the negative z-direction. The magnetic vector field $\vec{B}$ at a point having the coordinates (x, y) in the $z = 0$ plane is

(a) $\left(\dfrac{\mu_0 I}{2\pi}\right)\left(\dfrac{y\vec{i} - x\vec{j}}{x^2+y^2}\right)$ (b) $\left(\dfrac{\mu_0 I}{2\pi}\right)\left(\dfrac{x\vec{i} + y\vec{j}}{x^2+y^2}\right)$

(c) $\left(\dfrac{\mu_0 I}{2\pi}\right)\left(\dfrac{x\vec{j} - y\vec{i}}{x^2+y^2}\right)$ (d) $\left(\dfrac{\mu_0 I}{2\pi}\right)\left(\dfrac{x\vec{i} - y\vec{j}}{x^2+y^2}\right)$

12. A nonplanar loop of a conducting wire carrying a current I is placed as shown in the figure. Each of the straight sections of the loop is of length $2a$. The magnetic field due to this loop at the point P$(a, 0, a)$ points in the direction

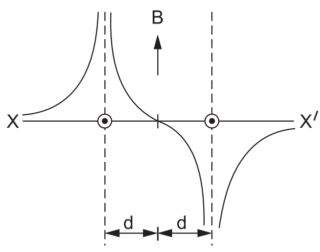


(a) $\dfrac{1}{\sqrt{2}}(-\vec{j}+\vec{k})$ (b) $\dfrac{1}{\sqrt{3}}(-\vec{j}+\vec{k}+\vec{i})$

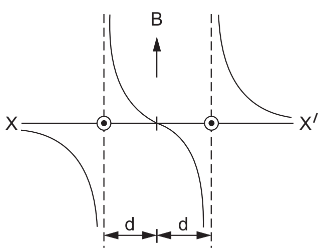
(c) $\dfrac{1}{\sqrt{3}}(\vec{i}+\vec{j}+\vec{k})$ (d) $\dfrac{1}{\sqrt{2}}(\vec{i}+\vec{k})$

13. Two long parallel wires are at a distance $2d$ apart. They carry steady equal currents flowing out of the plane of the paper, as shown. The variation of the magnetic field B along the line XX' is given by

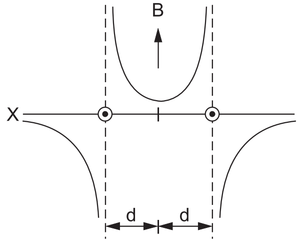
(a)



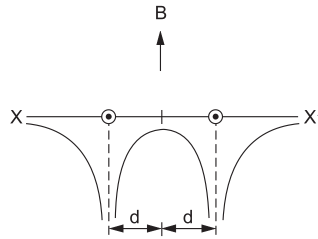
(b)



(c)



(d)



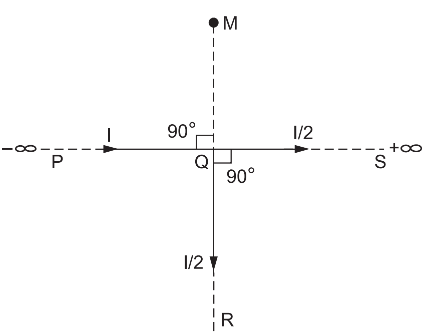
14. An a.c. voltage source of variable angular frequency ω and fixed amplitude V_0 is connected in series with a capacitance C and an electric bulb of resistance R (inductance zero). When ω is increased,

(a) the bulb glows dimmer
(b) the bulb glows brighter
(c) the total impedance of the circuit is unchanged
(d) the total impedance of the circuit increases

15. An ionised gas contains both positive and negative ions. If it is subjected simultaneously to an electric field along the $+x$-direction and a magnetic field along the $+z$-direction then

(a) the positive ions deflect towards the $+y$-direction and negative ions towards the $-y$-direction
(b) all the ions deflect towards the $+y$-direction
(c) all the ions deflect towards the $-y$-direction
(d) the positive ions deflect towards the $-y$-direction and negative ions towards the $+y$-direction

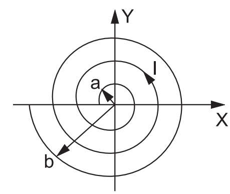
16. An infinitely long conductor PQR is bent to form a right angle as shown. A current I flows through PQR. The magnetic field strength due to this current at the point M is H_1. Now, another infinitely long straight conductor QS is connected at Q so that the current is $I/2$ in QR as well as in QS, the current in PQ remaining unchanged. The magnetic field strength at M is now H_2. The ratio H_1/H_2 equals



(a) $\frac{1}{2}$ (b) 1

(c) $\frac{2}{3}$ (d) 2

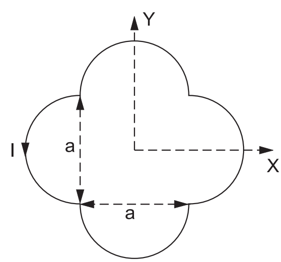
17. A long insulated copper wire is closely wound as a spiral of N turns. The spiral has the inner radius a and the outer radius b. The spiral lies in the xy-plane and a steady current I flows through the wire. The z-component of this magnetic field at the centre of the spiral is



(a) $\frac{\mu_0 NI}{2(b-a)} \ln\frac{b}{a}$ (b) $\frac{\mu_0 NI}{2(b-a)} \ln\frac{b+a}{b-a}$

(c) $\frac{\mu_0 NI}{2b} \ln\frac{b}{a}$ (d) $\frac{\mu_0 NI}{2b} \ln\frac{b+a}{b-a}$

18. A loop carrying a current I lies in the xy-plane as shown in the figure. The unit vector $\hat{k}$ is coming out of the plane of the paper. The magnetic moment of the current loop is



(a) $a^2 I\hat{k}$

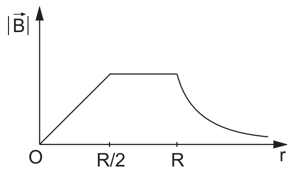
(b) $\left(\frac{\pi}{2}+1\right) a^2 I\hat{k}$

(c) $-\left(\frac{\pi}{2}+1\right) a^2 I\hat{k}$

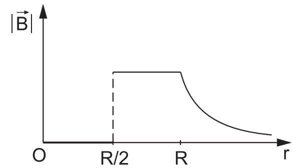
(d) $(2\pi+1)\, a^2 I\hat{k}$

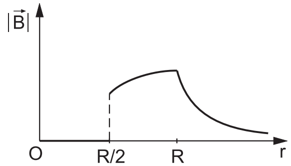
19. An infinitely long hollow conducting cylinder of inner radius $R/2$ and outer radius R carries a uniform current density along its length. The magnitude of the magnetic field, $|\vec{B}|$, as a function of the radial distance r from the axis is best represented by

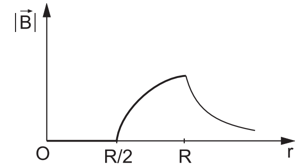
(a)



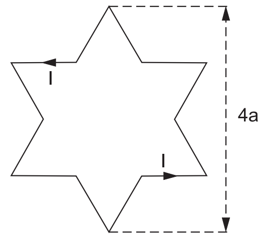
(b)



(c) 

(d) 

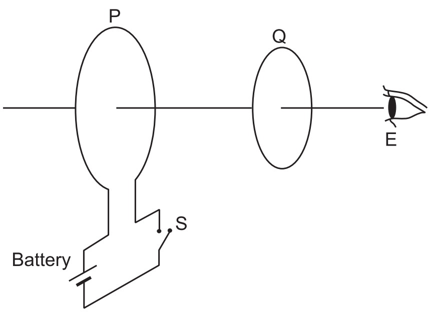
20. A symmetric star-shaped conducting wire loop is carrying a steady current I, as shown in the figure. The distance between any two diametrically opposite vertices of the star is $4a$. The magnitude of the magnetic field at the centre of the loop is



(a) $\dfrac{\mu_0 I}{4\pi a}\cdot 6(\sqrt{3}-1)$ (b) $\dfrac{\mu_0 I}{4\pi a}\cdot 3(\sqrt{3}-1)$

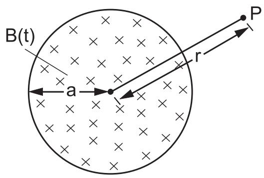
(c) $\dfrac{\mu_0 I}{4\pi a}\cdot 6(\sqrt{3}+1)$ (d) $\dfrac{\mu_0 I}{4\pi a}\cdot 3(2-\sqrt{3})$

21. As shown in the figure, P and Q are two coaxial conducting loops separated by some distance. When the switch S is closed, a clockwise current I_P flows in P (as seen by E) and an induced current I_{Q1} flows in Q. The switch remains closed for a long time. When S is opened, a current I_{Q2} flows in Q. Then the directions of I_{Q1} and I_{Q2} (as seen by E) are



(a) respectively clockwise and anticlockwise
(b) both clockwise
(c) both anticlockwise
(d) respectively anticlockwise and clockwise

22. A uniform but time-varying magnetic field $B(t)$ exists in a circular region of radius a and is directed into the plane of the paper, as shown. The magnitude of the induced electric field at the point P at a distance r from the centre of the circular region

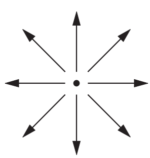


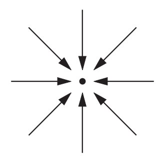
(a) is zero
(b) decreases as $\frac{1}{r}$
(c) increases as r
(d) decreases as $\frac{1}{r^2}$

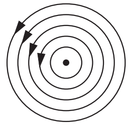
23. A coil having N turns is wound tightly in the form of a spiral with the inner and outer radii a and b respectively. When a current I passes through the coil, the magnetic flux density at the centre is

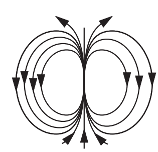
(a) $\frac{\mu_0 NI}{b}$
(b) $\frac{2\mu_0 NI}{a}$
(c) $\frac{\mu_0 NI}{2(b-a)}\ln\frac{b}{a}$
(d) $\frac{\mu_0 I^N}{2(b-a)}\ln\frac{b}{a}$

24. Which of the patterns given below is valid for an electric field as well as a magnetic field?

(a)


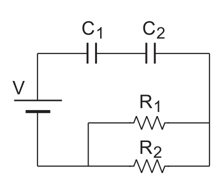
(b)


(c)


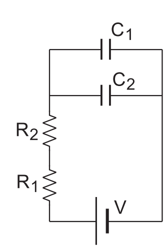
(d)


25. In the following circuits it is given that $R_1 = 1\,\Omega$, $R_2 = 2\,\Omega$, $C_1 = 2\mu F$ and $C_2 = 4\mu F$. The time constants (in μs) for the circuits I, II and III are respectively

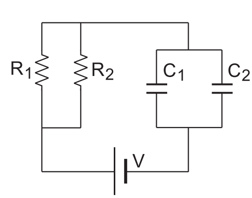
(a) 18, 8/9 and 4
(b) 18, 4 and 8/9
(c) 4, 8/9 and 18
(d) 8/9, 18 and 4



I

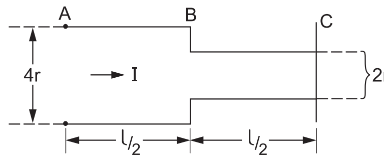


II



III

26. Consider a cylindrical element as shown in the figure. The current that flows through the element is I and the resistivity of the material of this cylinder is ρ. Choose the correct option.



(a) The power loss in the second half is four times the power loss in the first half.

(b) The voltage drop in the first half is twice the voltage drop in the second half.

(c) The current densities in the two halves are equal.

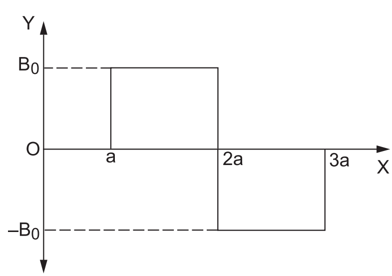
(d) The electric field in both the halves is the same.

27. A resistance of 2 Ω is connected across one gap of a metre bridge (the length of the wire being 100 cm), and an unknown resistance, greater than 2 Ω, is connected across the other gap. When these resistances are interchanged, the balance point shifts by 20 cm. Neglecting any corrections, the unknown resistance is

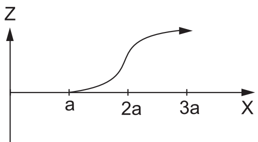
(a) 3 Ω (b) 4 Ω

(c) 5 Ω (d) 6 Ω

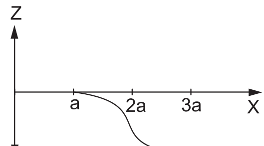
28. A magnetic field $\vec{B} = B_0\hat{j}$ exists in the region $a < x < 2a$ and $\vec{B} = -B_0\hat{j}$ exists in the region $2a < x < 3a$, where B_0 is a positive constant. A positive point charge moving with a velocity $\vec{v} = v_0\hat{i}$, where v_0 is a positive constant, enters the magnetic field at $x = a$. The trajectory of the charge in this region can be like

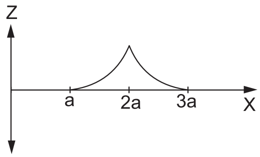


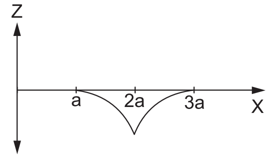
(a)

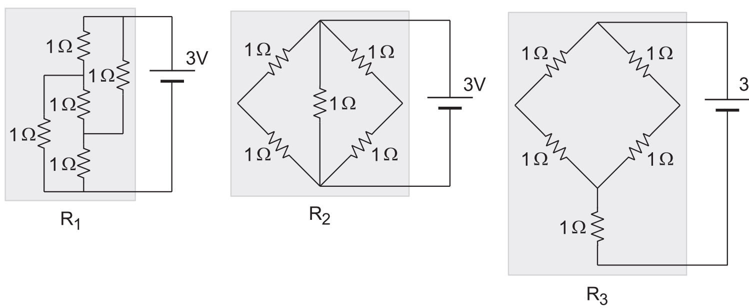


(b)



(c) 

(d) 

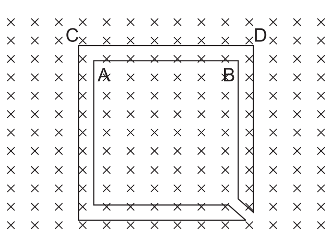
29. 

The figures above show three resistor configurations R_1, R_2 and R_3 connected to 3-V batteries. If the powers dissipated by the configurations R_1, R_2 and R_3 are respectively P_1, P_2 and P_3 then

(a) $P_1 > P_2 > P_3$　　(b) $P_1 > P_3 > P_2$

(c) $P_2 > P_1 > P_3$　　(d) $P_3 > P_2 > P_1$

30. The adjoining figure shows certain wire segments joined together to form a coplanar loop. The loop is placed in a perpendicular magnetic field in the direction going into the plane of the figure. The magnitude of the field increases with time, and I_1 and I_2 are the currents in the segments AB and CD. Then



(a) $I_1 > I_2$

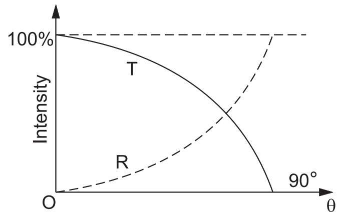
(b) $I_1 < I_2$

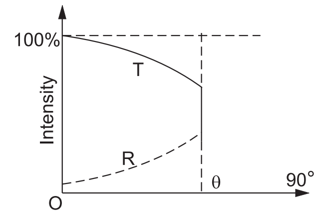
(c) I_1 is in the direction $\overrightarrow{BA}$ and I_2 is in the direction $\overrightarrow{CD}$

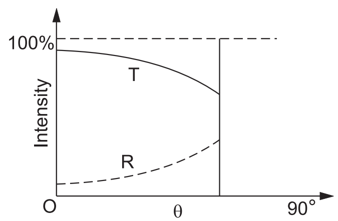
(d) I_1 is in the direction $\overrightarrow{AB}$ and I_2 is in the direction $\overrightarrow{DC}$

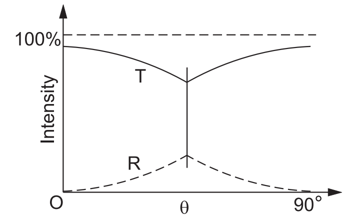
1.6 Ray Optics and Wave Optics

1. A light ray travelling in a glass medium is incident on a glass–air interface at an angle of incidence θ. The reflected (R) and transmitted (T) intensities, each as a function of θ, are plotted. The correct graph is

(a)


(b)


(c)


(d)


2. The image of an object formed by a plano-convex lens at a distance of 8 m behind the lens is real and is one-third the size of the object. The wavelength of light inside the lens is $\frac{2}{3}$ times the wavelength in free space. The radius of curvature of the curved surface is

(a) 1 m (b) 2 m
(c) 3 m (d) 4 m

3. A ray of light travelling in the direction $\frac{1}{2}(\hat{i}+\sqrt{3}\hat{j})$ is incident on a plane mirror. After reflection, it travels along the direction $\frac{1}{2}(\hat{i}-\sqrt{3}\hat{j})$. The angle of incidence is

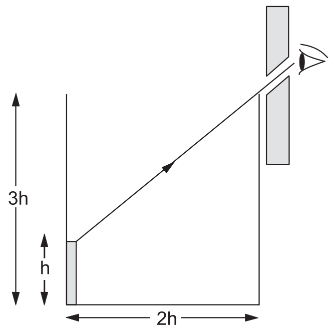
(a) 30° (b) 60°
(c) 40° (d) 75°

4. A biconvex lens of focal length 15 cm is in front of a plane mirror. The distance between the lens and the mirror is 10 cm. A small object is kept at a distance of 30 cm from the lens. The final image is

(a) virtual and at a distance of 16 cm from the mirror

(b) real and at a distance of 16 cm from the mirror
(c) virtual and at a distance of 20 cm from the mirror
(d) real and at a distance of 20 cm from the mirror

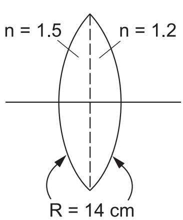
5. An observer can see through a pinhole the top end of a thin rod of height h, placed as shown in the figure. The beaker height is $3h$ and its radius is h. When the beaker is filled with a liquid up to a height $2h$, he can see the lower end of the rod. Then the refractive index of the liquid is



(a) $\frac{5}{2}$ (b) $\sqrt{\frac{5}{2}}$

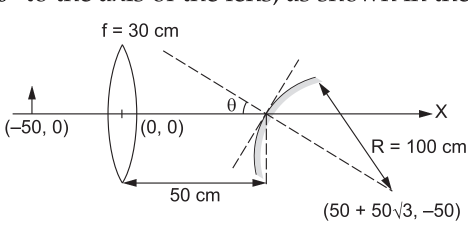
(c) $\sqrt{\frac{3}{2}}$ (d) $\frac{3}{2}$

6. A biconvex lens is formed with two thin plano-convex lenses, as shown in the figure. The refractive index n of the first lens is 1.5 and that of the second lens is 1.2. Both the curved surfaces are of the same radius of curvature $R = 14$ cm. For this biconvex lens, for an object distance of 40 cm the image distance will be



(a) –280.0 cm (b) 40.0 cm
(c) 21.5 cm (d) 13.3 cm

7. A small object is placed at 50 cm to the left of a thin convex lens of focal length 30 cm. A convex spherical mirror with radius of curvature 100 cm is placed to the right of the lens at a distance of 50 cm. The mirror is tilted such that the axis of the mirror is at an angle $\theta = 30°$ to the axis of the lens, as shown in the figure.

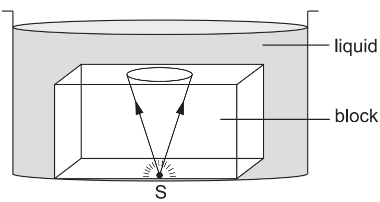


If the origin of the coordinate system is taken to be the centre of the lens, the coordinates (in cm) of the point (x, y) at which the image is formed are

(a) $(25, 25\sqrt{3})$ (b) $\left(\frac{125}{3}, \frac{25}{3}\right)$

(c) $(0, 0)$ (d) $(50 - 25\sqrt{3}, 25)$

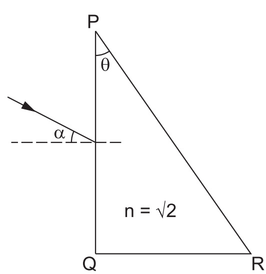
8. A point source S is placed at the bottom of a transparent block of height 10 mm and refractive index 2.72. It is immersed in a lower refractive-index liquid, as shown in the figure. It is found that the light emerging from the block to the liquid forms a circular bright spot of diameter 11.54 mm on the top of the block. The refractive index of the liquid is



(a) 1.21 (b) 1.30

(c) 1.36 (d) 1.42

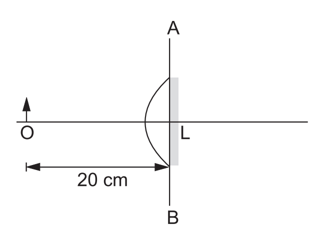
9. A parallel beam of light is incident from air at an angle α on the side PQ of a right-angled triangular prism of refractive index $n = \sqrt{2}$. The beam of light undergoes total internal reflection in the prism at the face PR when α has a minimum value of 45°. The angle θ of the prism is



(a) 15° (b) 22.5°

(c) 30° (d) 45°

10. A point object is placed at a distance of 20 cm from a plano-convex lens of focal length 15 cm. If the plane surface is silvered, the image will be formed at



(a) 60 cm left of AB

(b) 30 cm left of AB

(c) 12 cm left of AB

(d) 60 cm right of AB

11. 

A light beam is travelling from Region I to Region IV (refer to the figure). The refractive indices in Regions I, II, III and IV are n_0, $\frac{n_0}{2}$, $\frac{n_0}{6}$ and $\frac{n_0}{8}$ respectively. The angle of incidence (θ) for which the beam just misses entering Region IV is

(a) $\sin^{-1}\left(\frac{3}{4}\right)$ (b) $\sin^{-1}\left(\frac{1}{8}\right)$

(c) $\sin^{-1}\left(\frac{1}{4}\right)$ (d) $\sin^{-1}\left(\frac{1}{3}\right)$

12. A ball is dropped from a height of 20 m above the surface of water in a lake. The refractive index of water is 4/3. A fish inside the lake in the line of fall of the ball is looking at the ball. At an instant when the ball is 12.8 m above the water surface, the fish sees the speed of the ball as

(a) 9 m s^{-1} (b) 12 m s^{-1}

(c) 16 m s^{-1} (d) 21.33 m s^{-1}

13. A biconvex lens of focal length f forms a circular image of the sun of radius r in its focal plane. Therefore,

(a) $\pi r^2 \propto f$ (b) $\pi r^2 \propto f^2$

(c) if the lower half of the lens is covered with a black sheet, the area of the image is equal to $\frac{\pi r^2}{2}$

(d) if f is doubled, the intensity will increase

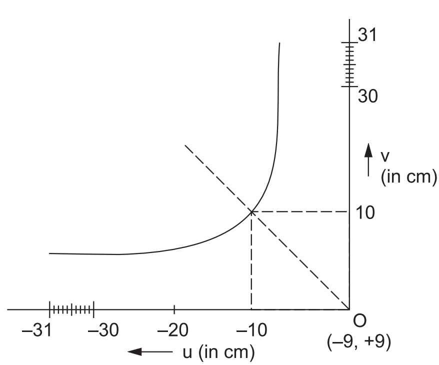
14. The graph of the image distance versus the object distance of a point from a convex lens is shown. Then the focal length of the lens is

(a) (0.50 ± 0.05) cm

(b) (0.50 ± 0.10) cm

(c) (5.00 ± 0.05) cm

(d) (5.00 ± 0.10) cm



15. In an experiment to determine the focal length (f) of a concave mirror by the u–v method, a student places the object pin A on the principal axis at a distance x from the pole P. The student looks at the pin and its inverted image from a distance keeping his/her eye in line with PA. When the student shifts his/her eye towards the left, the image appears to the right of the object pin. Then

(a) $x < f$ (b) $f < x < 2f$
(c) $x = 2f$ (d) $x > 2f$

16. A ray of light travelling in water is incident on its surface open to air. The angle of incidence is θ, which is less than the critical angle. Then there will be

(a) only a reflected ray and no refracted rays
(b) only a refracted ray and no reflected rays
(c) a reflected ray and a refracted ray, and the angle between them would be less than $180° - 2\theta$
(d) a reflected ray and a refracted ray, and the angle between them would be greater than $180° - 2\theta$

17. Two beams of red and violet lights are made to pass separately through a prism (the angle of the prism being 60°). In the position of the minimum deviation, the angle of refraction will be

(a) 30° for each light
(b) greater for the violet light
(c) greater for the red light
(d) the same for both the colours of light but not equal to 30°

18. In a double-slit experiment, instead of taking slits of equal widths, one slit is made twice as wide as the other. Then, in the interference pattern,

(a) the intensities of both the maxima and minima increase
(b) the intensity of the maxima increases and the minima has the zero intensity
(c) the intensity of the maxima decreases but that of the minima increases
(d) the intensity of the maxima decreases and the minima has the zero intensity

19. Young's double-slit experiment is carried out by using green, red and blue lights, one at a time. The fringe widths recorded are β_G, β_R and β_B respectively. Then

(a) $\beta_G > \beta_B > \beta_R$ (b) $\beta_B > \beta_G > \beta_R$
(c) $\beta_R > \beta_B > \beta_G$ (d) $\beta_R > \beta_G > \beta_B$

20. In Young's double-slit experiment using a monochromatic light of wavelength λ, the path difference (in terms of an integer) corresponding to any point having half the peak intensity is

(a) $(2n+1)\frac{\lambda}{2}$ (b) $(2n+1)\frac{\lambda}{4}$
(c) $(2n+1)\frac{\lambda}{8}$ (d) $(2n+1)\frac{\lambda}{16}$

1.7 Modern Physics

1. A pulse of light of 100 ns duration is absorbed completely by a small object initially at rest. The power of the pulse is 30 mW and the speed of light is 3×10^8 m s^{-1}. The final momentum of the object is

(a) 0.3×10^{-17} kg m s^{-1} (b) 1.0×10^{-17} kg m s^{-1}
(c) 3.0×10^{-17} kg m s^{-1} (d) 9.0×10^{-17} kg m s^{-1}

2. A metal surface is illuminated by light of two wavelengths 248 nm and 310 nm. The maximum speeds of the photoelectrons corresponding to those wavelengths are u_1 and u_2 respectively. If $u_1 : u_2 = 2 : 1$ and $hc = 1240$ eV nm, the work function of the metal is nearly

(a) 3.7 eV (b) 3.2 eV
(c) 2.8 eV (d) 2.5 eV

3. In a historical experiment to determine Planck constant, a metal surface was irradiated with lights of different wavelengths. The energies of the photoelectrons emitted were measured by applying a stopping potential. The relevant data for wavelengths (λ) of the incident light and the corresponding stopping potentials (V_0) are given below.

λ	V_0
0.3 μm	2.0 V
0.4 μm	1.0 V
0.5 μm	0.4 V

Given that $c = 3 \times 10^8$ m s^{-1} and $e = 1.6 \times 10^{-19}$ C, the value of Planck constant found in such an experiment is

(a) 6.0×10^{-34} (b) 6.4×10^{-34}
(c) 6.6×10^{-34} (d) 6.8×10^{-34}

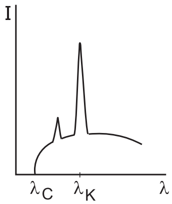
4. The electron in a hydrogen atom makes a transition from an excited state to the ground state. Which of the following statements is true?

(a) Its kinetic energy increases, and the potential and total energies decrease.
(b) Its kinetic energy decreases but the potential energy increases, and thus the total energy remains the same.
(c) Its kinetic and total energies decrease, and the potential energy increases.
(d) Its kinetic, potential and total energies decrease.

5. Electrons each having the energy 80 keV are incident on the tungsten target of an X-ray tube. The K-shell electrons of tungsten have –72.5 keV energy. The X-rays emitted by the tube contain only

(a) a continuous X-ray spectrum (breamsstrahlung) with a minimum wavelength of ~0.0155 nm
(b) a continuous X-ray spectrum (breamsstrahlung) with all wavelengths
(c) the characteristic X-ray spectrum of tungsten
(d) a continuous X-ray spectrum (breamsstrahlung) with a minimum wavelength of ~0.0155 nm and the characteristic X-ray spectrum of tungsten

6. The intensity (I) of X-rays from a Coolidge tube is plotted against the wavelength (λ), as shown in the figure. The minimum wavelength found is λ_C and the wavelength of the K_α line is λ_K. As the accelerating voltage is increased,



(a) $\lambda_K - \lambda_C$ increases
(b) $\lambda_K - \lambda_C$ decreases
(c) λ_K increases
(d) λ_K decreases

7. The wavelength of the first spectral line in the Balmer series of a hydrogen atom is 6561 Å. The wavelength of the second spectral line in the Balmer series of a singly ionised helium atom is

(a) 1215 Å
(b) 1640 Å
(c) 2430 Å
(d) 4687 Å

8. If λ_{Cu} is the wavelength of the K_α *x*-ray line of copper (atomic number: 29) and λ_{Mo} is the wavelength of the K_α *x*-ray line of molybdenum (atomic number: 42) then the value of the ratio $\lambda_{Cu}/\lambda_{Mo}$ is close to

(a) 1.99
(b) 2.14
(c) 0.50
(d) 0.48

9. Which of the following processes represents a gamma decay?

(a) ${}^{A}_{Z}X + \gamma \rightarrow {}^{A}_{Z-1}X + a + b$
(b) ${}^{A}_{Z}X + {}^{1}_{0}n \rightarrow {}^{A-3}_{Z-2}X + c$
(c) ${}^{A}_{Z}X \rightarrow {}^{A}_{Z}X + f$
(d) ${}^{A}_{Z}X + {}^{0}_{-1}e \rightarrow {}^{A}_{Z-1}X + g$

10. An accident in a nuclear laboratory resulted in depositions of a certain amount of a radioactive material of half-life 18 days inside the laboratory. Tests revealed that the radiation was 64 times the permissible level allowed for safe operation of the laboratory. What is the minimum number of days after which the laboratory can be considered safe for use?

(a) 64
(b) 90
(c) 108
(d) 120

11. A photoelectric material having the work function ϕ_0 is illuminated with light of wavelength $\lambda < \dfrac{hc}{\phi_0}$. The fastest photoelectron has a de Broglie wavelength λ_d. A change in wavelength of the incident light by $\Delta\lambda$ results in a change $\Delta\lambda_d$ in the de Broglie wavelength. Then the ratio $\Delta\lambda_d/\Delta\lambda$ is proportional to

(a) Δ_d^3/λ^2
(b) Δ_d^2/λ^2
(c) Δ_d/λ
(d) Δ_d^3/λ

12. The electrostatic energy of Z protons uniformly distributed throughout a spherical nucleus of radius R is given by

$$E = \frac{3}{5}\frac{Z(Z-1)e^2}{4\pi\varepsilon_0 R}.$$

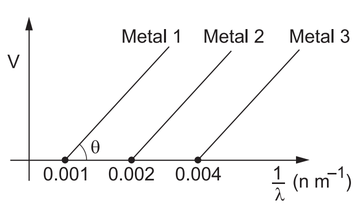
The measured masses of a neutron, ${}^{1}_{1}H$, ${}^{15}_{7}N$ and ${}^{15}_{8}O$ are 1.008665 u, 1.007825 u, 15.000109 u and 15.003065 u respectively. Given that the radii of both the ${}^{15}_{7}N$ and ${}^{15}_{8}O$ nuclei are the same; 1 u = 931.5 MeV/c^2, where c is the speed of light and $e^2/4\pi\varepsilon_0$ = 1.44 MeV fm. Assuming that the difference between the binding energies of ${}^{15}_{7}N$ and ${}^{15}_{8}O$ is purely due to their electrostatic energy, the radius of either of the nuclei is

(a) 2.85 fm (b) 3.03 fm
(c) 3.42 fm (d) 3.80 fm

13. Given a sample of radium–226 having a half-life of 4 days, find the probability that a nucleus disintegrates after two half-lives.

(a) 1 (b) $\frac{1}{2}$
(c) 1.5 (d) $\frac{3}{4}$

14. The graph between $1/\lambda$ and the stopping potentials (V) of three metals having the work functions ϕ_1, ϕ_2 and ϕ_3 in an experiment of photoelectric effect is plotted as shown in the figure. Which of the following statements is correct? (Here λ is the wavelength of the incident light.)



(a) The ratio of the work functions is $\phi_1 : \phi_2 : \phi_3 = 1:2:3$.
(b) The ratio of the work functions is $\phi_1 : \phi_2 : \phi_3 = 4:2:1$.
(c) $\tan\theta$ is directly proportional to hc/e, where h is Planck constant and c is the speed of light.
(d) Violet light can eject photoelectrons from Metals 2 and 3.

15. In the options given below, let E denote the rest-mass energy of a nucleus and n a neutron. The correct option is

(a) $E\left({}^{236}_{92}\text{U}\right) > E\left({}^{137}_{53}\text{I}\right) + E\left({}^{97}_{39}\text{Y}\right) + 2E(\text{n})$

(b) $E\left({}^{236}_{92}\text{U}\right) < E\left({}^{137}_{53}\text{I}\right) + E\left({}^{97}_{39}\text{Y}\right) + 2E(\text{n})$

(c) $E\left({}^{236}_{92}\text{U}\right) < E\left({}^{140}_{56}\text{Ba}\right) + E\left({}^{94}_{36}\text{Y}\right) + 2E(\text{n})$

(d) $E\left({}^{236}_{92}\text{U}\right) = E\left({}^{140}_{56}\text{Ba}\right) + E\left({}^{94}_{36}\text{Y}\right) + 2E(\text{n})$

16. The largest wavelength in the ultraviolet region of the hydrogen spectrum is 122 nm. The smallest wavelength in the infrared region of the hydrogen spectrum (to the nearest integer) is

(a) 802 nm
(b) 823 nm
(c) 1882 nm
(d) 1648 nm

17. Electrons with de Broglie wavelength λ fall on the target in an X-ray tube. The cut-off wavelength of the emitted X-rays is

(a) $\lambda_0 = \dfrac{2mc\lambda^2}{h}$
(b) $\lambda_0 = \dfrac{2h}{mc}$
(c) $\lambda_0 = \dfrac{2m^2c^2\lambda^3}{h^2}$
(d) $\lambda_0 = \lambda$

18. Which of the following statements is wrong in the context of X-rays generated from an X-ray tube?

(a) The wavelength of the characteristic X-rays decreases when the atomic number of the target increases.
(b) The cut-off wavelength of the continuous X-rays depends on the atomic number of the target.
(c) The intensity of the characteristic X-rays depends on the electrical power given to the X-ray tube.
(d) The cut-off wavelength of the continuous X-rays depends on the energy of the electrons in the X-ray tube.

19. A radioactive sample S_1, having an activity of 5 μC_i, has twice the number of nuclei as another sample S_2, which has an activity of 10 μC_i. The half-lives of S_1 and S_2 can be

(a) 20 years and 5 years respectively

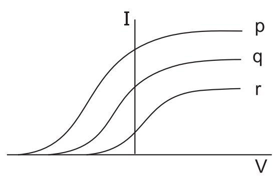
(b) 20 years and 10 years respectively

(c) 10 years each

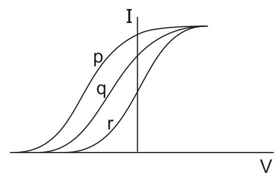
(d) 5 years each

20. Photoelectric-effect experiments are performed using three different metal plates p, q and r having the work functions $\phi_p = 2.0$ eV, $\phi_q = 2.5$ eV and $\phi_r = 3.0$ eV respectively. A light beam containing wavelengths of 550 nm, 450 nm and 350 nm with equal intensities illuminates each of the plates. The correct *I*–*V* graphs for the experiment is (taking $hc = 1240$ eV nm)

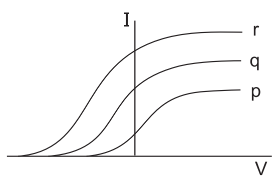
(a)



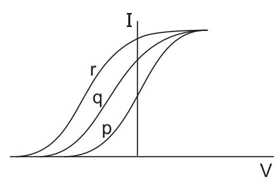
(b)



(c)



(d)



Answers

1.1 General Physics

1. d	**2.** d	**3.** c	**4.** a	**5.** c	**6.** b	**7.** c
8. b	**9.** c	**10.** c	**11.** a	**12.** a	**13.** d	**14.** c
15. a	**16.** a	**17.** c	**18.** a	**19.** d	**20.** c	**21.** b
22. b	**23.** d	**24.** d	**25.** d	**26.** c	**27.** c	**28.** d
29. a	**30.** b	**31.** c	**32.** b	**33.** c	**34.** c	**35.** c
36. d	**37.** c	**38.** a	**39.** b	**40.** b	**41.** b	**42.** d
43. a	**44.** a	**45.** d	**46.** d	**47.** d	**48.** d	**49.** b
50. d	**51.** a	**52.** b	**53.** d	**54.** c	**55.** d	**56.** b
57. c	**58.** d	**59.** b	**60.** c	**61.** d	**62.** b	**63.** c

1.2 Heat and Thermodynamics

1. a	**2.** a	**3.** a	**4.** a	**5.** d	**6.** d	**7.** d
8. b	**9.** c	**10.** b	**11.** d	**12.** c	**13.** d	**14.** a
15. b	**16.** c	**17.** a	**18.** a	**19.** c		

1.3 Sound Waves

1. b	**2.** b	**3.** c	**4.** d	**5.** a	**6.** b	**7.** b
8. d	**9.** b	**10.** a	**11.** a	**12.** a	**13.** a	**14.** a

1.4 Electrostatics

1. c	**2.** a	**3.** d	**4.** c	**5.** c	**6.** d	**7.** c
8. a	**9.** c	**10.** d	**11.** c	**12.** c	**13.** a	**14.** d
15. c	**16.** b	**17.** c	**18.** a	**19.** b	**20.** a	

1.5 Current Electricity and Magnetism

1. c	**2.** c	**3.** c	**4.** c	**5.** b	**6.** a	**7.** d
8. c	**9.** c	**10.** b	**11.** a	**12.** d	**13.** b	**14.** b
15. c	**16.** c	**17.** a	**18.** b	**19.** d	**20.** a	**21.** d
22. b	**23.** c	**24.** c	**25.** d	**26.** a	**27.** a	**28.** a
29. c	**30.** d					

1.6 Ray Optics and Wave Optics

1. c	**2.** c	**3.** a	**4.** b	**5.** b	**6.** b	**7.** a
8. c	**9.** a	**10.** c	**11.** b	**12.** c	**13.** b	**14.** d
15. b	**16.** c	**17.** a	**18.** a	**19.** d	**20.** b	

1.7 Modern Physics

1. b	**2.** a	**3.** b	**4.** a	**5.** d	**6.** a	**7.** a
8. b	**9.** c	**10.** c	**11.** a	**12.** c	**13.** d	**14.** c
15. a	**16.** b	**17.** a	**18.** b	**19.** a	**20.** a	

Hints and Solutions

1.1 General Physics

1. LC = 1 MSD – 1 VSD = 1 mm $-\dfrac{16}{20}$ mm = 0.2 mm.

2. $V = \dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{Q}{L}$.

$$\therefore\ [\varepsilon_0 LV] = [\varepsilon_0 L\Delta V] = [Q].$$

$$\therefore\quad [X] = \left[\frac{\varepsilon_0 L\Delta V}{\Delta t}\right] = \left[\frac{Q}{\Delta t}\right] = [\text{current}].$$

3. Diameter = 2.5 mm + $\dfrac{0.5\text{ mm}}{50}\times 20$ = 2.7 mm.

$$\therefore \text{percentage error in density} = \left(\frac{dm}{m}+\frac{3dr}{r}\right)\times 100$$

$$= 2+\left(3\times\frac{0.01}{2.70}\right)\times 100 = 3.1.$$

4.

$$\Delta d = \Delta l = \text{LC} = \frac{\text{pitch}}{\text{total cap division}} = \frac{0.5\text{ mm}}{100}.$$

$$Y = \frac{4MgL}{\pi d^2 l}.$$

$$\therefore\quad \frac{\Delta Y}{Y} = \frac{\Delta l}{l}+2\left(\frac{\Delta d}{d}\right) = \frac{0.5\text{ mm}/100}{0.25\text{ mm}}+2\cdot\left(\frac{0.5\text{ mm}/100}{0.5\text{ mm}}\right)$$

$$= \frac{0.5\times 10^{-2}}{0.25}+\frac{0.5\times 10^{-2}}{0.25}.$$

Hence, the error in the measurement of d and l are equal.

5. For balance with unknown resistance X,

$$\frac{X}{R} = \frac{l}{100\text{ cm}-l} = \frac{40\text{ cm}}{60\text{ cm}}$$

$$\Rightarrow\quad \frac{X}{90\ \Omega} = \frac{40}{60}$$

$$\Rightarrow\quad X = \frac{40}{60}\times 90\ \Omega = 60\ \Omega.$$

Now, $X = R\left(\dfrac{l}{100\text{ cm}-l}\right)$

$$\Rightarrow \quad \frac{\Delta X}{X} = \frac{\Delta l}{l} + \frac{\Delta l}{100\text{ cm} - l} = \frac{0.1}{40} + \frac{0.1}{60} = \frac{1}{240}.$$

$$\therefore \quad \Delta X = \frac{1}{240} \times 60\ \Omega = 0.25\ \Omega$$

$$\Rightarrow \quad X = (60 \pm 0.25)\,\Omega.$$

6. Main scale division (MSD) = 0.05 cm

and vernier scale division (VSD) = $\frac{4.9\text{ cm}}{100}$ = 0.049 cm.

∴ least count (LC) = 1 MSD – 1 VSD = 0.05 cm – 0.049 cm = 0.001cm.

$$\begin{aligned}\Rightarrow \quad \text{Diameter} &= \text{MS} + \text{VC} \times \text{LC}\\ &= (5.10 + 24 \times 0.001)\text{ cm}\\ &= 5.124\text{ cm}.\end{aligned}$$

7. With the first callipers,

MSD = 2.8 cm and VSD × LC = 7 × $\frac{1}{10}$ mm = 0.07 cm.

∴ the reading is 2.87 cm.

With the second callipers,

MSD = 2.8 cm and vernier scale reading = 7 × $\frac{-0.1}{10}$ cm = –0.07 cm.

∴ the reading is 2.80 + 0.10 – 0.07 = 2.83 cm.

8. $M = \frac{4}{3}\pi R^3 \rho = \text{constant}$

$$\Rightarrow \quad \frac{4}{3}\pi 3R^2\left(\frac{dR}{dt}\right)\rho + \frac{4}{3}\pi R^3\frac{d\rho}{dt} = 0$$

$$\Rightarrow \quad \frac{dR}{dt} = -\frac{R}{3\rho}\frac{d\rho}{dt}$$

$\Rightarrow \; v \propto R.$

9.
$$T = \sqrt{\frac{2L}{g}} + \frac{L}{v}$$

$$\Rightarrow \quad \frac{\delta T}{\delta L} = \frac{1}{\sqrt{2gL}} + \frac{1}{v}.$$

$$\therefore \quad \delta L = \frac{\delta T}{\frac{1}{\sqrt{2gL}} + \frac{1}{v}} = \frac{\delta T}{\frac{1}{20} + \frac{1}{300}} = \frac{150}{8}\delta T.$$

$$\therefore \quad \frac{\delta L}{L}\times 100\% = \frac{150}{8}\frac{\delta T}{L}\times 100\%$$

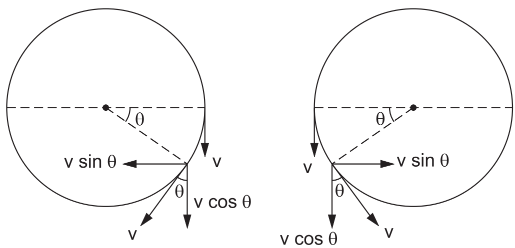
$$= \frac{150}{8}\times\frac{0.01}{20}\times 100\% = \frac{15}{16}\% \approx 1\%.$$

10. $\vec{S} = \vec{P} + b|\vec{R}| = \vec{P} + b|\vec{Q} - \vec{P}| = \vec{P}(1-b) + b\vec{Q}.$

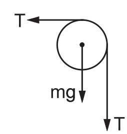
11. v is negative when the ball is falling and positive when it bounces up. Also, $v = -\sqrt{2g(d-h)}$ for downward motion and $v = \sqrt{g(d-2h)}$ for upward motion.

12. Relative velocity $= |\vec{v}_r| = 2v\sin\theta = 2v\sin\omega t.$

In each rotation, the relative speed becomes zero twice and reaches the maximum value twice.



13. For the block of mass M to be in equilibrium, the tension in the string is $T = Mg$. The force on the pulley by the clamp must balance all the other forces acting on it, i.e., the resultant of the forces shown in the figure. This has a magnitude of $\sqrt{(Mg+mg)^2 + (Mg)^2} = g\sqrt{(M+m)^2 + M^2}$.



14. The momentum of the two-particle system at $t = 0$ is given by

$$\vec{p}_i = m_1\vec{v}_1 + m_2\vec{v}_2.$$

A collision between the two does not affect the total momentum of the system.

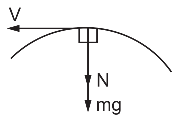
A constant external force $(m_1 + m_2)g$ acts on the system. The impulse given by this force in the time interval from $t = 0$ to $t = 2t_0$ is $(m_1 + m_2)g \times 2t_0$.

$\therefore$ the absolute change in momentum in this interval is

$$|(m_1\vec{v}_1' + m_2\vec{v}_2') - (m_1\vec{v}_1 + m_2\vec{v}_2)| = 2(m_1 + m_2)gt_0.$$

15. The blocks will have the same speed, say, equal to v, at the highest point of each track, as they all rise to the same height. Let R be the radius of curvature of a track and N be the normal reaction of the track at the highest point of the track.

Centripetal force $= N + mg = \dfrac{mv^2}{R}$.



N will be the maximum when R is the minimum. This occurs when the track is most sharply curved.

16. The linear acceleration of the bead is $a = L\,\alpha$.

$\therefore$ the reaction force on the bead due to the rod is $N = ma = mL\alpha$.

After the time t, the angular velocity of the bead is $\omega = \alpha t$.

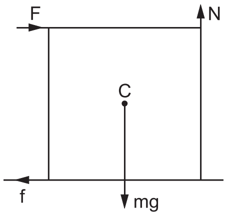
$\therefore$ the centripetal acceleration of the bead is $\omega^2 L = \alpha^2 t^2 L$.

$\therefore$ the force of friction at the limiting position is $\mu N = \mu mL\alpha$.

$\therefore$ for slipping,

$$\mu mL\alpha = m\alpha^2 t^2 L \qquad \text{or} \qquad t = \sqrt{\mu/\alpha}.$$

17. When F is applied, the normal reaction (N) of the floor moves to the right. The cube topples when N reaches its edge.

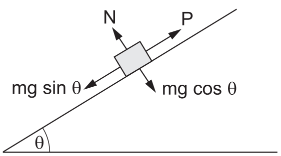


Here, $N = mg$ and the force of friction is $f = F$.

Taking the torque about the centre C,

$$F \times \frac{L}{2} + f \times \frac{L}{2} = N \times \frac{L}{2} \qquad \text{or} \qquad 2F = mg.$$

18. The force of friction, f, becomes zero when $P = mg\sin\theta$. For $P < mg\sin\theta$, f acts upwards along the incline.



19. Taking the torque about A,

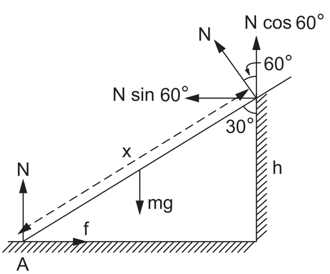
$$N\left(\frac{h}{\sin 60^\circ}\right) = mg\,\frac{l}{2}\cos 60^\circ$$

$$\Rightarrow \qquad \frac{2}{\sqrt{3}}Nh = \frac{1}{4}mgl \qquad \text{...(i)}$$

Again, $N + N\cos 60^\circ = mg$

$$\Rightarrow \qquad \frac{3}{2}N = mg$$

$$\Rightarrow \qquad N = \frac{2}{3}mg.$$



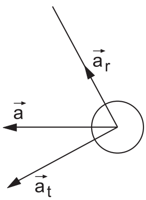
Substituting N in (i),

$$\frac{2}{\sqrt{3}} \cdot \frac{2}{3} mgh = \frac{1}{4} mgl$$

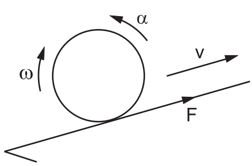
$$\Rightarrow \quad \frac{h}{l} = \frac{3\sqrt{3}}{16}.$$

Now, friction $= f = N \sin 60° = \frac{2}{3} mg \cdot \frac{\sqrt{3}}{2} = \frac{mg}{\sqrt{3}} = \frac{16\sqrt{3}}{3}$ N.

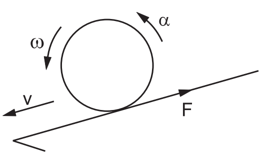
20. In the given position, the bob is moving along a circular path with some speed and hence has some radial acceleration $\vec{a}_r$. It also has some tangential acceleration $\vec{a}_t$. As $\vec{a}$ is the resultant of these two, it will be directed somewhere between them.



21. When the cylinder rolls up the incline, its angular velocity ω is clockwise and decreasing. This requires an anticlockwise angular acceleration α, which is provided by the force of friction (F) acting up the incline.



When the cylinder rolls down the incline, its angular velocity ω is anticlockwise and increasing. This requires an anticlockwise angular acceleration α, which is provided by the force of friction (F) acting up the incline.



22. The angular momentum (L) of the system is conserved; i.e., $L = I\omega$ = constant.

When the tortoise walks along a chord, it first moves closer to the centre and then away from the centre. Hence, I first decreases and then increases. As a result, ω will first increase and then decrease. Also, the change in ω will be a nonlinear function of time.

23. A centripetal force is provided by the component of the tension along the horizontal. So,

$$T \sin\theta = m\omega^2 r = \frac{Tr}{L} \Rightarrow \omega = \sqrt{\frac{T}{mL}} = \sqrt{\frac{324 \text{ N}}{(0.5 \text{ kg})(0.5 \text{ m})}} = 36 \text{ rad s}^{-1}.$$

24. Work done $= \int \vec{F} \cdot \vec{dr} = \int \vec{F} \cdot (dx\hat{i} + dy\hat{j})$

$$= \frac{K}{a^3} \int_a^0 x\, dx + \frac{K}{a^3} \int_0^a y\, dy \qquad [\because x^2 + y^2 = a^2]$$

$$= \frac{K}{a^3}\left(-\frac{a^2}{2} + \frac{a^2}{2}\right) = 0.$$

[Note: $\vec{F} = \frac{K}{a^3}(x\hat{i} + y\hat{j}) = \frac{K}{a^3}\vec{r}$ (radial).

Any radial force does zero work in circular motion.]

25. For energy conservation, $\frac{1}{2}mv^2 = mgR(1-\cos\theta)$, and for circular motion, $mg\cos\theta - N = \frac{mv^2}{R}$.

Simplifying, $N = mg(3\cos\theta - 2)$.

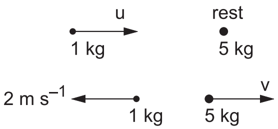
Initially, $\theta = 0$, i.e., $N = mg$ (radially inward).

As θ increases, N decreases. At $\theta = \cos^{-1}\left(\frac{2}{3}\right) \approx 48°$, N reduces to zero, and afterwards N acts radially outward.

26. The area under the F–t curve is 4.5 kg m s^{-1} = p.

$\therefore$ KE $= \frac{p^2}{2m} = 5.06$ J.

27. $u = -2$ m s^{-1} + $5v$.



$v + 2$ m s^{-1} = $-1(0 - u) = u$.

Solving, $u = 3$ m s^{-1} and $v = 1$ m s^{-1}.

KE of CM = $\frac{1}{2}$ (6 kg) (0.5 m s^{-1})^2 = 0.75 J.

28. $h = \frac{1}{2}gt^2 \Rightarrow 5 = \frac{1}{2}(10)t^2 \Rightarrow t = 1\,\text{s}$.

From $x = v_x t$ we have $v_{\text{ball}} = 20$ m s^{-1} and $v_{\text{bullet}} = 100$ m s^{-1}.

Conserving linear momentum, $0.01\,v = 0.01 \times 100$ m s^{-1} + 0.2×20 m s^{-1}
$\Rightarrow v = 500$ m s^{-1}.

29. At the highest point, the velocity of the 1st particle is $\vec{v}_1 = (u_0\cos\alpha)\hat{i}$.

At the position of collision $\left(H = \frac{u_0^2\sin^2\alpha}{2g}\right)$, the velocity of the 2nd particle is

$$\vec{v}_2 = \sqrt{u_0^2 - 2gH}\;\hat{j} = \sqrt{u_0^2 - 2g\left(\frac{u_0^2\sin^2\alpha}{2g}\right)}\;\hat{j} = (u_0\cos\alpha)\,\hat{j}.$$

Conserving linear momentum,

$$2m\vec{v} = m\vec{v}_1 + m\vec{v}_2 = m(u_0\cos\alpha)(\hat{i} + \hat{j})$$

$$\Rightarrow \quad \vec{v} = \frac{u_0\cos\alpha}{2}(\hat{i} + \hat{j}). \quad \therefore\ \theta = 45°.$$

30. Kinetic energy $= K = \frac{1}{2} mv^2$.

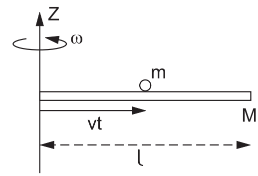
$\therefore \frac{dK}{dt} = \frac{1}{2} m\left(2v \cdot \frac{dv}{dt}\right)$.

31. $v_{CM} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = \frac{(10\text{ kg})(14\text{ m s}^{-1}) + (4\text{ kg})(0)}{10\text{ kg} + 4\text{ kg}} = 10\text{ m s}^{-1}$.

32. Angular momentum $= L = I\omega = m(vt)^2\omega = mv^2\omega t^2$.

$\therefore$ torque $= \tau = \frac{dL}{dt} = 2mv^2\omega t$.

So, $\tau \propto t$, which represents a straight line passing through the origin.



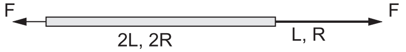
33. $\overrightarrow{L_O}$ has a constant magnitude and a constant direction (perpendicular to the plane of rotation); $\overrightarrow{L_P}$ has a constant magnitude, but its direction continuously changes with time.

34. At $t = \frac{T}{8} = \frac{1}{8}\left(\frac{2\pi}{\omega}\right) = \frac{\pi}{4\omega}$,

the x-coordinate of P is $(\omega R)\left(\frac{\pi}{4\omega}\right) = \frac{\pi R}{4} > R\cos 45°$.

$\therefore$ both P and Q land in the unshaded region.

35. If x_1 and x_2 be the elongations in the thick and thin wires respectively, the Young modulus

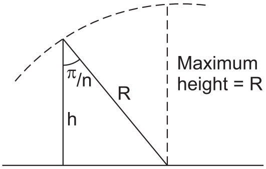


$$Y = \frac{F/A_1}{x_1/L_1} = \frac{F/A_2}{x_2/L_2}.$$

$$\therefore \quad \frac{x_2}{x_1} = \frac{A_1}{A_2} \cdot \frac{L_2}{L_1} = \frac{2}{1}.$$

36. $\cos\left(\frac{\pi}{n}\right) = \frac{h}{R}$

$\Rightarrow \Delta = R - h = h\left[\frac{1}{\cos\left(\frac{\pi}{n}\right)} - 1\right]$.



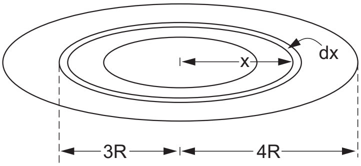
37. For a satellite in a circular orbit around the earth, the time period T depends on the radius r as $T^2 \propto r^3$. Here, for the geostationary satellite, $T_1 = 24$ h and $r_1 = 36000$ km. For the spy satellite, $r_2 = 6400$ km and if the time period be T_2,

$$\frac{T_2^2}{T_1^2} = \frac{r_2^3}{r_1^3} \text{ or } T_2 = T_1\left(\frac{r_2}{r_1}\right)^{3/2} = (24\text{ h})\left(\frac{6400\text{ km}}{36000\text{ km}}\right)^{3/2} \approx 2\text{ h}.$$

38. Mass per unit area $= \sigma = \dfrac{M}{\pi(4R)^2 - (3R)^2}$,

area of ring $= 2\pi x dx$,

and mass of ring $= dm = 2\pi\sigma x dx$.



$$\therefore\ V_p = -G\int_{3R}^{4R} \frac{dm}{[(4R)^2 + x^2]^{1/2}}.$$

39. For the object to escape, $\text{KE} + \left(-\dfrac{GMm}{r}\right) = 0$.

For circular motion, $\dfrac{mv^2}{r} = \dfrac{GMm}{r^2}$.

So, $\dfrac{GMm}{r} = mv^2 =$ required KE.

40. The value of g at a distance r (where $r < R$) from the centre of a planet is

$$g = g_0\frac{r}{R} = \frac{4}{3}G\pi r\rho.$$

The force required to keep the wire at rest is

$$F = \text{weight of the wire} = \int dm{\cdot}g$$

$$= \int_{\frac{4R}{5}}^{R} (\lambda dr)\left(\frac{4}{3}\pi G\rho r\right) = \frac{4}{3}G\pi\rho\lambda\left.\frac{r^2}{2}\right|_{4R/5}^{R}$$

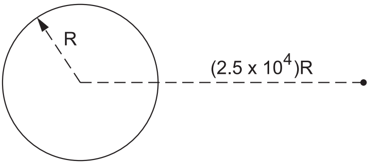
$$= \frac{4}{3}G\pi\rho\lambda\frac{9R^2}{50}.$$

Taking ρ = density of the earth $= \dfrac{M_e}{\frac{4}{3}\pi R_e^3}$ and $R = \dfrac{R_e}{10}$,

we obtain $\quad F = \dfrac{9}{5000}\, gR_e\lambda = 108$ N.

41. The escape speed from the earth is

$$v_e = \sqrt{2gR_e} = \sqrt{\frac{2GM_e}{R}}$$
$$= 11.2 \text{ km s}^{-1}.$$



If v_0 be the escape speed from the earth–sun system then

$$\frac{1}{2}mv_0^2 - \frac{GmM_e}{R} - \frac{GM_S m}{(2.5\times10^4)R} \geqslant 0.$$

For the rocket to just escape,

$$\frac{1}{2}mv_0^2 = \frac{GmM_e}{R} + \frac{GM_S m}{(2.5\times 10^4)R}$$

$$\Rightarrow \qquad \frac{v_0^2}{2} = \frac{GM_e}{R} + \frac{G\times 3\times 10^5 M_e}{(2.5\times 10^4)R} = \frac{13GM_e}{R}$$

$$\Rightarrow \qquad v_0 = \sqrt{13\left(\frac{2GM_e}{R}\right)} = \sqrt{13}\ v_e = \sqrt{13}\times 11.2\ \text{km s}^{-1}$$

$$= 40.4\ \text{km s}^{-1} \approx 40\ \text{km s}^{-1}.$$

42. Conserving the angular momentum of the system,

$$MR^2\,\omega = MR^2.\frac{8}{9}\omega + \frac{M}{8}\left(\frac{3}{5}R\right)^2\times\frac{8\omega}{9} + \frac{M}{8}x^2\left(\frac{8\omega}{9}\right) \Rightarrow x = \frac{4}{5}R.$$

43. The frequency $f = \frac{1}{2\pi}\sqrt{\frac{K}{m}}$ of a spring–block system is independent of the external fields (e.g., gravity).

44. Time of flight $= T = \frac{2u\sin\theta}{g}$, i.e., $1\ \text{s} = \frac{2u\cdot\sin 45^\circ}{10}\cdot$

$\therefore u = 5\sqrt{2}\ \text{m s}^{-1} = \sqrt{50}\ \text{m s}^{-1}.$

45. Net upward buoyancy force on the cylinder

= weight of the liquid displaced by it

= $\rho g V$

= (upward force on the bottom) – (downward force on the top).

$\therefore$ the force on the bottom is $\rho g V + (h\rho g)\pi R^2 = \rho g(V + \pi R^2 h)$.

46. When the coin slips into the water, the wooden block moves up and l decreases. When the coin was floating, it displaced water equal to its own weight. When inside the water, it displaces water equal to its own volume. As its density is greater than that of water, it displaced more water in the first case. Hence, h decreases when it falls into the water.

47. $mg + (V_l\rho)g = \frac{V_0}{2}\times\rho\times g \quad \Rightarrow V_l = \frac{V_0}{2} - \frac{m}{\rho}\cdot$

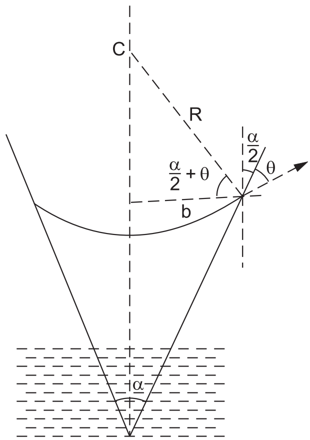
Hence, $V_l < \frac{V_0}{2}\cdot$

48. From the geometry of the figure,

$$\frac{b}{R} = \cos\left(\theta + \frac{\alpha}{2}\right)\cdot$$

If p_0 = atmospheric pressure then

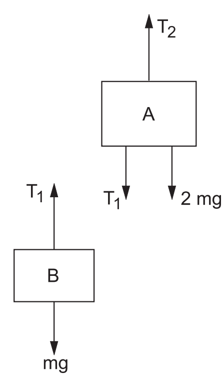
$$p_0 - \frac{2s}{R} + h\rho g = p_0$$



$$\Rightarrow \qquad \frac{2s}{R} = h\rho g$$

$$\Rightarrow \qquad h = \frac{2s}{R\rho g} = \frac{2s}{b\rho g}\cos\left(\theta + \frac{\alpha}{2}\right).$$

49. From the free-body diagram (FBD) of block A,



$$T_2 = T_1 + 2mg.$$

Similarly, for the block B,

$$T_1 = mg \text{ and } T_2 = 3mg.$$

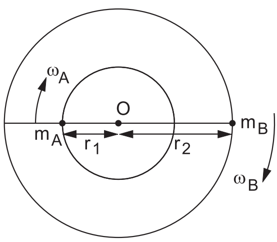
When the string is cut, $T_1 = 0$.

$\therefore$ for B, $mg = ma_B \Rightarrow a_B = g$.

For A, $3mg - 2mg = 2ma_A \Rightarrow a_A = \dfrac{g}{2}$.

50. The gravitational attraction provides the required centripetal force.

Thus, $\dfrac{Gm_1 m_2}{(r_1 + r_2)^2} = m_A \omega_A^2 r_1 = m_B \omega_B^2 r_2$.



Further, the system rotates about the centre of mass O. So,

$$m_A r_1 = m_B r_2$$

$$\Rightarrow \qquad \omega_A = \omega_B.$$

$$\therefore \qquad T_A = T_B.$$

51. Since the mass of the sphere is unchanged, we have

$$\left(\frac{2}{5}MR^2\right)_{\text{sphere}} = \left(\frac{1}{2}MR_0^2 + MR_0^2\right)_{\text{disc}}$$

or $$\frac{2}{5}R^2 = \frac{3}{2}R_0^2.$$

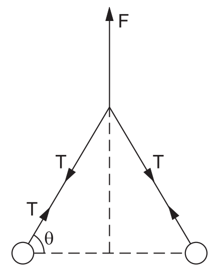
$\therefore$ the radius of the disc is $R_0 = \dfrac{2R}{\sqrt{15}}\cdot$

52. $$2T\sin\theta = F$$

and $$T\cos\theta = m\cdot a_x.$$

$\therefore$ $$2\tan\theta = \frac{F}{ma_x}$$

$\Rightarrow$ $$a_x = \frac{F}{2m\tan\theta} = \frac{F}{2m}\left(\frac{x}{\sqrt{a^2-x^2}}\cdot\right)$$



53. By the law of conservation of mechanical energy,

$$\frac{1}{2}mv^2\left(1+\frac{K^2}{R^2}\right) = mgh = mg\left(\frac{3v^2}{4g}\right)$$

$\Rightarrow$ $$1+\frac{K^2}{R^2} = \frac{3}{2}\cdot$$

$\therefore$ $\dfrac{K^2}{R^2} = \dfrac{1}{2}$, which is true for a disc.

54. The gravitational system represents a solid sphere of radius R having a uniform density of $\rho = \rho_0$.

For $r < R$, force $= \dfrac{G\frac{4}{3}\pi r^3\rho m}{r^2} = \dfrac{mv^2}{r}\cdot$

$\therefore$ $v \propto r$ (a straight line from $r = 0$ to $r = R$).

For $r > R$, $F = \dfrac{G\frac{4}{3}\pi R^3\rho m}{r^2} = \dfrac{mv^2}{r}$, i.e., $v \propto \dfrac{1}{\sqrt{r}}\cdot$

$\therefore$ the correct plot is the option (c).

55. The critical speed at A is $\sqrt{5gL}$.

Conserving energy from A to C,

$$\frac{1}{2}m\left(\sqrt{5gL}\right)^2 = \frac{1}{2}mv_c^2 + mgL(1-\cos\theta)$$

or $\cos\theta = -\frac{7}{8}$ $\left[\because v_C = \frac{v_A}{2} = \frac{1}{2}\sqrt{5gL}\right]$

$= -0.87.$

since $\cos\frac{3\pi}{4} = -\frac{1}{\sqrt{2}} = -0.71,$

we get $\frac{3\pi}{4} < \theta < \pi.$

56. The pressure inside a spherical soap bubble exceeds the outer (atmospheric) pressure p_0 by $4S/R$, where S is the surface tension of the soap solution, and R is the radius.

Thus, the pressure inside the bubble is $p = p_0 + \frac{4S}{R}$.

$\because$ $R_2 > R_1$, $\therefore$ $p_2 < p_1$.

Hence, air flows from 1 to 2.

57. By the law of conservation of mechanical energy,

$$\frac{1}{2}kx^2 = \frac{1}{2}(4k)\,y^2$$

$$\Rightarrow \quad \frac{y}{x} = \frac{1}{2}.$$

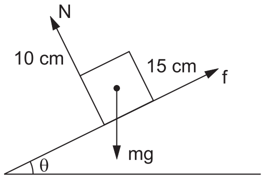
58. From the given x–t graph of SHM, displacement $= x = A\sin\omega t$.

$\therefore$ $v = A\omega\cos\omega t$ and $a = -A\omega^2\sin\omega t$.

Given that $A = 1$ cm, $\omega = \frac{2\pi}{8}$ rad s^{-1} and $t = \frac{4}{3}$ s

$$\Rightarrow \quad a = -(1\text{ cm})\left(\frac{2\pi}{8}\text{ s}^{-1}\right)^2 \sin\left(\frac{2\pi}{8}\cdot\frac{4}{3}\right) = -\frac{\sqrt{3}\,\pi^2}{32}\text{ cm s}^{-2}.$$

59. The block slides down the plane when



$$mg\sin\theta \geq f = \mu N = \mu mg\cos\theta$$

or $\tan\theta \geq \mu = \sqrt{3} = 1.732.$

The block topples without sliding when

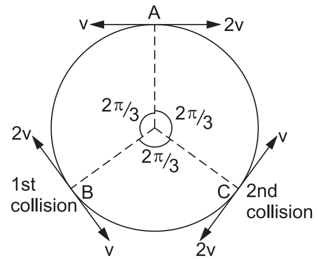
$$f\left(\frac{15}{2}\right) \geq N(5) \qquad \text{(taking torque about CM)}$$

or $\mu N\left(\frac{15}{2}\right) \geq N(5)$

or $\mu \geq \frac{2}{3} = 0.67$ or $\tan\theta \geq 0.67.$

Thus, the block will neither slip nor topple till $\theta \leq \tan^{-1}(0.67)$, and when θ exceeds this, it topples before sliding occurs.

60. The angular displacement of the faster particle is 2θ and that of the slower particle is θ. So,



$$2\theta + \theta = 2\pi$$

$$\Rightarrow \qquad \theta = \frac{2\pi}{3}.$$

Hence, the first collision will occur at B, where the velocities are interchanged due to equal masses and their elastic collision. The second collision will occur at C and the third at A. Thus, two collisions will occur before meeting at A.

61. The springs are in series; so restoring force is the same in both the springs. Hence,

$$k_1 x_1 = k_2 x_2.$$

Given that $\qquad x_1 + x_2 = A.$

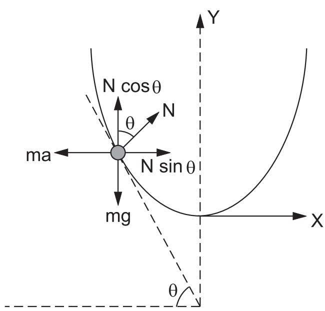
$$\therefore \qquad x_1 = \frac{Ak_2}{k_1 + k_2}.$$

62. Forces acting on the bead as shown in the free-body diagram are

(i) weight (mg)

(ii) normal reaction (N)

(iii) pseudo force (ma).



Resolving N, we get for balance

$$N \sin\theta = ma \text{ and } N\cos\theta = mg.$$

$$\therefore \qquad \tan\theta = \frac{a}{g}$$

$$\Rightarrow \qquad \frac{dy}{dx} = \frac{a}{g}.$$

But $\qquad y = kx^2.$

So, $\qquad \frac{dy}{dx} = 2kx$

$$\Rightarrow \qquad x = \frac{a}{2gk}.$$

63. When the rod is turned through an angle θ, the restoring torque (opposing θ) is

$$\tau = -2k\left(\frac{L}{2}\theta\right)\frac{L}{2} = I\frac{d^2\theta}{dt^2}.$$

$$\therefore \qquad \frac{d^2\theta}{dt^2} = \frac{\frac{kL^2}{2}(-\theta)}{\frac{ML^2}{12}} = -\left(\frac{6k}{M}\right)\theta.$$

This represents an angular SHM for which

$$\omega = \sqrt{\frac{6k}{M}} = \frac{2\pi}{T} \text{ or } f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{6k}{M}}.$$

1.2 Heat and Thermodynamics

1. Let ρ_{Fe} and ρ_{Hg} denote respectively the densities of iron and mercury at 0 °C, and m be the mass of the block.

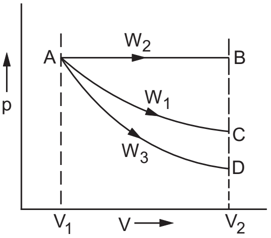
$\therefore$ the volume of the block is m/ρ_{Fe} and that of the displaced mercury is $k_1 m/\rho_{Fe}$.

$$\therefore \left(\frac{k_1 m}{\rho_{Fe}}\right)\rho_{Hg} = m \quad \text{or} \quad k_1 = \frac{\rho_{Fe}}{\rho_{Hg}}. \quad \text{Also, } \rho_t = \frac{\rho_0}{1+\gamma t}.$$

$$\therefore \; k_2 = \frac{\rho_{Fe}}{1+(\gamma_{Fe} \times 60\text{ °C})} \times \frac{1+(\gamma_{Hg} \times 60\text{ °C})}{\rho_{Hg}} = k_1 \frac{1+(60\text{ °C})\gamma_{Hg}}{1+(60\text{ °C})\gamma_{Fe}}$$

$$\text{or } \frac{k_1}{k_2} = \frac{1+(60\text{ °C})\,\gamma_{Fe}}{1+(60\text{ °C})\,\gamma_{Hg}}.$$

2. The three processes are plotted on a p–V diagram. AB is isobaric, AC is isothermal, and AD is adiabatic. In each case, the work done is equal to the area under the curve.



3. From the first law of thermodynamics, $Q = \Delta U + W$. For a cyclic process, $\Delta U = 0$. Also, $Q = 5$ J.

$W = W_{AB} + W_{BC} + W_{CA}$.

$W_{AB} = p\Delta V = (10 \text{ N m}^{-2})(1 \text{ m}^3) = 10$ J and $W_{BC} = 0$.

$\therefore \quad 5 \text{ J} = 10 \text{ J} + W_{CA} \qquad \text{or} \qquad W_{CA} = -5$ J.

4. $TV^{\gamma-1}$ = constant and γ for helium (a monatomic gas) is $\frac{5}{3}$.

$\therefore \quad T_1(5.6)^{2/3} = T_2(0.7)^{2/3}$, hence $T_2 = 4T_1$.

$$\therefore \quad W = \frac{nR}{\gamma-1}(T_2 - T_1) = \frac{1}{4}\frac{R}{2/3}(4T_1 - T_1) = \frac{9}{8}RT_1.$$

5. $\dfrac{V_{\text{rms(He)}}}{V_{\text{rms(A}_r)}} = \dfrac{\sqrt{3RT/m_{\text{He}}}}{\sqrt{3RT/m_{\text{Ar}}}} = \sqrt{\dfrac{m_{\text{Ar}}}{m_{\text{He}}}} = \sqrt{\dfrac{40}{4}} = \sqrt{10} = 3.16.$

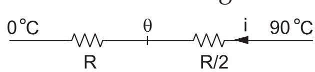
6. $\Delta Q = nC_{\text{P}}\Delta T = n\left(\dfrac{\gamma}{\gamma-1}R\right)\Delta T = 2\left(\dfrac{5}{2}R\right)\Delta T = 2\left(\dfrac{5}{2}\times\dfrac{25}{3}\right)\times 5\text{ J} = 208\text{ J}.$

7. At low pressures, the number of molecules per unit volume decreases, so that the 'point mass' approximation holds. Also, at high temperatures, the average speeds increase.

8. (Internal energy) $\propto$ (absolute temperature).
$\therefore\ U_{\text{A}} = U_{\text{B}}$.
In the isothermal process AB, $W = nRT \ln (V_{\text{f}}/V_{\text{i}}) = p_0V_0 \ln (V_{\text{B}}/V_{\text{A}})$.

9. $pV = nRT$. At constant pressure, $p\Delta V = nR(\Delta T)$.
Dividing, $\dfrac{\Delta V}{V} = \dfrac{\Delta T}{T}$ or $\delta = \dfrac{\Delta V}{V(\Delta T)} = \dfrac{1}{T}$ or $\delta \propto \dfrac{1}{T}$.

10. As the three rods are made of the same material and have the same dimensions, they have the same thermal resistance, say R. Let θ be the temperature of the junction.
The circuit becomes as shown in the figure below.

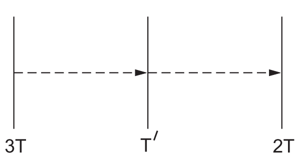


$$\therefore \qquad i = \frac{90\text{ °C} - \theta}{R/2} = \frac{\theta - 0\text{ °C}}{R}$$

or $\quad 180\text{ °C} - 2\theta = \theta \quad$ or $\quad \theta = 60\text{ °C}$.

11. A furnace behaves as a black body.

12. In the steady state, absorption and emission by the middle plate will be the same. Thus,



$$\sigma A(3T)^4 - \sigma A(T')^4 = \sigma A(T')^4 - \sigma A(2T)^4$$

$$\Rightarrow \qquad T' = \left(\frac{97}{2}\right)^{1/4} T.$$

13. By the gas laws: $\quad pV = nRT = \dfrac{m}{M}RT$

$$\Rightarrow \qquad pM = \frac{m}{V}RT = \rho RT$$

$$\Rightarrow \qquad \frac{\rho_1}{\rho_2} = \frac{p_1}{p_2}\times\frac{M_1}{M_2} = \frac{4}{3}\times\frac{2}{3} = \frac{8}{9}.$$

14. Let the thermal resistances in the two configurations be R_1 and R_2 respectively.

In Configuration I,

$$R_1 = \frac{L}{KA} + \frac{L}{2KA} = \frac{3L}{2KA}.$$

In Configuration II,

$$\frac{1}{R_2} = \frac{1}{\frac{L}{KA}} + \frac{1}{\frac{L}{2KA}} = \frac{3KA}{L}$$

$$\Rightarrow \quad R_2 = \frac{L}{3KA}.$$

$$\because \quad \Delta Q_1 = \Delta Q_2,$$

$$\therefore \quad \left(\frac{\Delta\theta}{R_1}\right)t_1 = \left(\frac{\Delta\theta}{R_2}\right)t_2$$

$$\Rightarrow \quad t_2 = \left(\frac{R_2}{R_1}\right)t_1 = 2 \text{ s}.$$

15. Heat generated = (3 kW) (3 h) = $3 \times 10^3 \times 3 \times 3600$ J = 324×10^5 J.

Heat absorbed by water = $mc_w\Delta\theta$ = (120)(4200)(20) J = 100.8×10^5 J.

Heat absorbed by the coolant,

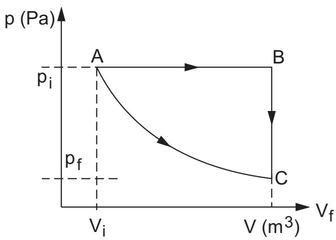
$$Pt = (324 \times 10^5 - 100.8 \times 10^5) \text{ J}$$

$$\Rightarrow \quad P = \frac{223.2 \times 10^5 \text{ J}}{3 \times 3600 \text{ s}} = 2067 \text{ W}.$$

16. Given that p^3V^5 = constant $\Rightarrow pV^{5/3}$ = constant.

$\therefore$ $\gamma = 5/3$ (for a monatomic gas).

For the isobaric process AB,



$$\Delta Q_1 = nc_p\,\Delta T = n\left(\frac{5}{2}R\right)\Delta T$$

$$= \frac{5}{2}p\,\Delta V$$

$$= \frac{5}{2}(10^5 \text{Pa})(7 \times 10^{-3} \text{m}^3)$$

$$= 1750 \text{ J (heat absorbed)}.$$

For the isochoric process BC,

$$\Delta Q_2 = nc_V\Delta T = n\left(\frac{3}{2}\right)R\,\Delta T$$

$$= \frac{3}{2}(Vdp) = \frac{3}{2}(8 \times 10^{-3} \text{ m}^3)\left(\frac{1}{32} - 1\right) \times 10^5 \text{ Pa}$$

$$= -1162.5 \text{ J (heat expelled)}.$$

So, the net heat supplied to the system is 1750 J + (–1162.5 J)

$$= 587.5 \text{ J} \approx 588 \text{ J}.$$

17. In the steady state,

the rate of energy loss = rate of energy incidence

$$\Rightarrow \quad \pi R^2 \cdot I = \sigma \cdot 4\pi R^2 \cdot (T^4 - T_0^4)$$

$$\Rightarrow \quad I = 4\sigma(T^4 - T_0^4).$$

$$\therefore \quad T^4 - T_0^4 = 40 \times 10^8\ \text{K}^4$$

$$\Rightarrow \quad T^4 - 81 \times 10^8\ \text{K}^4 = 40 \times 10^8\ \text{K}^4$$

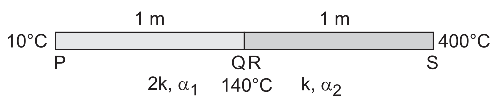
$$\Rightarrow \quad T^4 = 121 \times 10^8\ \text{K}^4$$

$$\Rightarrow \quad T \approx 330\ \text{K}.$$

18. For an element dx at a distance x from the end P, the change in length

$$dl = \alpha_1\, dx(\theta - 10) \text{ or } \Delta l = \int dl,$$

where θ is the temperature at the distance x from the end P.



$$\text{Temperature gradient in PQ} = \frac{\theta - 10}{x} = \frac{140 - 10}{1}$$

$$\Rightarrow \quad \theta = 130x + 10$$

$$\Rightarrow \quad \Delta l = \int_0^1 (130x)\alpha_1\, dx = 130\,\alpha_1 \frac{x^2}{2}\Big|_0^1 = 65 \times 1.2 \times 10^{-5}\ \text{m}$$

$$= 78.0 \times 10^{-5}\ \text{m} = 0.78\ \text{mm}.$$

19. Given that pT^2 = constant. But $pV = nRT$.

$$\therefore \quad \left(\frac{nRT}{V}\right)T^2 = \text{constant}.$$

$$\Rightarrow \quad T^3 = KV. \qquad \text{...(1)}$$

Differentiating, $3T^2dT = KdV$. ...(2)

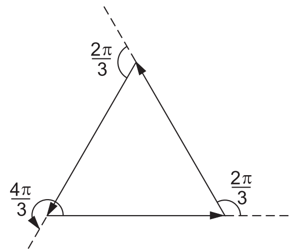
Dividing (2) by (1), $\dfrac{dV}{V} = \dfrac{3dT}{T}$.

$$\therefore \quad \gamma = \frac{1}{V}\frac{dV}{dT} = \frac{3}{T}.$$

1.3 Sound Waves

1. After 2 s, the pulses will overlap completely. The string becomes straight and therefore does not have any potential energy. Its entire energy must be kinetic.

2. Conclude from the vector triangle (equilateral).



3. A stationary wave has the equation of the form

$$y = A \sin kx \sin \omega t.$$

Here, for y_1, we have $k_1 = \frac{\pi}{L}$ and $\omega_1 = \omega$.

$$\therefore \qquad v_1 = \frac{\omega_1}{k_1} = \frac{\omega L}{\pi}.$$

For y_2, we have $k_2 = \frac{2\pi}{L}$ and $\omega_2 = 2\omega$.

$$\therefore \qquad v_2 = \frac{\omega_2}{k_2} = \frac{\omega L}{\pi} = v_1.$$

Thus, the wave velocities are the same in both the cases. Also, they have the same amplitude. The frequency for y_2 is twice the frequency for y_1.

Now, since energy $\propto$ (frequency)^2,

$$\therefore \qquad E_2 = 4E_1.$$

4. $\nu = \frac{1}{2l}\sqrt{\frac{T}{m}} = \frac{1}{2l}\sqrt{\frac{T}{A\rho}} = \frac{1}{2l}\sqrt{\frac{T}{\pi R^2 \rho}} = \frac{1}{2lR}\sqrt{\frac{T}{\pi\rho}}.$

As T and ρ are the same for both the wires, $\nu \propto \frac{1}{lR}$.

Here, $LR = L \times 2r = 2L \times r$. $\quad \therefore\ \nu_1 = \nu_2$.

5. The frequency of vibration of a string is given by

$$n = \frac{N}{2l}\sqrt{\frac{T}{m}},$$

where N = number of loops (or segments) = number of antinodes. The other symbols have their usual meanings.

Here, $\qquad n = \frac{5}{2l}\sqrt{\frac{9g}{m}} = \frac{3}{2l}\sqrt{\frac{Mg}{m}} \quad$ or $\quad M = 25$ kg.

6. $\frac{\lambda}{4} = L + e$

$\Rightarrow L = \frac{\lambda}{4} - e = \frac{v}{4f} - 0.6R = 16.4\text{ cm} - 1.2\text{ cm} = 15.2\text{ cm}.$

7. $\dfrac{V_{\text{sound}}}{2l_{\text{pipe}}} = 2\left(\dfrac{1}{2l_{\text{string}}}\right)\sqrt{\dfrac{T}{m}},$

mass of the string = ml_s = 10 g.

8. $n_o = \dfrac{n_s V}{V - v_s}.$

Here, $\quad f_1 = \dfrac{n_s V}{340 \text{ m s}^{-1} - 34 \text{ m s}^{-1}}$ and $f_2 = \dfrac{n_s V}{340 \text{ m s}^{-1} - 17 \text{ m s}^{-1}}.$

$\therefore \quad \dfrac{f_1}{f_2} = \dfrac{340 \text{ m s}^{-1} - 17 \text{ m s}^{-1}}{340 \text{ m s}^{-1} - 34 \text{ m s}^{-1}} = \dfrac{19}{18}.$

9. When an observer approaches a stationary source, which is emitting sound waves of frequency n_s, he hears a frequency n, given by

$$n = n_s\left(1 + \frac{v}{V}\right),$$

where v = velocity of the observer and V = velocity of sound.

Here, let v_A and v_B be the velocities of the trains A and B.

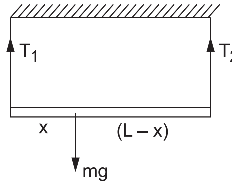
For A, we have $n = 5.5 \text{ kHz} = (5 \text{ kHz})\left(1 + \dfrac{v_A}{V}\right)$ or $\dfrac{v_A}{V} = 0.1.$

For B, we have $n = 6 \text{ kHz} = (5 \text{ kHz})\left(1 + \dfrac{v_B}{V}\right)$ or $\dfrac{v_B}{V} = 0.2.$

$\therefore \quad \dfrac{v_B}{v_A} = 2 \quad$ or $\quad v_B : v_A = 2 : 1.$

10. $f' = \left(\dfrac{v + v_o}{v - v_s}\right) f = \left(\dfrac{320 + 10}{320 - 10}\right)(8 \times 10^3 \text{ Hz}) = 8.5 \text{ kHz}.$

11. $T_1 + T_2 = mg,\ T_1 x = T_2 (L - x),$



$f_1 = \dfrac{1}{2l}\sqrt{\dfrac{T_1}{\mu}}$ and $f_2 = \dfrac{2}{2l}\sqrt{\dfrac{T_2}{\mu}},$

where μ = linear mass density and $f_1 = f_2$.

$\therefore \quad T_1 = 4T_2 \quad$ or $\quad T_1 x = \dfrac{T_1}{4}(L - x) \quad$ or $\quad x = \dfrac{L}{5}.$

12. The prongs of the tuning fork are kept in a vertical plane in order to set up longitudinal standing waves in the air column.

13. The frequency of the third harmonic of the closed pipe is

$$f = 3\left(\frac{v}{4l}\right) = \frac{3 \times 340 \text{ m s}^{-1}}{4 \times 0.75 \text{ m}} = 340 \text{ Hz}.$$

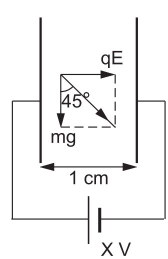
Since the increase in tension reduces the beat frequency from 4 s^{-1} to 2 s^{-1},

$\therefore \quad n - 340 \text{ Hz} = 4 \text{ Hz} \quad \Rightarrow \quad n = 344 \text{ Hz}.$

14.
$$y = A \sin(\omega t \pm \phi).$$
$$\because \quad A = 10 \text{ cm and } y = 5 \text{ cm},$$
$$\therefore \quad \sin(\omega t \pm \phi) = \frac{1}{2} = \sin 30°$$
$$\Rightarrow \quad \omega t \pm \phi = 30°.$$
Here,
$$\omega = \frac{2\pi}{T} = 2\pi\left(\frac{V}{\lambda}\right) = 2\pi\left(\frac{10 \text{ cm s}^{-1}}{50 \text{ cm}}\right) = \frac{2\pi}{5} \text{ rad s}^{-1}.$$
$\therefore$ the velocity of the point P is
$$v_P = \frac{dy}{dt} = A\omega\cos(\omega t \pm \phi) = (10 \text{ cm})\left(\frac{2\pi}{5} \text{ s}^{-1}\right)(\cos 30°)$$
$$= 2\pi\sqrt{3} \text{ cm s}^{-1} = \frac{\pi\sqrt{3}}{50} \text{ m s}^{-1}.$$
$$\therefore \quad \vec{v}_P = \frac{\pi\sqrt{3}}{50}\,\hat{j} \text{ m s}^{-1}.$$

1.4 Electrostatics

1. The tangent drawn to a line of force at any point on it must give the direction of the electric intensity at that point. Consider a point midway between any two of the charges. Here, the resultant intensity is only the intensity due to the third charge and must point away from the third charge. This is satisfied only in the sketch (c).
2. Only the option (a) has the dimensions of force.
3. $qE = \frac{4}{3}\pi r^3 \rho g = 6\pi\eta r v.$
4. Flux = (field) (projected area) = $E_0 a^2$.
5.
$$qE = mg$$
$$\Rightarrow \quad eE = m_P g$$
$$\Rightarrow \quad (1.602 \times 10^{-19} \text{ C})\,\frac{x}{10^{-2}\text{m}} = (1.672 \times 10^{-27} \text{ kg})\,(10 \text{ m s}^{-2})$$
$$\Rightarrow \quad x \approx 1 \times 10^{-9} \text{ V}.$$



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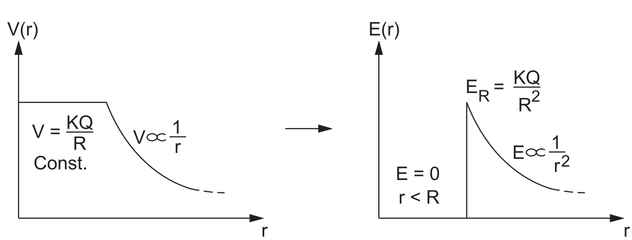


7. The electric field at a point outside the dielectric sphere is $E = \dfrac{Q}{4\pi\varepsilon_0 r^2}$ and that at a point inside is $E = E_{\text{surface}} \times \dfrac{r}{R}$, where r is the distance from the centre.

$\therefore\ E_1 = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{R^2},\ E_2 = \dfrac{1}{4\pi\varepsilon_0}\dfrac{2Q}{R^2}$ and $E_3 = \dfrac{1}{4\pi\varepsilon_0}\dfrac{4Q}{4R^2}\dfrac{R}{2R}$.

$\therefore\ E_1 : E_2 : E_3 = 2 : 4 : 1.$

8. The $\pm q$ charges appearing on the inner surfaces of A are bound charges. As B is without charge initially and is isolated, the charges on A will not be affected on closing the switch S. No charge will flow into B.

9. The initial energy of the system is given by $E_i = \frac{1}{2}CV_1^2 + \frac{1}{2}CV_2^2$.

When the capacitors are joined, they reach a common potential equal to $\dfrac{\text{total charge}}{\text{total capacity}} = \dfrac{CV_1 + CV_2}{2C} = \dfrac{V_1 + V_2}{2} = V.$

The final energy of the system is $E_f = \frac{1}{2}(2C)V^2$.

$\therefore$ the decrease in energy is $E_i - E_f = \frac{1}{4}C(V_1 - V_2)^2$.

10. Fractional loss in energy $= \dfrac{\Delta U}{U}$,

where $\Delta U = \dfrac{1}{2}\dfrac{C_1C_2}{C_1 + C_2}(V_1 - V_2)^2 = \dfrac{1}{2}\dfrac{2\times 8}{10}(V - 0)^2 = \dfrac{8}{10}V^2.$

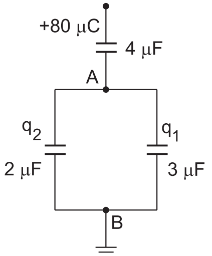
Initial energy $= U = \frac{1}{2}C_1V_1^2 = \frac{1}{2}(2)V^2 = V^2$.

$\therefore$ per cent dissipation $= \dfrac{\Delta U}{U} \times 100\%.$

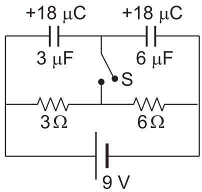
11. The potential difference across AB is

$$V = \frac{q_1}{C_1} = \frac{q_2}{C_2} = \frac{q_1 + q_2}{C_1 + C_2} = \frac{80\ \mu C}{5\ \mu F} = 16\ V.$$

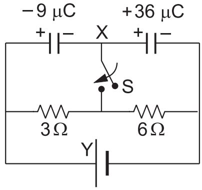
$$\therefore\quad q_1 = (16\ V)(3\ \mu F) = 48\ \mu C.$$



12. Consider the charge distribution when (i) S is open, (ii) S is closed.



(i)



(ii)

Applying the junction rule at X, the charge that flows from Y to X is (36 – 9) μC = 27μC.

13. If λ is the linear charge density given to the inner cylinder then according to Gauss's law,

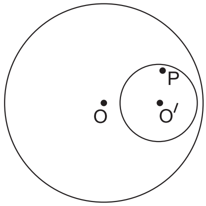
$$E = \frac{\lambda}{2\pi\varepsilon_0 r} \text{ for } a < r < b.$$

$$\therefore \text{ potential difference} = \Delta V = -\int \vec{E}\cdot\vec{dr} = \frac{\lambda}{2\pi\varepsilon_0}\ln\frac{b}{a}.$$

14. Solution not required.

15. By symmetry, the xy-plane through the origin is equipotential; so at the points A(–a, 0, 0) and B(0, a, 0), $V_A = V_B = 0$. Hence, $W = 0$.

16. Let P be a point in the cavity.



Let $\overrightarrow{OP} = \vec{r}_1$, $\overrightarrow{O'P} = \vec{r}_2$ and $\overrightarrow{OO'} = \vec{a}$.

The field at P due to the entire sphere is $\vec{E}_1 = \frac{\rho}{3\varepsilon_0}(\overrightarrow{OP})$, and that due to the cavity is $\vec{E}_2 = -\frac{\rho}{3\varepsilon_0}(\overrightarrow{O'P})$.

$$\therefore \text{ the net field at P is } \vec{E} = \vec{E}_1 + \vec{E}_2 = \frac{\rho}{3\varepsilon_0}(\overrightarrow{OP} - \overrightarrow{O'P}) = \frac{\rho}{3\varepsilon_0}(\overrightarrow{OP} + \overrightarrow{PO'})$$

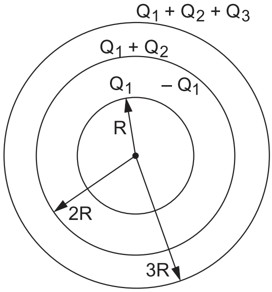
$$= \frac{\rho}{3\varepsilon_0}(\overrightarrow{OO'}) = \text{constant}.$$

Thus, the field in the cavity is nonzero and uniform.

17. $$F_{BC} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{\left(\frac{q}{3}\right)\left(\frac{2q}{3}\right)}{(2R\sin 60°)^2} = \frac{q^2}{54\pi\varepsilon_0 R^2}.$$

18. The charges enclosed in the cubical surface are (i) half the charge on the disc, (ii) the point charge at $\left(\frac{a}{4}, -\frac{a}{4}, 0\right)$, and (iii) the charge on the rod from $\frac{a}{4}$ to $\frac{a}{2}$, which is $\frac{8\,C}{4}$. Thus, the electric flux is $\phi = \frac{1}{\varepsilon_0}\Sigma q = \frac{1}{\varepsilon_0}\left[3\,C - 7\,C + \frac{8\,C}{4}\right] = -\frac{2\,C}{\varepsilon_0}$.

19. The charge distribution on the outer surfaces of the three concentric conducting shells are shown in the figure. For the charge densities to be equal,



$$\frac{Q_1}{4\pi R^2} = \frac{Q_1 + Q_2}{4\pi(2R)^2} = \frac{Q_1 + Q_2 + Q_3}{4\pi(3R)^2} = \text{constant}.$$

$\therefore\ Q_1 = K, Q_1 + Q_2 = 4K$ and $Q_1 + Q_2 + Q_3 = 9K$.

$\therefore\ Q_2 = 3K$ and $Q_3 = 5K$.

$\therefore\ Q_1 : Q_2 : Q_3 = 1 : 3 : 5$.

20. Effective capacitance $= C = \dfrac{\varepsilon_0 A}{d - y + \frac{y}{K}}$.

$\because\ A = 1$ and $K = 2$,

$$\therefore\ C = \frac{\varepsilon_0}{d - \frac{y}{2}}.$$

Now, $\dfrac{dy}{dt} = -V$

$$\Rightarrow \int_{d/3}^{y} dy = -\int_0^t V\,dt \quad \Rightarrow\ y = \frac{d}{3} - Vt.$$

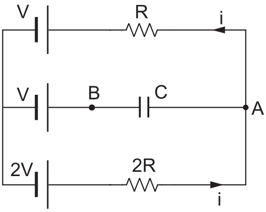
$$\therefore\quad C = \frac{\varepsilon_0}{d - \frac{1}{2}\left(\frac{d}{3} - Vt\right)} = \frac{\varepsilon_0}{\frac{5}{6}d + \frac{Vt}{2}} = \frac{6\varepsilon_0}{5d + 3Vt}.$$

$$\therefore\quad \text{time constant} = \tau = RC = \frac{6\varepsilon_0 R}{5d + 3Vt}.$$

1.5 Current Electricity and Magnetism

1. As the sheet is square, $R = \rho\frac{l}{A} = \rho\frac{L}{Lt} = \frac{\rho}{t}$.
2. G_1 acts as a voltmeter due to the high resistance R_1 in series. The small resistance R_2 in parallel with G_2 acts as a shunt.

3. Moving anticlockwise from A,



$$iR + V - 2V + 2iR = 0$$

or $\quad 3iR = V \quad$ or $\quad i = \dfrac{V}{3R}.$

Then, $V_A - V_B = iR + V - V = iR$

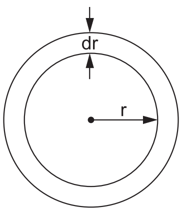
$$= \frac{V}{3} = \text{potential drop across C.}$$

4. As I is independent of R_6, no current flows through R_6. This requires that the junction of R_1 and R_2 is at the same potential as the junction of R_3 and R_4. This must satisfy the condition $R_1/R_2 = R_3/R_4$, as in the Wheatstone bridge.

5. $X(48\text{ cm} + 2\text{ cm}) = (10\ \Omega)\ (52\text{ cm} + 1\text{ cm})$

$$\Rightarrow \quad X = \frac{(10\ \Omega)\ (53\text{ cm})}{50\text{ cm}} = 10.6\ \Omega.$$

6. For an infinite line charge,



$$E = \frac{\lambda}{2\pi\varepsilon r} = -\frac{dV}{dr}$$

or $$dV = -\frac{\lambda}{2\pi\varepsilon}\frac{dr}{r}.$$

The current through the elemental cylindrical shell of radius r and thickness dr, is

$$I = \frac{|dV|}{dR} = \frac{\dfrac{\lambda}{2\pi\varepsilon}\dfrac{dr}{r}}{\dfrac{dr}{\sigma \cdot 2\pi r l}} = \frac{\sigma\lambda l}{\varepsilon}.$$

Since the current is radially outward,

$$I = \frac{d}{dt}(\lambda l) = -\frac{\lambda\sigma l}{\varepsilon} \Rightarrow \frac{d\lambda}{\lambda} = -\frac{\sigma}{\varepsilon}\,dt$$

$$\Rightarrow \quad \lambda = \lambda_0 e^{-\left(\frac{\sigma}{\varepsilon}\right)t}.$$

$$\therefore \quad j(t) = \frac{I}{A} = \frac{I}{2\pi r l} = \frac{\sigma\lambda}{2\pi\varepsilon r} = \frac{\sigma\lambda_0}{2\pi\varepsilon r}\,e^{-\left(\frac{\sigma}{\varepsilon}\right)t}.$$

This corresponds to the graph (a).

7. Power $= P = \dfrac{V^2}{R}.$

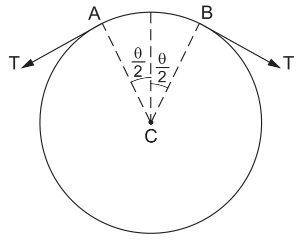
As resistance increases with temperature, the equality $\dfrac{1}{R_{100}} = \left(\dfrac{1}{R_{40}}\right) + \left(\dfrac{1}{R_{60}}\right)$ does not hold exactly. Hence, $\dfrac{1}{R_{100}} > \dfrac{1}{R_{60}} > \dfrac{1}{R_{40}}$ is the best option.

8. The effective current is $i = q \cdot \dfrac{\omega}{2\pi}$ and the area is $A = \pi r^2$.

The magnetic moment is $\mu = Ai = \dfrac{1}{2} q\omega r^2$.

The angular momentum is $L = I\omega = mr^2\omega$. $\therefore \dfrac{\mu}{L} = \dfrac{q}{2m}$.

9. $L = 2\pi r \quad \Rightarrow \quad r = \dfrac{L}{2\pi}$.



(Length of the element AB) $= dl = r\theta = \left(\dfrac{L}{2\pi}\right)\theta$.

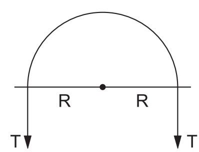
(Outward ampere force on AB) $\Rightarrow BI\,dl$.

(Inward force on AB due to tension T)

$= 2T \sin(\theta/2) = T\theta$.

Equating each of the two forces for equilibrium $T = \dfrac{BIL}{2\pi}$.

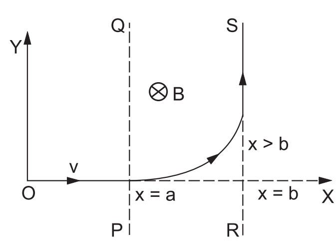
Alternative method



$2T = IB(2R) = IB\left(\dfrac{L}{\pi}\right)$

$\Rightarrow \quad T = \dfrac{IBL}{2\pi}$.

10. In the figure, the z-axis points out of the paper, and the magnetic field is directed into the paper, existing in the region between PQ and RS. The particle moves in a circular path of radius r in the magnetic field. It can just enter the region $x > b$ for $r \geq b - a$.



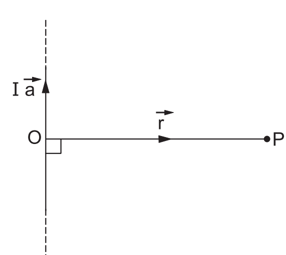
Now, $mv = Bqr$

or $\quad r = \dfrac{mv}{Bq} \geq b - a$

or $\quad v \geq \dfrac{q(b-a)B}{m}$

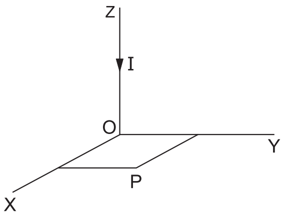
or $\quad v_{\text{min}} = \dfrac{q(b-a)B}{m}$.

11. To find the magnetic field at a point P due to a long straight wire carrying the current I, let $\vec{a}$ be a unit vector in the direction of current flow, and $\vec{r}$ be the vector, normal to $\vec{a}$, from the wire to P. Then the magnetic field at P is



$$\vec{B} = \frac{\mu_0 I}{2\pi} \cdot \frac{\vec{a} \times \vec{r}}{r^2}.$$

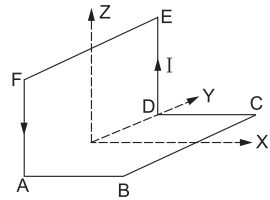
Here, $\vec{a} = -\vec{k}$ and $\vec{r} = x\vec{i} + y\vec{j}$.



$$\therefore \quad \vec{B} = \frac{\mu_0 I}{2\pi} \cdot \frac{(-\vec{k}) \times (x\vec{i} + y\vec{j})}{r^2}.$$

$$= \left(\frac{\mu_0 I}{2\pi}\right)\left(\frac{y\vec{i} - x\vec{j}}{x^2 + y^2}\right).$$

12. The conductors BC and EF will together produce a magnetic field at P, which lies in the xz-plane and is equally inclined to the x- and z-axes. The conductors AB and CD will together produce a magnetic field at P directed along the z-axis and of the magnitude B_0 (say). The conductors DE and FA will produce a field at P with the same magnitude B_0, and directed along the x-axis. Thus, the field at P will have equal components in the x- and z-directions, and no component in the y-direction. It must therefore point in the direction $(1/\sqrt{2})\,(\vec{i} + \vec{k})$.



13. The magnetic field lines due to a straight current constitute concentric circles. Hence, the magnetic field between the two wires act in opposite directions. Similarly, the magnetic field to the left of the first wire and that to the right of the second wire are oppositely directed. Hence the correct graph is (b).

14. Impedance $= Z = \left[R^2 + \left(\frac{1}{\omega C}\right)^2\right]^{1/2}$ and current $= I = \frac{V}{Z}$.

When ω increases, Z decreases and I increases.

As power $\propto I^2$, the bulb glows brighter.

15. In the electric field $\vec{E}$, force $= q\vec{E} \Rightarrow \vec{V} \propto q\vec{E} \Rightarrow \vec{V} \propto qE\hat{i}$.

In the magnetic field $\vec{B}$, force $\vec{F} = q(\vec{V} \times \vec{B}) \Rightarrow \vec{F} \propto q(qE\hat{i}) \times B\hat{k}$.

$$\Rightarrow \quad F \propto q^2 EB(-\hat{j}) \qquad [\because\ \hat{i} \times \hat{k} = -\hat{j}].$$

As $F \propto q^2$, so both the positive and the negative ions get deflected along the $-y$-direction.

16. The magnetic field strength at M due to QR is zero in all cases. When QS carries $I/2$ current, the field strength at M due to QS is $H_1/2$. Then

$$H_2 = H_1 + \frac{H_1}{2} = \frac{3H_1}{2} \quad \text{or} \quad \frac{H_1}{H_2} = \frac{2}{3}.$$

17. $B_0 = \int_a^b \frac{\mu_0 IN}{2r(b-a)}\, dr = \frac{\mu_0 IN}{2(b-a)} \int_a^b \frac{dr}{r} = \frac{\mu_0 IN}{2(b-a)} \ln \frac{b}{a}.$

18. Area of the current loop, $A = a^2 + 4 \times \frac{\pi(a/2)^2}{2} = a^2 + \frac{\pi a^2}{2}.$

$\therefore$ magnetic moment $= \vec{m} = I\vec{A} = I\left(\frac{\pi}{2} + 1\right)a^2 \hat{k}.$

19. In the cavity $\left(\text{where } r < \frac{R}{2}\right)$, $B = 0$.

Outside the cylinder (where $r > R$), $B = \frac{\mu_0 I}{2\pi r}.$

In the material of the cylinder $\left(\text{where } \frac{R}{2} < r < R\right)$,

$$B \cdot 2\pi r = \mu_0 \left[\pi r^2 - \pi\left(\frac{R}{2}\right)^2\right] J$$

$$\Rightarrow \qquad B = \frac{\mu_0 J}{2r}\left(r^2 - \frac{R^2}{4}\right).$$

For $\quad r = \frac{R}{2}$, $B = 0$; and for $r = R$,

$$B = \frac{\mu_0 J}{2R}\left(R^2 - \frac{R^2}{4}\right) = \frac{3}{8}\mu_0 JR.$$

Thus, $\quad B = 0$ for $r < \frac{R}{2}$;

$B = \frac{\mu_0 J}{2}\left(r - \frac{R^2}{4r}\right)$ ... for $\frac{R}{2} \le r < R$ (nonlinear increase);

and B decreases inversely with r for $r > R$.

20. The total magnetic field at the centre is

$$B = 12 \times \text{field due to each wire}$$

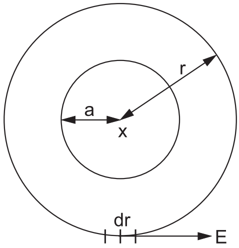
$$= 12\left(\frac{\mu_0 I}{4\pi a}\right)(\sin 60° - \sin 30°).$$

21. When S is closed, the clockwise current I_P flowing through P produces a magnetic field B directed from E to Q and then to P. This field B, by Lenz's law, causes an anticlockwise current I_{Q1} in Q. When S is opened, B decreases. This induces a clockwise current I_{Q2} in Q.

22. Construct a concentric circle of radius r. The induced electric field (E) at any point on this circle is equal to that at P. For this circle,

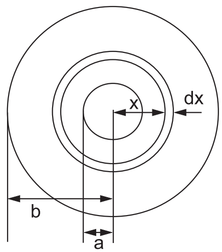
$$\oint \vec{E} \cdot d\vec{\ell} = 2\pi r E = \text{induced emf} = \frac{d\Phi}{dt} = \pi a^2 \dot{B}(t).$$

As the RHS is independent of r, we have $E \propto \frac{1}{r}$.



23. The number of turns per unit length is $\frac{N}{b-a}$.

Consider an elemental ring of radius x and width dx.



The number of turns in the ring is $dN = \frac{N\,dx}{b-a}$.

The magnetic field at the centre due to the ring is

$$dB = \frac{\mu_0 (dN) I}{2x} = \frac{\mu_0 I}{2} \cdot \frac{N\,dx}{b-a} \cdot \frac{1}{x}.$$

$\therefore$ the field at the centre is $\int dB = \frac{\mu_0 NI}{2(b-a)} \int_a^b \frac{dx}{x} = \frac{\mu_0 NI}{2(b-a)} \cdot \ln \frac{b}{a}$.

24. The pattern in the option (c) corresponds to a magnetic field (due to a straight steady current) as well as an electric field (inside a solenoid carrying a time-varying electric current).

25. For the circuit I, $\tau_1 = C_{eq} \times R_{eq} = \left(\frac{C_1 C_2}{C_1 + C_2}\right)\left(\frac{R_1 R_2}{R_1 + R_2}\right) = \frac{8}{6} \times \frac{2}{3} = \frac{8}{9}\,\mu\text{s}.$

For the circuit II, $\tau_2 = (C_1 + C_2)(R_1 + R_2) = 6 \times 3 = 18\,\mu\text{s}.$

For the circuit III, $\tau_3 = (C_1 + C_2)\left(\frac{R_1 R_2}{R_1 + R_2}\right) = 6 \times \frac{2}{3} = 4\,\mu\text{s}.$

26. $R = \rho\left(\frac{l}{A}\right)$; so $\frac{R_1}{R_2} = \frac{A_2}{A_1} = \frac{\pi r^2}{\pi 4 r^2} = \frac{1}{4}$.

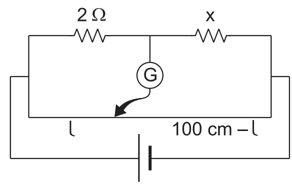
Ratio of power loss $= \frac{P_1}{P_2} = \frac{I^2 R_1}{I^2 R_2} = \frac{1}{4}$.

Ratio of voltage drop $= \frac{V_1}{V_2} = \frac{IR_1}{IR_2} = \frac{1}{4}$.

Ratio of current density $= \frac{J_1}{J_2} = \frac{I/A_1}{I/A_2} = \frac{A_2}{A_1} = \frac{1}{4}$.

Ratio of electric fields $= \frac{E_1}{E_2} = \frac{V_1/l_1}{V_2/l_2} = \frac{V_1}{V_2} = \frac{1}{4}$.

27. For balance,



$$\frac{2\,\Omega}{x} = \frac{l}{100\text{ cm} - l} \qquad ...(1)$$

and $$\frac{x}{2\,\Omega} = \frac{l + 20\text{ cm}}{80\text{ cm} - l}. \qquad ...(2)$$

Solving (1) and (2),

$$x = 3\,\Omega.$$

28. The magnetic force on a moving charge is given by $\vec{F} = q(\vec{v} \times \vec{B})$.

For $a < x < 2a$, $\vec{B} = B_0\hat{j}$. So, $\vec{F_1} = qvB(\hat{k})$

$\Rightarrow$ the path is circular.

Similarly, for $2a < x < 3a$, $\vec{B} = -B\hat{j}$.

$\therefore$ $\vec{F_2} = -qvB(\hat{k})$, and the correct graph is (a).

29. Power dissipation $= \dfrac{V^2}{R}\cdot$

The equivalent resistance in the three configurations are

$$R_1 = 1\,\Omega,\ R_2 = \frac{1}{2}\,\Omega \text{ and } R_3 = 2\,\Omega.$$

$$\therefore\ P_1 = \frac{V^2}{1},\ P_2 = \frac{2V^2}{1} \text{ and } P_3 = \frac{V^2}{2}\cdot$$

$$\therefore\ P_2 > P_1 > P_3.$$

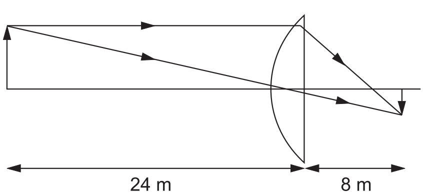
30. According to Lenz's law, a current will be induced in the anticlockwise direction so as to oppose the increasing magnetic flux into the paper. Thus, I_1 flows from A to B, and I_2 flows from D to C.

1.6 Ray Optics and Wave Optics

1. After the total internal reflection, there is no refracted ray.

2. The refractive index of the given lens material is

$$\mu = \frac{\lambda_a}{\lambda_m} = \frac{3}{2}\cdot$$



Now, $\dfrac{1}{f} = (\mu - 1)\dfrac{1}{R} = \dfrac{1}{2R}\cdot$

From the lens formula,

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\Rightarrow\quad \frac{1}{2R} = \frac{1}{8\text{ m}} - \frac{1}{-24\text{ m}} = \frac{1}{6\text{ m}}$$

$$\Rightarrow\quad R = 3\text{ m}.$$

3. If θ be the angle between the directions of the incident ray and the reflected ray then

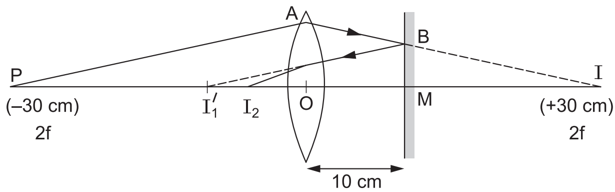
$$\cos\theta = \frac{1}{2}(\hat{i}+\sqrt{3}\hat{j})\cdot\frac{1}{2}(\hat{i}-\sqrt{3}\hat{j})$$

$$= \frac{1}{4}(1-3) = -\frac{1}{2} = \cos 120^\circ$$

$$\Rightarrow \qquad \theta = 120^\circ$$

$$\therefore \quad \text{angle of incidence} = i = \frac{180^\circ - \theta}{2} = 90^\circ - 60^\circ = 30^\circ.$$

4.



(i) The real image of P is at I, where OI = 30 cm and MI = 30 – 10 = 20 cm.

(ii) The incident ray AB gets reflected by the mirror and would meet the axis at I_1', where $I_1'M = MI = 20$ cm.

(iii) $I_1'O = I_1'M - OM = 20 - 10 = 10$ cm.

Finally, I_1' will act as a virtual object with $u = OI_1' = (+10 \text{ cm})$ for the convex lens.

For the lens formula, $u = OI_1' = +10$ cm,

$v = ?, f = +15$ cm.

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \quad \Rightarrow \quad \frac{1}{v}-\frac{1}{10}=\frac{1}{15}$$

$$\Rightarrow \qquad v = 6 \text{ cm} = OI_2.$$

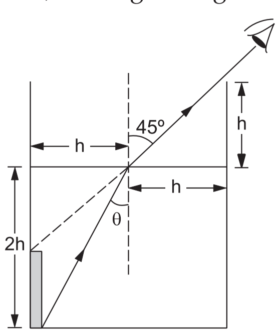
$\therefore$ distance from the mirror,

$$= I_2M = I_2O + OM = 6 \text{ cm} + 10 \text{ cm} = 16 \text{ cm}.$$

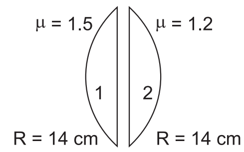
5. The line of sight of the observer remains constant, making an angle of 45° with the normal.

$$\sin\theta = \frac{h}{\sqrt{h^2+(2h)^2}} = \frac{1}{\sqrt{5}};$$

$$\mu = \frac{\sin 45^\circ}{\sin\theta} = \frac{1/\sqrt{2}}{1/\sqrt{5}} = \sqrt{\frac{5}{2}}.$$



6. From the lens-maker's formula,



$$\frac{1}{f_1} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$= (1.5 - 1)\left(\frac{1}{14} - \frac{1}{\infty}\right) = \frac{1}{28}.$$

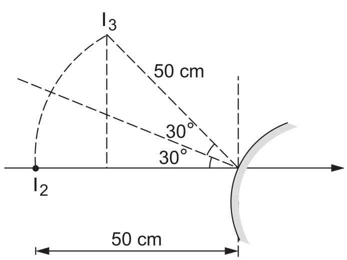
Similarly, $$\frac{1}{f_2} = (1.2 - 1)\left(\frac{1}{\infty} - \frac{1}{-14}\right) = \frac{1}{70}.$$

$$\Rightarrow \quad \frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{28} + \frac{1}{70} = \frac{1}{20}.$$

Now, from the lens formula,

$$\frac{1}{v} = \frac{1}{20\text{ cm}} - \frac{1}{40\text{ cm}} = \frac{1}{40\text{ cm}} \quad \Rightarrow v = 40\text{ cm}.$$

7. For the first image formed by the lens, u = –50 cm, f = +30 cm, and so v = 75 cm. This is at 25 cm to the right of the mirror.



For final image by the mirror,

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \quad \Rightarrow \frac{1}{25} + \frac{1}{v} = \frac{1}{50}$$

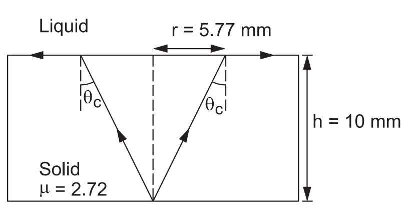
$\Rightarrow$ v = –50 cm, which is at I_2 when the mirror is straight. When tilted by 30°, the final image is at I_3, for which

$x = 50 - 50 \cos 60° = 25$ cm

$y = 50 \sin 60° = 25\sqrt{3}$ cm.

8. $$\tan\theta_c = \frac{r}{h} = \frac{5.77}{10} \approx \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \theta_c = 30°.$$



Now, $$\frac{\mu_l}{\mu_s} = \sin\theta_c$$

$$\Rightarrow \quad \mu_l = \sin 30° \times \mu_s = \frac{1}{2} \times 2.72$$

$$= 1.36.$$

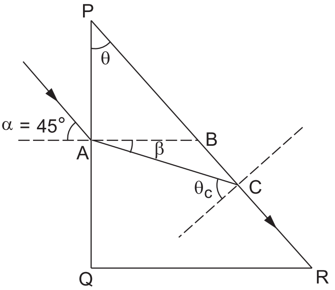
9. From Snell's law, $1\cdot(\sin 45°) = \sqrt{2}\sin\beta$

$\Rightarrow \quad \beta = 30°$. For critical angle θ_c,

$$\sin\theta_c = \frac{1}{\sqrt{2}} \text{ or } \theta_c = 45°.$$

Now, $\theta + (90° + \beta) + (90° - 45°) = 180°$,

$\Rightarrow \quad \theta = 45° - \beta = 15°$.



10. For a silvered lens, equivalent focal length

$$\frac{1}{f} = \frac{2}{f_l} + \frac{1}{f_m} = \frac{2}{15\text{ cm}} + \frac{1}{\infty} \quad \Rightarrow f = \frac{15}{2}\text{ cm.}$$

The silvered lens behaves as a concave mirror, so applying mirror formula:

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$\Rightarrow \frac{1}{-20\text{ cm}} + \frac{1}{v} = -\frac{2}{15\text{ cm}}$$

$$\Rightarrow \quad \frac{1}{v} = \frac{1}{20\text{ cm}} - \frac{2}{15\text{ cm}} = \frac{-5}{60\text{ cm}}$$

$$\Rightarrow \quad v = -12\text{ cm (left of AB).}$$

11. Let i be the angle of incidence at the interface of Regions III and IV. From Snell's law,

$$n_0 \sin\theta = \frac{n_0}{6}\sin i.$$

For TIR, $\quad i \geq i_c$ (critical angle)

$$\Rightarrow \quad \sin i \geq \frac{n_0/8}{n_0/6}$$

$$\text{or} \quad \sin i \geq \frac{3}{4}.$$

$$\therefore \quad n_0 \sin\theta = \frac{n_0}{6}\cdot\frac{3}{4} \quad \Rightarrow \sin\theta = \frac{1}{8}.$$

12.

$$v_{\text{ball}}^2 = 2 \times (10\text{ m s}^{-2})\,(7.2\text{ m})$$

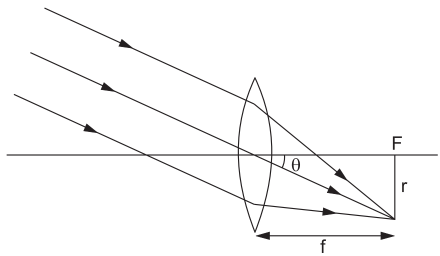
$$\Rightarrow \quad v_{\text{ball}} = 12\text{ m s}^{-1}.$$

To the fish, the image of the ball will appear as elevated, so

$$X_{\text{image of ball}} = \mu_w \times X_{\text{ball}}.$$

Differentiating, $\quad v_{\text{image}} = \frac{4}{3}(v_{\text{ball}}) = \frac{4}{3}(12\text{ m s}^{-1}) = 16\text{ m s}^{-1}.$

13. From the ray diagram,



$$\tan\theta = \frac{r}{f}$$

$$\Rightarrow \quad \pi r^2 = (\pi \tan^2\theta)\, f^2.$$

$$\therefore \quad \pi r^2 \propto f^2.$$

14. From the $u \sim v$ graph, $|u| = |v| = 10$ cm.

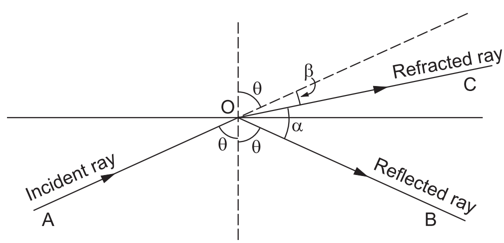
From lens formula, $f = 5.0$ cm.

The least count of the scale on the x- and y-axes is 0.1 cm.

$\Rightarrow$ The option is (5.00 ± 0.10) cm.

15. Due to parallax, shift of image is more than that of the object as observed. So the image is nearer to the eye and $v > u$. Hence, the object lies between centre and focus $\Rightarrow f < x < 2f$.

16. From the figure showing the incident ray (AO), reflected ray (OB) and refracted ray (OC), we have



$$\alpha + \beta = 180° - 2\theta$$

$$\Rightarrow \quad \alpha < (180° - 2\theta).$$

17. For minimum deviation $r_1 = r_2 = r = \dfrac{A}{2} = 30°$; which is same for all wavelengths.

18. In interference between waves of equal amplitudes a, the minimum intensity is zero and the maximum intensity is proportional to $4a^2$. For waves of unequal amplitudes a and A $(A > a)$, the minimum intensity is nonzero and the maximum intensity is proportional to $(a + A)^2$, which is greater than $4a^2$.

19. Fringe width $\beta = \dfrac{D\lambda}{d}$, so $B \propto \lambda \Rightarrow \beta_R > \beta_G > V_B$.

20. If I_0 = intensity of each coherent source then

$$I = 2I_0\,(1 + \cos\phi) \text{ and } I_{max} = 4I_0.$$

$$\text{Now, } I = \frac{I_{max}}{2} = 2I_0 = 2I_0(1 + \cos\phi), \quad \text{so } \cos\phi = 0$$

$$\Rightarrow \quad \phi = \frac{2\pi}{\lambda}\Delta = (2n+1)\frac{\pi}{2}, \text{ hence } \Delta = (2n+1)\frac{\lambda}{4}.$$

1.7 Modern Physics

1. $t = 100\ \text{ns} = 100 \times 10^{-9}\ \text{s}$, $P = 30\ \text{mW} = 30 \times 10^{-3}\ \text{W}$, $c = 3 \times 10^{8}\ \text{m s}^{-1}$.

$$\text{Momentum} = \frac{P \times t}{c} = \frac{(30 \times 10^{-3}\,W)\,(100 \times 10^{-9}\,\text{s})}{(3 \times 10^{8}\ \text{m s}^{-1})} = 1.0 \times 10^{-17}\ \text{m s}^{-1}.$$

2. For $\lambda_1 = 248\ \text{nm}$, $E_1 = \dfrac{hc}{\lambda} = \dfrac{1240\ \text{eV nm}}{248\ \text{nm}} = 5\ \text{eV}$.

For $\lambda_2 = 310\ \text{nm}$, $E_2 = \dfrac{1240\ \text{eV nm}}{310\ \text{nm}} = 4\ \text{eV}$.

$$\frac{(\text{KE})_1}{(\text{KE})_2} = \left(\frac{u_1}{u_2}\right)^2 = 4 = \frac{5\ \text{eV} - \phi_0}{4\ \text{eV} - \phi_0}$$

$\Rightarrow \quad \phi_0 = 3.7\ \text{eV}.$

3. $$\frac{hc}{\lambda} = \phi + eV_0 \qquad \text{or} \qquad hc\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right) = e\,(V_1 - V_2)$$

$$\Rightarrow \quad h = \frac{e\,(V_1 - V_2)\,\lambda_1\lambda_2}{c\,(\lambda_2 - \lambda_1)} = \frac{(1.6 \times 10^{-19}\,\text{C})\,(2\text{V} - 1\text{V})\,(0.3 \times 0.4) \times 10^{-12}\,\text{m}^2}{(3 \times 10^{8}\ \text{m s}^{-1})\,(0.4 - 0.3) \times 10^{-6}\,\text{m}}$$

$$= 6.4 \times 10^{-34}\ \text{J s}.$$

Alternatively: $V_0 = \dfrac{hc}{e\lambda} - \dfrac{\phi}{e}$, so the slope of $V_0 \sim \dfrac{1}{\lambda}$ graph is a constant $= \dfrac{hc}{e}$.

Thus, $$\frac{hc}{e} = \frac{(2\text{V} - 1\text{V})}{\left(\frac{1}{0.3} - \frac{1}{0.4}\right)} \times 10^{-6}\,\text{m} = 1.2 \times 10^{-6}\,\text{Vm}$$

$$\Rightarrow \quad h = \frac{e}{c}\,(1.2 \times 10^{-6}\ \text{Vm}) = 6.4 \times 10^{-34}\ \text{J s}.$$

4. In H-atom, $E_{\text{total}} = -\dfrac{K}{n^2}$, $\text{KE} = \dfrac{K}{n^2}$ and potential energy $= -\dfrac{2K}{n^2}$.

For transition from excited state ($n > 1$) to ground state ($n = 1$), kinetic energy of electron increases $\left(\dfrac{K}{n^2} \rightarrow K\right)$ whereas potential energy and total energy decrease (become more negative).

5. As the energy of the incident electrons is greater than the magnitude of the energy of the K-shell electrons, the target atoms will have vacancies in the K shell (K-shell electrons will be knocked out). This will cause emission of the entire characteristic spectrum of tungsten. Along with this, continuous X-rays will always be present, with

$$\lambda_c = \frac{1242\ \text{nm eV}}{80 \times 10^{3}\ \text{eV}} \cong 0.0155\ \text{nm}.$$

6. λ_K does not depend on the accelerating voltage. λ_c decreases with increase in accelerating voltage. Thus, as the accelerating voltage is increased, the difference $\lambda_K - \lambda_c$ will increase.

7. $$\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\Rightarrow \quad \frac{1}{6561} = R\left(\frac{1}{4} - \frac{1}{9}\right) = \frac{5R}{36}, \ \frac{1}{\lambda} = 4R\left(\frac{1}{4} - \frac{1}{16}\right) = \frac{3R}{4}.$$

8. $$\frac{\lambda_{Cu}}{\lambda_{Mo}} = \left(\frac{Z_{Mo} - 1}{Z_{Cu} - 1}\right)^2.$$

9. In a gamma decay process, there is no change in either A or Z.

10. Activity $A = A_0\left(\frac{1}{2}\right)^{t/T}$

or $$\frac{A_0}{64} = A_0\left(\frac{1}{2}\right)^{t/T}$$

or $$\left(\frac{1}{2}\right)^6 = \left(\frac{1}{2}\right)^{t/18\text{ days}} \quad \Rightarrow t = 108 \text{ days}.$$

11. $$\frac{hc}{\lambda} = \phi_0 + \text{KE}_{\text{max}} = \phi_0 + \frac{p^2}{2m} = \phi_0 + \frac{h^2}{2m\lambda_d^2}$$

$$\Rightarrow \quad \frac{h^2}{2m\lambda_d^2} = \frac{hc}{\lambda} - \phi_0.$$

Differentiating, $-\dfrac{3h^2\, d\lambda_D}{2m\lambda_D^3} = -\dfrac{hc}{\lambda^2}d\lambda$

$$\Rightarrow \quad \frac{d\lambda_D}{d\lambda} \propto \frac{\lambda_D^3}{\lambda^2}.$$

12. Consider the reaction ${}_0^1n + {}_8^{15}\text{O} \rightarrow {}_7^{15}\text{N} + {}_1^1\text{H} + \text{Q}$,

so $Q = (\Delta M)c^2 = (M_n + M_O - M_N - M_p)\, c^2$

$= 0.003796 \times 931.5 = 3.5359$ MeV. ...(1)

From the given relation

$$\Delta Q = \Delta E = E_{{}_8^{15}\text{O}} - E_{{}_7^{15}\text{N}}$$

$$= \frac{3}{5}\frac{e^2}{4\pi\varepsilon_0 R}(8 \times 7 - 7 \times 6)$$

$$= \frac{3}{5}(1.44 \text{ MeV}\cdot\text{fm}) \times \frac{14}{R} \qquad ...(2)$$

Solving equations (1) and (2), $R = 3.42$ fm.

13. Probability P for the decay is

$$P = 1 - e^{-\lambda t}$$

For $$t = 2T_{1/2} = 2\left(\frac{\ln 2}{\lambda}\right)$$

$$P = 1 - e^{-\lambda \cdot 2\left(\frac{\ln 2}{\lambda}\right)}$$

$$= 1 - e^{-\ln 4} = 1 - \frac{1}{4} = \frac{3}{4}.$$

14. From Einstein's photoelectric equation

$$\frac{hc}{\lambda} = \phi + eV_s$$

$$\Rightarrow \quad V_s = \left(\frac{hc}{e}\right)\frac{1}{\lambda} - \frac{\phi}{e}.$$

This represents a straight line with slope $\tan\theta = \dfrac{hc}{e}$.

Work function $$\phi = h\nu_0 = \frac{hc}{\lambda_0}.$$

For metal 1, $$\frac{1}{\lambda_{01}} = \frac{\phi_1}{hc} = 0.001 \text{ nm}^{-1}.$$

For metal 2, $$\frac{1}{\lambda_{02}} = \frac{\phi_2}{hc} = 0.002 \text{ nm}^{-1}.$$

For metal 3, $$\frac{1}{\lambda_{03}} = \frac{\phi_3}{hc} = 0.004 \text{ nm}^{-1}.$$

$$\therefore \quad \phi_1 : \phi_2 : \phi_3 = 1 : 2 : 4.$$

$$\lambda_{01} = 1000 \text{ nm}.$$

$$\lambda_{02} = \frac{1000}{2} \text{ nm} = 500 \text{ nm}.$$

$$\lambda_{03} = \frac{1000}{4} \text{ nm} = 250 \text{ nm}.$$

$\because \lambda_v \le 400$ nm, photoelectrons can be ejected by metal 1 and metal 2 only and not by metal 3.

15. During a nuclear reaction, the rest mass energy of the main nucleus (U) must be greater than the rest mass energy of daughter nuclei in which it breaks up (as the conservation of momentum is always followed).

16. The largest wavelength (λ_1) in UV region of H-spectrum is the first line in Lyman series for the transition $n = 2$ to $n = 1$:

$$\frac{1}{122} = R\left(1 - \frac{1}{4}\right) = \frac{3}{4}R. \qquad \text{...(1)}$$

The smallest wavelength (λ_2) in IR region is the last line in Paschen series for the transition $n = \infty$ to $n = 3$:

$$\frac{1}{\lambda_2} = R\left(\frac{1}{9} - \frac{1}{\infty}\right) = \frac{R}{9}. \qquad \text{...(2)}$$

$\therefore$ from (1) and (2), $\lambda_2 = 823.5$ nm.

17. For X-ray emission, cut-off wavelength

$$\lambda_0 = \frac{hc}{eV} \qquad \text{...(1)}$$

But $$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2m\,(eV)}} \quad \Rightarrow eV = \frac{h^2}{2m\lambda^2}.$$

Equating eV, $$\frac{hc}{\lambda_0} = \frac{h^2}{2m\lambda^2}$$

$\Rightarrow$ $$\lambda_0 = \frac{2m\,c\lambda^2}{h}.$$

18. $\lambda_{\text{cut off}} = \frac{hc}{eV}$, which is independent of the atomic number (Z)

$\Rightarrow$ option (b) is wrong.

19. $$A = \left|\frac{dN}{dt}\right| = \lambda N = \frac{\ln 2}{T} N$$

$\Rightarrow$ $$5\,\mu\,C_i = \frac{\ln 2}{T_1} \cdot 2N_0$$

$$10\,\mu\,C_i = \frac{\ln 2}{T_2} \cdot N_0$$

$\Rightarrow$ $$T_1 = 4T_2.$$

20. Energy of photon corresponding to wavelength λ is

$$E = \frac{hc}{\lambda} = \frac{1240 \text{ eV nm}}{\lambda}.$$

$\therefore$ $$\lambda_1 = 550 \text{ nm}, E_1 = \frac{1240}{550} \text{ eV} = 2.25 \text{ eV}$$

$$\lambda_2 = 450 \text{ nm}, E_2 = \frac{1240}{450} \text{ eV} = 2.75 \text{ eV}$$

$$\lambda_3 = 350 \text{ nm}, E_3 = \frac{1240}{350} \text{ eV} = 3.5 \text{ eV}$$

Since emission occurs when $E > \phi$,

$\phi_p = 2$ eV, all the three components cause photoemission

$\phi_q = 2.5$ eV, last two cause emission

$\phi_r = 3$ eV, only the last causes emission

$\therefore$ $I_p > I_q > I_r$.

❑

2

Multiple-Choice Questions: Type 2

Each question in this chapter has ***four*** *options (a, b, c and d).* ***One or more*** *options may be correct.*

2.1 General Physics

1. Consider (i) a vernier callipers in which each 1 cm on the main scale is divided into 8 equal divisions and (ii) a screw gauge with 100 divisions on its circular scale. In the vernier callipers 5 divisions of the vernier scale coincides with 4 divisions on the main scale and in the screw gauge, one complete rotation of the circular scale moves it by two divisions on the linear scale. Now, which of the following statements is/are correct?

   (a) If the pitch of the screw gauge is twice the least count of the vernier callipers, the least count of the screw gauge is 0.01 mm.

   (b) If the pitch of the screw gauge is twice the least count of the vernier callipers, the least count of the screw gauge is 0.005 mm.

   (c) If the least count of the linear scale of the screw gauge is twice the least count of the vernier callipers, the least count of the screw gauge is 0.01 mm.

   (d) If the least count of the linear scale of the screw gauge is twice the least count of the vernier callipers, the least count of the screw gauge is 0.005 mm.

2. A student uses a simple pendulum of exactly 1-m length to determine g, the acceleration due to gravity. He uses a stopwatch with the least count of 1 second for this and records 40 seconds for 20 oscillations. For this observation, which of the following statements is/are true?

(a) The error ΔT in measuring T, the time period, is 0.05 second.
(b) The error ΔT in measuring T, the time period, is 1 second.
(c) The percentage error in the determination of g is 5.
(d) The percentage error in the determination of g is 2.5.

3. In an experiment to determine the acceleration due to gravity g, the formula used for the time period of a periodic motion is $T = 2\pi\sqrt{\dfrac{7(R-r)}{5g}}\cdot$ The values of R and r are measured to be (60 ± 1) mm and (10 ± 1) mm respectively. In five successive measurements, the time period is found to be 0.52 s, 0.56 s, 0.54 s, 0.59 s and 0.57 s. The least count of the watch used for measurement of time period is 0.01 s. Which of the following statements is/are true?

(a) The error in the measurement of r is 10%.
(b) The error in the measurement of T is 3.57%.
(c) The error in the measurement of T is 2%.
(d) The error in the measurement of g is 11%.

4. Planck constant h, speed of light c and gravitational constant G are used to form a unit of length L and a unit of mass M. Then the correct option(s) is/are

(a) $M \propto \sqrt{c}$ (b) $M \propto \sqrt{G}$
(c) $L \propto \sqrt{h}$ (d) $L \propto \sqrt{G}$

5. The length-scale (l) depends on the permittivity (ε) of the dielectric material, Boltzmann constant (k_B), the absolute temperature (T), the number per unit volume (n) of a certain charged particle, and the charge (q) carried by each of the particles. Which of the following expressions for l is/are dimensionally correct?

(a) $l = \sqrt{\dfrac{nq^2}{\varepsilon k_B T}}$ (b) $l = \sqrt{\dfrac{\varepsilon k_B T}{nq^2}}$

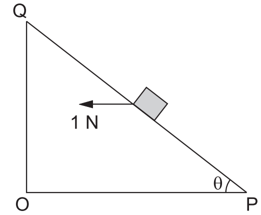
(c) $l = \sqrt{\dfrac{q^2}{\varepsilon n^{2/3} k_B T}}$ (d) $l = \sqrt{\dfrac{q^2}{\varepsilon n^{1/3} k_B T}}$

6. In terms of potential difference V, electric current I, permittivity ε_0, permeability μ_0 and speed of light c, the dimensionally correct equation(s) is/are

(a) $\mu_0 I^2 = \varepsilon_0 V^2$ (b) $\varepsilon_0 I = \mu_0 V$

(c) $I = \varepsilon_0 c V$ (d) $\mu_0 c I = \varepsilon_0 V$

7. A small block of mass 0.1 kg lies on a fixed inclined plane PQ which makes an angle θ with the horizontal. A horizontal force of 1 N is applied on this block through its centre of mass as shown in the figure. The block remains stationary if (take $g = 10$ m s^{-2})



(a) $\theta = 45°$

(b) $\theta > 45°$ and a frictional force acts on the block towards P

(c) $\theta > 45°$ and a frictional force acts on the block towards Q

(d) $\theta < 45°$ and a frictional force acts on the block towards Q

8. A flat plate is moving normal to its plane through a gas under the action of a constant force F. The gas is kept at a very low pressure. The speed v of the plate is much less than the average speed u of the gas molecules. Which of the following options is/are true?

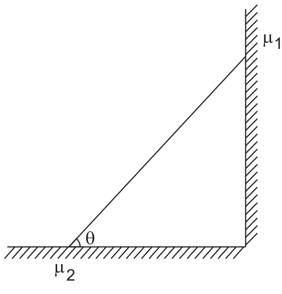
(a) The resistive force experienced by the plate is proportional to v.

(b) The pressure difference between the leading and trailing faces of the plate is proportional to uv.

(c) The plate will continue to move with constant nonzero acceleration at all times.

(d) At a later time, the external force F balances the resistive force.

9. In the figure, a ladder of mass m is shown leaning against a wall. It is in static equilibrium, making an angle θ with the horizontal floor. The coefficient of friction between the wall and the ladder is μ_1 and that between the floor and the ladder is μ_2. The normal reaction of the wall on the ladder is $\mathcal{N}_1$ and that of the floor is $\mathcal{N}_2$. If the ladder is about to slip then



(a) $\mu_1 = 0, \mu_2 \neq 0$ and $\mathscr{N}_2 \tan\theta = \dfrac{mg}{2}$

(b) $\mu_1 \neq 0, \mu_2 = 0$ and $\mathscr{N}_1 \tan\theta = \dfrac{mg}{2}$

(c) $\mu_1 \neq 0, \mu_2 = 0$ and $\mathscr{N}_2 = \dfrac{mg}{1+\mu_1\mu_2}$

(d) $\mu_1 = 0, \mu_2 \neq 0$ and $\mathscr{N}_1 \tan\theta = \dfrac{mg}{2}$

10. A particle of mass m is attached to one end of a massless spring of force constant k, lying on a frictionless horizontal plane. The other end of the spring is fixed. The particle starts moving horizontally from its equilibrium position at time $t = 0$ with an initial velocity u_0. When the speed of the particle is $0.5u_0$, it collides elastically with a rigid wall. After this collision,

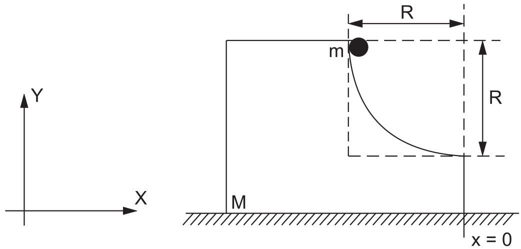
(a) the speed of the particle when it returns to its equilibrium position is u_0

(b) the time at which the particle passes through the equilibrium position for the first time is $t = \pi\sqrt{\dfrac{m}{k}}$

(c) the time at which the maximum compression in the spring occurs is $t = \dfrac{4\pi}{3}\sqrt{\dfrac{m}{k}}$

(d) the time at which the particle passes through the equilibrium position for the second time is $t = \dfrac{5\pi}{3}\sqrt{\dfrac{m}{k}}$

11. A block of mass M has a circular cut with a frictionless surface as shown. The block rests on a horizontal frictionless surface of a fixed table. Initially the right edge of the block is at $x = 0$, in a coordinate system fixed to the table. A point mass m is released from rest at the topmost point of the path as shown and it slides down. When the mass loses contact with the block, its position is at x and the velocity is v. At that instant, which of the following options is/are correct?



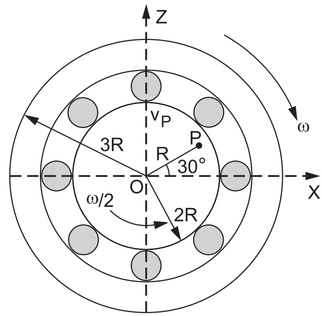
(a) The x-component of displacement of the centre of mass of the block M is $-\dfrac{mR}{M+m}$.

(b) The position of the point mass is $x = -\sqrt{2}\dfrac{mR}{M+m}$.

(c) The velocity of the point mass m is $v = \sqrt{\dfrac{2gR}{1+\dfrac{m}{M}}}$.

(d) The velocity of the block M is $v = -\dfrac{m}{M}\sqrt{2gR}$.

12. The figure shows a system consisting of (i) a ring of outer radius $3R$ rolling clockwise without slipping on a horizontal surface with angular speed ω and (ii) an inner disc of radius $2R$ rotating anticlockwise with angular speed $\dfrac{\omega}{2}$. The ring and the disc are separated by frictionless ball bearings. The system is in the xz-plane. The point P on the inner disc is at a distance R from the origin, where OP makes an angle of 30° with this horizontal. Then with respect to this horizontal surface



(a) the point O has linear velocity $3R\omega\hat{i}$

(b) the point P has linear velocity $\left(\dfrac{11}{4}R\omega\hat{i} + \dfrac{\sqrt{3}}{4}R\omega\hat{k}\right)$

(c) the point P has linear velocity $\left(\dfrac{13}{4}R\omega\hat{i} - \dfrac{\sqrt{3}}{4}R\omega\hat{k}\right)$

(d) the point P has linear velocity $\left(\left(3 - \dfrac{\sqrt{3}}{4}\right)R\omega\hat{i} + \dfrac{1}{4}R\omega\hat{k}\right)$

13. The position vector $\vec{r}$ of a particle of mass m is given by the equation $\vec{r}(t) = \alpha t^3\hat{i} + \beta t^2\hat{j}$, when $\alpha = \dfrac{10}{3}\,\mathrm{m\,s^{-3}}$, $\beta = 5\,\mathrm{m\,s^{-2}}$ and $m = 0.1$ kg. At $t = 1$ s, which of the following statements is/are true about the particle?

(a) The velocity $\vec{v}$ is given by $\vec{v} = (10\hat{i} + 10\hat{j})\,\mathrm{m\,s^{-1}}$.

(b) The angular momentum $\vec{L}$ with respect to the origin is given by $\vec{L} = -\frac{5}{3}\hat{k}$ N m s.

(c) The force $\vec{F}$ is given by $\vec{F} = (\hat{i} + 2\hat{j})$ N.

(d) The torque $\vec{\tau}$ with respect to the origin is given by $\vec{\tau} = -\left(\frac{20}{3}\right)\hat{k}$ N m.

14. Two solid cylinders P and Q of the same mass and same radius start rolling down a fixed inclined plane from the same height at the same time. The cylinder P has most of its mass concentrated near its surface, while the cylinder Q has most of its mass concentrated near the axis. Which of the following statements is/are correct?

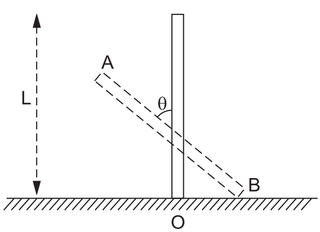
(a) Both the cylinders P and Q reach the ground at the same time.

(b) The cylinder P has larger linear acceleration than the cylinder Q.

(c) Both the cylinders reach the ground with the same translational kinetic energy.

(d) The cylinder Q reaches the ground with large angular speed.

15. A rigid uniform bar AB of length L is slipping from its vertical position on a frictionless floor (as shown in the figure). At some instant of time, the angle made by the bar with the vertical is θ. Which of the following statements about the motion is/are correct?



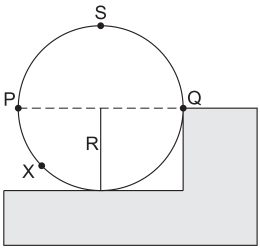
(a) The trajectory of the point A is a parabola.

(b) The instantaneous torque about the point in contact with the floor is proportional to sin θ.

(c) The mid-point of the bar will fall vertically downwards.

(d) When the bar makes an angle θ with the vertical, the displacement of its mid-point from the initial position is proportional to (1 – cos θ).

16. A wheel of radius R and mass M is placed at the bottom of a fixed step of height R as shown in the figure. A constant force is continuously applied on the surface of the wheel so that it just climbs the step without slipping. Consider the torque τ about an axis normal to the plane of the paper passing through the point Q. Which of the following options is/are correct?



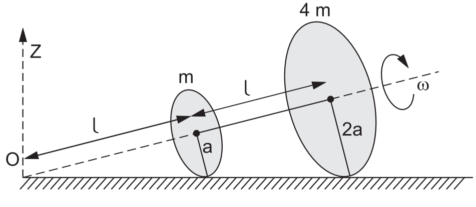
(a) If the force is applied tangentially at point X then $\tau \neq 0$ but the wheel never climbs the step.

(b) If the force is applied normal to the circumference at point P then τ is zero.

(c) If the force is applied normal to the circumference at point X then τ is constant.

(d) If the force is applied at point P tangentially then τ decreases continuously as the wheel climbs.

17. Two thin circular discs of mass m and $4m$, having radii of a and $2a$ respectively, are tightly fixed by a massless rigid rod of length $l = \sqrt{24}\, a$ through their centres. This assembly is laid on a firm and flat surface, and set rolling without stopping on the surface so that the angular speed about the axis of the rod is ω. The angular momentum of the entire assembly about the point O is $\vec{L}$ (see the figure). Which of the following statements is/are true?



(a) The magnitude of the angular momentum of the assembly about its centre of mass is $17ma^2\omega/2$.

(b) The magnitude of the z-component of $\vec{L}$ is $55ma^2\omega$.

(c) The magnitude of angular momentum of centre of mass of the assembly about the point O is $81ma^2\omega$.

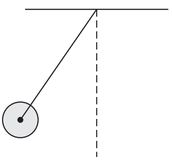
(d) The centre of mass of the assembly rotates about the z-axis with an angular speed of $\omega/5$.

18. Two spherical planets P and Q have the same uniform density ρ, masses M_P and M_Q and surface areas A and $4A$ respectively. A spherical planet R also has uniform density ρ and its mass is $(M_P + M_Q)$. The escape velocities from the planets P, Q and R are v_P, v_Q and v_R respectively. Then

(a) $v_Q > v_R > v_P$ (b) $v_R > v_Q > v_P$

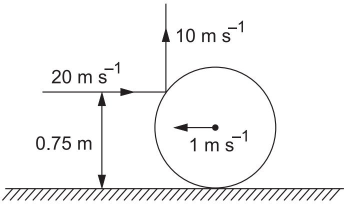
(c) $\dfrac{v_R}{v_P} = 3$ (d) $\dfrac{v_P}{v_Q} = \dfrac{1}{2}$

19. A metal rod of length L and mass m is pivoted at one end. A thin disc of mass M and radius R ($< L$) is attached at its centre to the free end of the rod. Consider the two ways the disc is attached. Case A: the disc is not free to rotate about its centre. Case B: the disc is free to rotate about its centre. The rod–disc system performs SHM in a vertical plane after being released from the same displaced position. Which of the following statements is/are true?



(a) Restoring torque in Case A = restoring torque in Case B
(b) Restoring torque in Case A < restoring torque in Case B
(c) Angular frequency in Case A > angular frequency in Case B
(d) Angular frequency in Case A < angular frequency in Case B

20. A thin ring of mass 2 kg and radius 0.5 m is rolling without slipping on a horizontal plane with velocity 1 m s^{-1}. A small ball of mass 0.1 kg moving with velocity 20 m s^{-1} in the opposite direction hits the ring at a height of 0.75 m and goes vertically up with a velocity of 10 m s^{-1}. Immediately after the collision



(a) the ring has pure rotation about the stationary CM
(b) the ring comes to a complete stop
(c) friction between the ring and the ground is to the left
(d) there is no friction between the ring and the ground

21. Two spherical bodies, each of mass M, are kept with their centres at a distance $2L$ apart. A particle of mass m is projected from the mid-point of the line joining their centres, perpendicular to the line. The gravitational constant is G. Which of the following statements is/are true?

(a) The minimum initial velocity of the mass m to escape the gravitational field of the two bodies is $4\sqrt{\dfrac{GM}{L}}$.

(b) The minimum initial velocity of the mass m to escape the gravitational field of the two bodies is $2\sqrt{\dfrac{GM}{L}}$.

(c) The minimum initial velocity of the mass m to escape the gravitational field of the two bodies is $\sqrt{\dfrac{2GM}{L}}$.

(d) The total mechanical energy of the mass m remains constant.

22. A spherical body of radius R consists of a fluid of constant density and is in equilibrium under its own gravity. If $p(r)$ is the pressure at $r(r < R)$ then the correct option(s) is/are

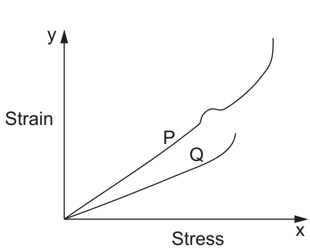
(a) $p(r = 0) = 0$

(b) $\dfrac{p\left(r = \frac{3R}{4}\right)}{p\left(r = \frac{2R}{3}\right)} = \dfrac{63}{80}$

(c) $\dfrac{p\left(r = \frac{3R}{5}\right)}{p\left(r = \frac{2R}{5}\right)} = \dfrac{16}{21}$

(d) $\dfrac{p\left(r = \frac{R}{2}\right)}{p\left(r = \frac{R}{3}\right)} = \dfrac{20}{27}$

23. While plotting stress versus strain curves for two materials P and Q, a student by mistake puts strain on the y-axis and stress on the x-axis as shown in the figure. Which of the following statements is/are correct?



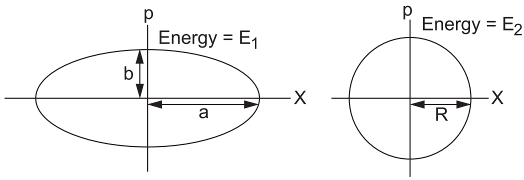
(a) P has more tensile strength than Q.

(b) P is more ductile than Q.

(c) P is more brittle than Q.

(d) The Young modulus of P is more than that of Q.

24. Two independent harmonic oscillators of equal mass are oscillating about the origin with angular frequencies ω_1 and ω_2 and have total energies E_1 and E_2 respectively. The variations of their momenta p with positions x are shown in the figure.



If $\frac{a}{b} = n^2$ and $\frac{a}{R} = n$ then the correct equation(s) is/are

(a) $E_1\omega_1 = E_2\omega_2$

(b) $\frac{\omega_2}{\omega_1} = n^2$

(c) $\omega_1\omega_2 = n^2$

(d) $\frac{E_1}{\omega_1} = \frac{E_2}{\omega_2}$

25. A block of mass M is connected by a massless spring with stiffness constant K to a rigid wall and moves without friction on a horizontal surface. The block oscillates with a small amplitude A about an equilibrium position x_0. Consider two cases: (i) when the block is at x_0, and (ii) when the block is at $x = x_0 + A$. In both the cases, a particle of mass m ($< M$) is gently placed on the block after which they stick to each other. Which of the following statements is/are true about the motion after the mass m is placed on the mass M?

(a) The amplitude of oscillation in the first case changes by a factor of $\sqrt{\frac{M}{M+m}}$, whereas in the second case it remains unchanged.

(b) The final time period of oscillation in both the cases is same.

(c) The total energy decreases in both the cases.

(d) The instantaneous speed at x_0 of the combined masses decreases in both the cases.

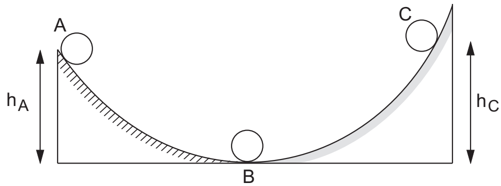
26. A solid cylinder of mass m and radius r is rolling on a rough inclined plane of inclination θ. The coefficient of friction between the cylinder and the incline is μ. Then

(a) frictional force is always $\mu mg\cos\theta$

(b) friction is a dissipative force

(c) by decreasing θ, frictional force decreases
(d) friction opposes translation and supports rotation

27. A ball moves on a fixed track as shown in the figure. From A to B, the ball rolls without slipping. Surface BC is frictionless. K_A, K_B and K_C are kinetic energies of the ball at A, B and C respectively. Then

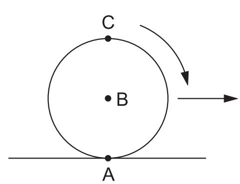


(a) $h_A > h_C$; $K_B > K_C$
(b) $h_A > h_C$; $K_C > K_A$
(c) $h_A = h_C$; $K_B = K_C$
(d) $h_A < h_C$; $K_B > K_C$

28. If the resultant of all the external forces acting on a system of particles is zero then from an inertial frame, one can surely say that

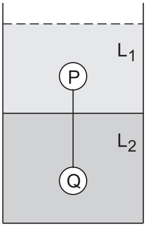
(a) the linear momentum of the system does not change in time
(b) the kinetic energy of the system does not change in time
(c) the angular momentum of the system does not change in time
(d) the potential energy of the system does not change in time

29. A sphere is rolling without slipping on a fixed horizontal plane surface. In the figure, A is the point of contact, B is the centre of the sphere and C is its topmost point. Then,



(a) $\vec{v}_C - \vec{v}_A = 2(\vec{v}_B - \vec{v}_C)$
(b) $\vec{v}_C - \vec{v}_B = \vec{v}_B - \vec{v}_A$
(c) $|\vec{v}_C - \vec{v}_A| = 2|\vec{v}_B - \vec{v}_C|$
(d) $|\vec{v}_C - \vec{v}_A| = 4|\vec{v}_B|$

30. Two spheres P and Q of equal radii have densities ρ_1 and ρ_2 respectively. The spheres are connected by a massless string and placed in liquids L_1 and L_2 of densities σ_1 and σ_2 and viscosities η_1 and η_2 respectively. They float in equilibrium with sphere P in L_1 and sphere Q in L_2, the string being taut (see the figure). If the sphere P alone in L_2 has terminal velocity $\vec{v}_P$ and the sphere Q alone in L_1 has terminal velocity $\vec{v}_Q$ then



(a) $\frac{|\vec{v}_P|}{|\vec{v}_Q|} = \frac{\eta_1}{\eta_2}$ (b) $\frac{|\vec{v}_P|}{|\vec{v}_Q|} = \frac{\eta_2}{\eta_1}$

(c) $\vec{v}_P \cdot \vec{v}_Q > 0$ (d) $\vec{v}_P \cdot \vec{v}_Q < 0$

31. A solid sphere of radius R and density ρ is attached to one end of a massless spring of force constant k. The other end of the spring is connected to another solid sphere of radius R and density 3ρ. The complete arrangement is placed in a liquid of density 2ρ and is allowed to reach equilibrium. Now, which of the following statements is/are correct?

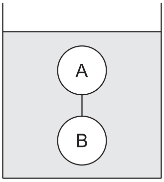
(a) The net elongation of the spring is $\frac{4\pi R^3 \rho g}{3k}$.

(b) The net elongation of the spring is $\frac{8\pi R^3 \rho g}{3k}$.

(c) The lighter sphere is partly submerged.

(d) The lighter sphere is completely submerged.

32. Two solid spheres A and B of equal volumes but of different densities d_A and d_B are connected by a string. They are fully immersed in a fluid of density d_F. They get arranged into an equilibrium as shown in the figure with a tension in the string. The arrangement is possible only if



(a) $d_A < d_F$ (b) $d_B > d_F$

(c) $d_A > d_F$ (d) $d_A + d_B = 2d_F$

2.2 Heat and Thermodynamics

1. A container of fixed volume has a mixture of one mole of hydrogen and one mole of helium in equilibrium at temperature T. Assuming the gases to be ideal, which of the following statements is/are correct?

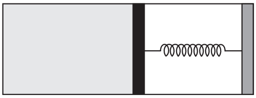
(a) The average energy per mole of the gas mixture is $2RT$.

(b) The ratio of the speed of sound in the gas mixture to that in the helium gas is $\sqrt{\frac{6}{5}}$.

(c) The ratio of the rms speed of helium atoms to that of the hydrogen molecules is $\frac{1}{2}$.

(d) The ratio of the rms speed of helium atoms to that of the hydrogen molecules is $\frac{1}{\sqrt{2}}$.

2. An ideal monatomic gas is confined in a horizontal cylinder by a spring-loaded piston (as shown in the figure). Initially the gas is at temperature T_1, pressure p_1 and volume V_1 and the spring is in its relaxed state. The gas is then heated very slowly to temperature T_2, pressure p_2 and volume V_2. During this process the piston moves out by a distance x. Now, which of the following statements is/are correct? (Ignore the friction between the piston and the cylinder.)



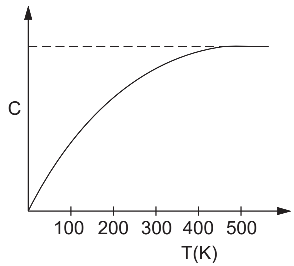
(a) If $V_2 = 2V_1$ and $T_2 = 3T_1$ then the energy stored in the spring is $\frac{1}{4}p_1V_1$.

(b) If $V_2 = 2V_1$ and $T_2 = 3T_1$ then the change in internal energy is $3p_1V_1$.

(c) If $V_2 = 3V_1$ and $T_2 = 4T_1$ then the work done by the gas is $\frac{7}{3}p_1V_1$.

(d) If $V_2 = 3V_1$ and $T_2 = 4T_1$ then the heat supplied to the gas is $\frac{17}{6}p_1V_1$.

3. The variation of specific heat capacity (C) of a solid as a function of temperature (T) is shown in the figure. The temperature is increased continuously from 0 to 500 K at a constant rate. Ignoring any volume change, find which of the following statements is/are correct to a reasonable approximation.

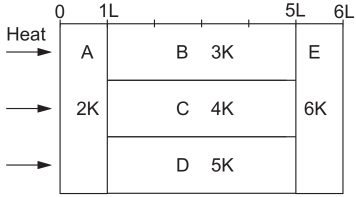


(a) The rate at which heat is absorbed in the range 0–100 K varies linearly with temperature.
(b) Heat absorbed in increasing the temperature from 0 to 100 K is less than the heat required for increasing the temperature from 400 to 500 K.
(c) There is no change in the rate of heat absorption in the range 400–500 K.
(d) The rate of heat absorption increases in the range 200–300 K.

4. A human body has a surface area of approximately 1 m^2. The normal body temperature is 10 K above the surrounding room temperature T_0. Take the room temperature T_0 to be 300 K. For T_0 = 300 K, the value of σT_0 = 460 W m^{-2} (where σ is the Stefan–Boltzmann constant). Which of the following options is/are correct?

(a) The amount of energy radiated by the body in 1 second is close to 60 J.
(b) If the surrounding temperature reduces by a small amount $\Delta T_0 << T_0$ then to maintain the same body temperature the same (living) human being needs to radiate $\Delta W = 4\sigma T_0^3 \Delta T_0$ more energy per unit time.
(c) Reducing the exposed surface area of the body (e.g., by curling up) allows humans to maintain the same body temperature by reducing the energy lost by radiation.
(d) If the body temperature rises significantly then the peak in the spectrum of electromagnetic radiation emitted by the body would shift to longer wavelengths.

5. A composite block is made of slabs A, B, C, D and E of different thermal conductivities (given in terms of a constant K) and sizes (given in terms of length L) as shown in the figure. All slabs are of the same width. Heat Q flows only from left to right through the blocks. Then in the steady state,



(a) heat flow through slabs A and E are same
(b) heat flow through slab E is maximum
(c) temperature difference across slab E is smallest
(d) heat flow through C = heat flow through B + heat flow through D

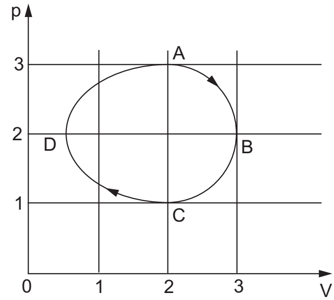
6. In a dark room with ambient temperature T_0, a black body is kept at a temperature T. Keeping the temperature of the black body constant (at T), sunrays are allowed to fall on the black body through a hole in the roof of the dark room. Assuming that there is no change in the ambient temperature of the room, which of the following statements is/are true?

(a) The quantity of radiation absorbed by the black body in unit time will increase.
(b) Since emissivity = absorptivity, the quantity of radiation emitted by the black body in unit time will increase.
(c) The black body radiates more energy in unit time in the visible spectrum.
(d) The reflected energy in unit time by the black body remains the same.

7. C_V and C_p denote the molar heat capacities of a gas at constant volume and constant pressure respectively. Then,

(a) $C_p - C_V$ is larger for a diatomic ideal gas than for a monatomic ideal gas
(b) $C_p + C_V$ is larger for a diatomic ideal gas than for a monatomic ideal gas
(c) C_p / C_V is larger for a diatomic ideal gas than for a monatomic ideal gas
(d) $C_p \cdot C_V$ is larger for a diatomic ideal gas than for a monatomic ideal gas

8. The figure shows the p-V plot of an ideal gas taken through a cycle ABCDA. The part ABC is a semicircle and CDA is half of an ellipse. Then,



(a) the process during the path A → B is isothermal

(b) heat flows out of the gas during the path B → C → D

(c) work done during the path A → B → C is zero

(d) positive work is done by the gas in the cycle ABCDA

2.3 Sound Waves

1. A horizontally stretched string fixed at two ends is vibrating in its fifth harmonic according to the equation

$$y(x, t) = (0.01 \text{ m}) \sin[(62.8 \text{ m}^{-1})x] \cos[(628 \text{ s}^{-1})t].$$

Assuming $\pi = 3.14$, which of the following statements is/are correct?

(a) The number of nodes is 5.

(b) The length of the string is 0.25 m.

(c) The maximum displacement of the mid-point of the string from its equilibrium position is 0.01 m.

(d) The fundamental frequency is 100 Hz.

2. Two loudspeakers M and N are located 20 m apart and emit sound at frequencies 118 Hz and 121 Hz respectively. A car is initially at a point P, 1800 m away from the mid-point Q of the line MN and moves towards Q constantly at 60 km h^{-1} along the perpendicular bisector of MN. It crosses Q and eventually reaches the point R, 1800 m away from Q. Let $\nu(t)$ represent the beat frequency measured by a person sitting in the car at time t. Let ν_P, ν_Q and ν_R be the beat frequencies measured at locations P, Q and R respectively.

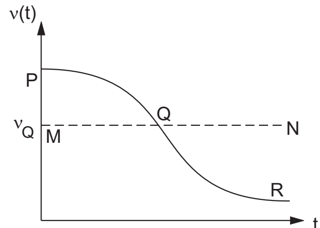


Fig. A

The speed of sound in air is 330 m s^{-1}. Which of the following statements is/are true regarding the sound heard by the person?

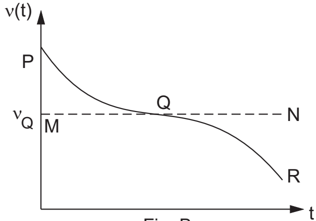


Fig. B

(a) Figure A represents schematically the variation of the beat frequency with time.

(b) Figure B represents schematically the variation of beat frequency with time.

(c) The rate of change of beat frequency is maximum when the car passes through Q.

(d) $\nu_P + \nu_B = 2\nu_Q$.

3. Two vehicles, each moving with speed u on the same horizontal straight road, are approaching each other. Wind blows along the road with velocity w. One of these vehicles blows a whistle of frequency f_1. An observer in the other vehicle hears the frequency of the whistle to be f_2. The speed of sound in still air is v. Identify the correct statement(s).

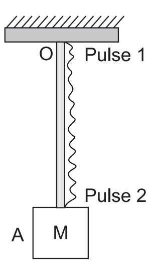
(a) If the wind blows from the observer to the source then $f_2 > f_1$.

(b) If the wind blows from the source to the observer then $f_2 > f_1$.

(c) If the wind blows from the observer to the source then $f_2 < f_1$.

(d) If the wind blows from the source to the observer then $f_2 < f_1$.

4. A block M hangs vertically from the bottom end of a uniform rope of constant mass per unit length. The top end of the rope is attached to a fixed rigid support at O. A transverse wave pulse (Pulse 1) of wavelength λ_O is produced at point O on the rope. The pulse takes time T_{OA} to reach point A. If the wave pulse of wavelength λ_O is produced at point A (Pulse 2) without disturbing the position of M, it takes time T_{AO} to reach the point O. Which of the following options is/are correct?



(a) The time $T_{AO} = T_{OA}$.
(b) The velocities of the two pulses (Pulse 1 and Pulse 2) are the same at the mid-point of the rope.
(c) The wavelength of Pulse 1 becomes longer when it reaches point A.
(d) The velocity of any pulse along the rope is independent of its frequency and wavelength.

5. One end of a taut string of length 3 m along the x-axis is fixed at $x = 0$. The speed of the waves in the string is 100 m s^{-1}. The other end of the string is vibrating in the y-direction so that stationary waves are set up in the string. The possible waveform(s) of these stationary waves is/are

(a) $y(t) = A\sin\frac{\pi x}{6}\cos\frac{50\pi t}{3}$ (b) $y(t) = A\sin\frac{\pi x}{3}\cos\frac{100\pi t}{3}$

(c) $y(t) = A\sin\frac{5\pi x}{6}\cos\frac{250\pi t}{3}$ (d) $y(t) = A\sin\frac{5\pi x}{2}\cos 250\pi t$

6. A student is performing an experiment using a resonance column and a tuning fork of frequency 244 s^{-1}. He is told that the air in the tube has been replaced by another gas (assume that the column remains filled with the gas). If the minimum height at which resonance occurs is (0.350 ± 0.005) m, the gas in the tube is (useful information: $\sqrt{167\,RT} = 640\,J^{1/2}\,mol^{-1/2}$; $\sqrt{140\,RT} = 590\,J^{1/2}\,mol^{-1/2}$, the molar mass M in grams is given in each option; take the value of $\sqrt{\frac{10}{M}}$ for each gas as given there)

(a) neon $\left(M = 20, \sqrt{\frac{10}{20}} = \frac{7}{20}\right)$ (b) nitrogen $\left(M = 28, \sqrt{\frac{10}{28}} = \frac{3}{5}\right)$

(c) oxygen $\left(M = 32, \sqrt{\frac{10}{32}} = \frac{9}{16}\right)$ (d) argon $\left(M = 36, \sqrt{\frac{10}{36}} = \frac{17}{32}\right)$

7. A person blows into the open end of a long pipe. As a result, a high-pressure pulse of air travels down the pipe. When this pulse reaches the other end of the pipe

(a) a high-pressure pulse starts travelling up the pipe, if the other end of the pipe is open

(b) a low-pressure pulse starts travelling up the pipe, if the other end of the pipe is open
(c) a low-pressure pulse starts travelling up the pipe, if the other end of the pipe is closed
(d) a high-pressure pulse starts travelling up the pipe, if the other end of the pipe is closed

8. Function $X = A\sin^2 \alpha x + B\cos^2 \alpha x + C\sin \alpha x \cos \alpha x$ represents SHM

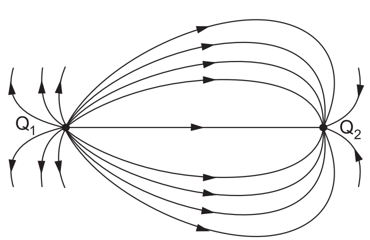
(a) for any value of A, B and C (except $C = 0$)
(b) if $A = -B$, $C = 2B$, amplitude = $|B\sqrt{2}|$
(c) if $A = B$, $C = 0$
(d) if $A = B$, $C = 2B$, amplitude = $|B|$

9. A student performed the experiment to measure the speed of sound in air using resonance air-column method. Two resonances in the air column were obtained by lowering the water level. The resonance with the shorter air column is the first resonance and that with the longer air column is the second resonance. Then,

(a) the intensity of the sound heard at the first resonance was more than that at the second resonance
(b) the prongs of the tuning fork were kept in a horizontal plane above the resonance tube
(c) the amplitude of vibration of the ends of the prongs is typically around 1 cm
(d) the length of the air column at the first resonance was somewhat shorter than 1/4th of the wavelength of the sound in air

2.4 Electrostatics

1. A few electric field lines for a system of two charges Q_1 and Q_2 fixed at two different points on the x-axis are shown in the figure. These lines suggest that



(a) $|Q_1| > |Q_2|$ (b) $|Q_1| < |Q_2|$

(c) at a finite distance to the left of Q_1 the electric field is zero

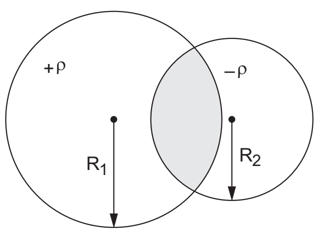
(d) at a finite distance to the right of Q_2 the electric field is zero

2. A spherical metal shell A of radius R_A and a solid metal sphere B of radius R_B ($< R_A$) are kept far apart and each is given charge $+Q$. If they are connected by a thin metal wire then

(a) $E_A^{\text{inside}} = 0$ (b) $Q_A > Q_B$

(c) $\dfrac{\sigma_A}{\sigma_B} = \dfrac{R_B}{R_A}$ (d) $E_A^{\text{on surface}} < E_B^{\text{on surface}}$

3. Two nonconducting spheres of radii R_1 and R_2 and carrying uniform volume charge densities $+\rho$ and $-\rho$ respectively are placed such that they partially overlap, as shown in the figure. At all points in the overlapping region,



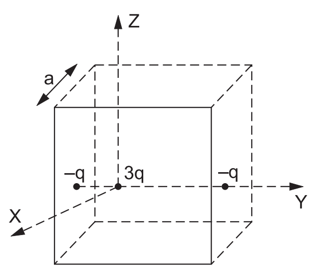
(a) the electrostatic field is zero

(b) the electrostatic potential is constant

(c) the electrostatic field is constant in magnitude

(d) the electrostatic field has the same direction

4. A cubical region of side a has its centre at the origin. It encloses three fixed point charges: $-q$ at $\left(0, -\frac{a}{4}, 0\right)$, $+3q$ at $(0, 0, 0)$ and $-q$ at $\left(0, +\frac{a}{4}, 0\right)$. Choose the correct option(s).



(a) The net electric flux crossing the plane $x = +\frac{a}{2}$ is equal to the net electric flux crossing the plane $x = -\frac{a}{2}$.

(b) The net electric flux crossing the plane $y = +\frac{a}{2}$ is more than the net electric flux crossing the plane $y = -\frac{a}{2}$.

(c) The net electric flux crossing the entire region is $\frac{q}{\varepsilon_0}$.

(d) The net electric flux crossing the plane $z = +\frac{a}{2}$ is equal to the net electric flux crossing the plane $x = +\frac{a}{2}$.

5. Two nonconducting solid spheres of radii R and $2R$, having uniform volume charge densities ρ_1 and ρ_2 respectively touch each other. The net electric field at a distance $2R$ from the centre of the smaller sphere, along the line joining the centres of the spheres is zero. The ratio ρ_1/ρ_2 can be

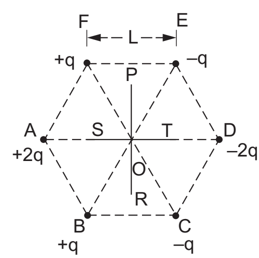
(a) -4

(b) $-\frac{32}{25}$

(c) $\frac{32}{25}$

(d) 4

6. Six point charges are kept at the vertices of a regular hexagon of side L and centre O, as shown in the figure. Given that $K = \frac{1}{4\pi\varepsilon_0}\frac{q}{L^2}$, which of the following statements is/are correct?



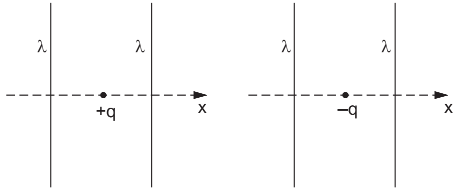
(a) The electric field at O is $6K$ along OD.

(b) The potential at O is zero.

(c) The potential at all points on the line PR is the same.

(d) The potential at all points on the line ST is the same.

7. The figures below depict two situations, in each of which two infinitely long static line charges of constant positive line charge density λ are kept parallel to each other. In their resulting electric fields, point charges $+q$ and $-q$ are kept in equilibrium between the lines. The point charges are confined to move in the x-direction only. If they are given a small displacement about their equilibrium positions then which of the following statements is/are correct?



(a) Both charges execute simple harmonic motion.

(b) Both charges will continue to move in the directions of their displacements.

(c) Charge $+q$ executes SHM while charge $-q$ continues to move in the direction of its displacement.

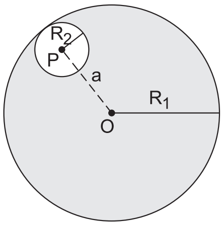
(d) Charge $-q$ executes SHM while charge $+q$ continues to move in the direction of its displacement.

8. Let $E_1(r)$, $E_2(r)$ and $E_3(r)$ be the respective electric fields at a distance r from a point charge Q, an infinitely long wire with a constant linear charge density λ, and an infinite plane with a uniform surface charge density σ. If $E_1(r_0) = E_2(r_0) = E_3(r_0)$ at a given distance r_0 then

(a) $Q = 4\sigma\pi r_0^2$ (b) $r_0 = \dfrac{\lambda}{2\pi\sigma}$

(c) $E_1\left(\dfrac{r_0}{2}\right) = 2E_2\left(\dfrac{r_0}{2}\right)$ (d) $E_2\left(\dfrac{r_0}{2}\right) = 4E_3\left(\dfrac{r_0}{2}\right)$

9. Consider a uniform spherical charge distribution of radius R_1, centred at the origin O. In this distribution, an eccentric spherical cavity of radius R_2, centred at P with distance OP $= a = R_1 - R_2$ (see the figure) is made. If the electric field inside the cavity at position $\vec{r}$ is $\vec{E}(\vec{r})$ then which of the following statements is/are correct?



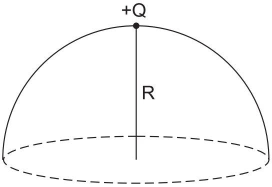
(a) $\vec{E}$ is uniform, its magnitude is independent of R_2 but its direction depends on $\vec{r}$.

(b) $\vec{E}$ is uniform, its magnitude depends on R_2 but its direction depends on $\vec{r}$.

(c) $\vec{E}$ is uniform, its magnitude is independent of a but its direction depends on $\vec{a}$.

(d) $\vec{E}$ is uniform and both its magnitude and direction depends on $\vec{a}$.

10. A point charge $+Q$ is placed just outside an imaginary hemispherical surface of radius R as shown in the figure. Which of the following statements is/are correct?



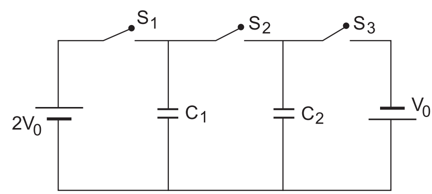
(a) The electric flux passing through the curved surface of the hemisphere is $-\frac{Q}{2\varepsilon_0}\left(1-\frac{1}{\sqrt{2}}\right)$.

(b) The component of electric field normal to the flat surface is constant over the surface.

(c) The total flux through the curved and the flat surfaces is $\frac{Q}{\varepsilon_0}$.

(d) The circumference of the flat surface is all equipotential.

11. In the circuit shown in the figure, there are two parallel plate capacitors, each of capacitance C. The switch S_1 is pressed first to fully charge the capacitor C_1 and then released. The switch S_2 is then pressed to charge the capacitor C_2. After a pause, S_2 is released and then S_3 is pressed. After some time



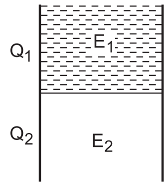
(a) the charge on the upper plate of C_1 is $2CV_0$

(b) the charge on the upper plate of C_1 is CV_0

(c) the charge on the upper plate of C_1 is 0

(d) the charge on the upper plate of C_2 is $-CV_0$

12. A parallel plate capacitor has a dielectric slab of dielectric constant K between the plates that covers $\frac{1}{3}$rd of the area of its plates as shown in the figure. The total capacitance of the capacitor is C while that of the portion with dielectric in between is C_1. When the capacitor is charged, the plate area covered by the dielectric gets charge Q_1 and the rest of the area gets charge Q_2. The electric field in the dielectric is E_1 and that on the other portion is E_2. Choose the correct option(s), ignoring the edge effects.



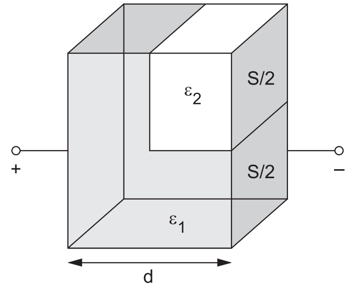
(a) $\frac{E_1}{E_2}=1$

(b) $\frac{E_1}{E_2}=\frac{1}{K}$

(c) $\frac{Q_1}{Q_2}=\frac{3}{K}$

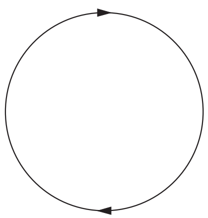
(d) $\frac{C}{C_1}=\frac{2+K}{K}$

13. A parallel plate capacitor having plates of area A and plate separation d, has capacitance C_1 in air. When two dielectrics of different relative permittivities ($\varepsilon_1 = 2$ and $\varepsilon_2 = 4$) are introduced between the two plates as shown in the figure, the capacitance becomes C_2. The ratio C_2/C_1 is



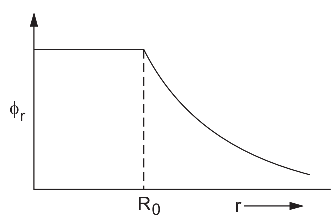
(a) $\frac{6}{5}$ (b) $\frac{5}{3}$ (c) $\frac{7}{5}$ (d) $\frac{7}{3}$

14. In the given diagram, a field-line of a particular force-field is shown. It can never represent



(a) an electrostatic field
(b) a magnetostatic field
(c) a gravitational field of a mass at rest
(d) an induced electric field

15. The electrostatic potential (ϕ_r) of a spherical symmetric system kept at origin is shown in the adjacent figure, and given as



$$\phi_r = \frac{q}{4\pi\varepsilon_0 r} \cdots (r \geq R_0),$$
$$\phi_r = \frac{q}{4\pi\varepsilon_0 R_0} \cdots (r \leq R_0).$$

Which of the following options is/are correct?

(a) For spherical region $r \leq R_0$, the total electrostatic energy stored is zero.
(b) Within $r = 2R_0$, the total charge is q.
(c) There will be no charge anywhere except at $r = R_0$.
(d) The electric field is discontinuous at $r = R_0$.

16. Under the influence of the coulomb field of charge $+Q$, a charge $-q$ is moving around it in an elliptical orbit. Find out the correct statement(s).

(a) The angular momentum of the charge $-q$ is constant

(b) The linear momentum of the charge $-q$ is constant

(c) The angular velocity of the charge $-q$ is constant

(d) The linear speed of the charge $-q$ is constant

2.5 Current Electricity and Magnetism

1. An incandescent bulb has a thin filament of tungsten that is heated to a high temperature by passing an electric current. The hot filament emits black-body radiation. The filament is observed to break up at random locations after a sufficiently long time of operation due to nonuniform evaporation of tungsten from the filament. If the bulb is powered at a constant voltage, which of the following statements is/are true?

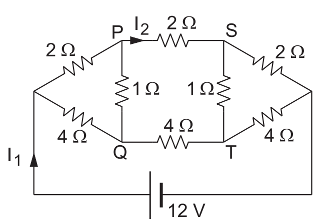
(a) The temperature distribution over the filament is uniform.

(b) The resistance over small sections of the filament decreases with time.

(c) The filament emits more light at higher band frequencies before it breaks up.

(d) The filament consumes less electrical power towards the end of the life of the bulb.

2. For the resistance network shown in the figure, choose the correct option(s).



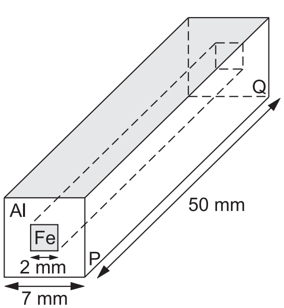
(a) The current through PQ is zero.

(b) $I_1 = 3A$.

(c) The potential at S is less than that at Q.

(d) $I_2 = 2A$.

3. In an aluminium (Al) bar of square cross section, a square hole is drilled and is filled with iron (Fe) as shown in the figure. The electrical resistivities of Al and Fe are 2.7×10^{-8} Ω m and 1.0×10^{-7} Ω m respectively. The electrical resistance between the two faces P and Q of the composite bar is



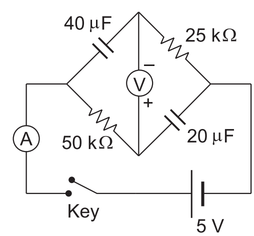
(a) $\dfrac{2475}{64}\,\mu\,\Omega$ (b) $\dfrac{1875}{64}\,\mu\,\Omega$

(c) $\dfrac{1875}{49}\,\mu\,\Omega$ (d) $\dfrac{2475}{132}\,\mu\,\Omega$

4. The heater of an electric kettle is made of a wire of length L and diameter d. It takes 4 minutes to raise the temperature of 0.5 kg of water by 40 K. This heater is replaced by a new heater having two wires of the same material, each of length L and diameter $2d$. The way these wires are connected is given in the options. How much time in minutes will it take to raise the temperature of the same amount of water by 40 K?

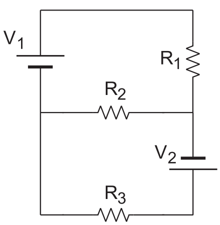
(a) 4, if the wires are in parallel
(b) 2, if the wires are in series
(c) 1, if the wires are in series
(d) 0.5, if the wires are in parallel

5. In the given circuit, the key is pressed at time t = 0. Which of the following statements is/are true?



(a) The voltmeter displays –5 V as soon as the key is pressed, and displays +5 V after a long time.
(b) The voltmeter will display 0 V at time t = ln 2 s.
(c) The current in the ammeter becomes $1/e$ of the initial value after 1 s.
(d) The current in the ammeter become zero after a long time.

6. Two ideal batteries of emf V_1 and V_2 and three resistors R_1, R_2 and R_3 are connected as shown in the figure. The current in resistance R_2 would be zero if



(a) $V_1 = V_2$ and $R_1 = R_2 = R_3$
(b) $V_1 = V_2$ and $R_1 = 2R_2 = R_3$
(c) $V_1 = 2V_2$ and $2R_1 = 2R_2 = R_3$
(d) $2V_1 = V_2$ and $2R_1 = R_2 = R_3$

7. Consider two identical galvanometers and two identical resistors with resistance R. If the internal resistance of the galvanometers $R_g < R/2$, which of the following statements about any one of the galvanometers is/are true?

(a) The maximum voltage range is obtained when all the components are connected in series.

(b) The maximum voltage range is obtained when the two resistors and one galvanometer are connected in series, and the second galvanometer is connected in parallel to the first galvanometer.

(c) The maximum current range is obtained when all the components are connected in parallel.

(d) The maximum current range is obtained when the two galvanometers are connected in series and the combination is connected in parallel with both the resistors.

8. A steady current I flows along an infinitely long hollow cylindrical conductor of radius R. This cylinder is placed coaxially inside an infinitely long solenoid of radius $2R$. The solenoid has n turns per unit length and carries a steady current I. Consider a point P at a distance r from the common axis. Which of the following statements is/are correct?

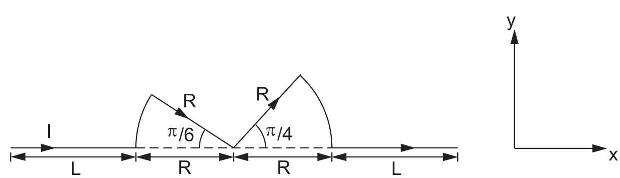
(a) In the region $0 < r < R$, the magnetic field is nonzero.

(b) In the region $R < r < 2R$, the magnetic field is along the common axis.

(c) In the region $R < r < 2R$, the magnetic field is tangential to the circle of radius r, centred on the axis.

(d) In the region $r > 2R$, the magnetic field is nonzero.

9. A conductor (shown in the figure) carrying a constant current I is kept in the xy-plane in a uniform magnetic field $\vec{B}$. If $\vec{F}$ is the magnitude of the total magnetic force acting on the conductor, the correct statement(s) is/are



(a) If $\vec{B}$ is along $\hat{z}$, $F \propto (L + R)$ (b) If $\vec{B}$ is along $\hat{x}$, $F = 0$

(c) If $\vec{B}$ is along $\hat{y}$, $F \propto (L + R)$ (d) If $\vec{B}$ is along $\hat{z}$, $F = 0$

10. Consider the motion of a positive point charge in a region where there are simultaneous uniform electric and magnetic fields $\vec{E} = E_0\hat{j}$ and $\vec{B} = B_0\hat{j}$. At time $t = 0$, this charge has velocity $\vec{v}$ in the xy-plane, making an angle θ with the x-axis. Which of the following options is/are correct for time $t > 0$?

(a) If $\theta = 0$, the charge moves in a circular path in the xz-plane.

(b) If $\theta = 0$, the charge undergoes helical motion with a constant pitch along the y-axis.

(c) If $\theta = 10°$, the charge undergoes helical motion with its pitch increasing with time.

(d) If $\theta = 90°$, the charge undergoes linear but accelerated motion along the y-axis.

11. An electron and a proton are moving on straight parallel paths with the same velocity. They enter a semi-infinite region of uniform magnetic field perpendicular to the velocity. Which of the following statements is/are true?

(a) They will never come out of this magnetic field region.

(b) They will come out travelling along parallel paths.

(c) They will come out at the same time.

(d) They will come out at different times.

12. A particle of mass M and positive charge Q, moving with a constant velocity $\vec{u_1} = (4\hat{i})$ m s^{-1}, enters a region of uniform static magnetic field normal to the xy-plane. The region of the magnetic field extends from $x = 0$ to $x = L$ for all values of y. After passing through the region, the particle emerges into the other side after 10 ms with a velocity $\vec{u_2} = 2(\sqrt{3}\,\hat{i} + \hat{j})$ m s^{-1}. Which of the following statements is/are correct?

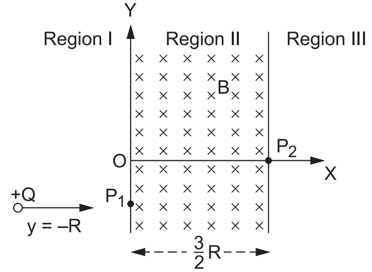
(a) The direction of the magnetic field is along the $-z$ axis.

(b) The direction of the magnetic field is along the $+z$ axis.

(c) The magnitude of the magnetic field is $\dfrac{50\pi M}{3Q}$ units.

(d) The magnitude of the magnetic field is $\dfrac{100\pi M}{3Q}$ units.

13. A uniform magnetic field B exists in the region between $x = 0$ and $x = \frac{3R}{2}$ (Region II in the figure), pointing normally into the plane of the paper. A particle with charge $+Q$ and momentum p directed along the x-axis enters Region II from Region I at a point P_1 ($y = -R$). Which of the following options is/are correct?



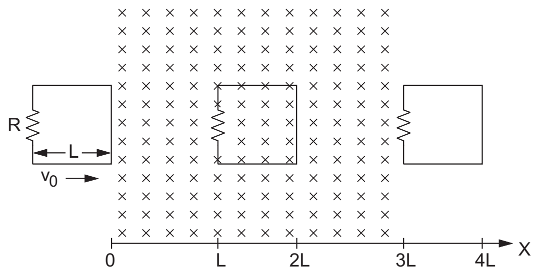
(a) For $B = \frac{8}{13}\frac{p}{QR}$, the particle will enter Region III through the point P_2 on the x-axis.

(b) For $B > \frac{2}{3}\frac{p}{QR}$, the particle will re-enter Region I.

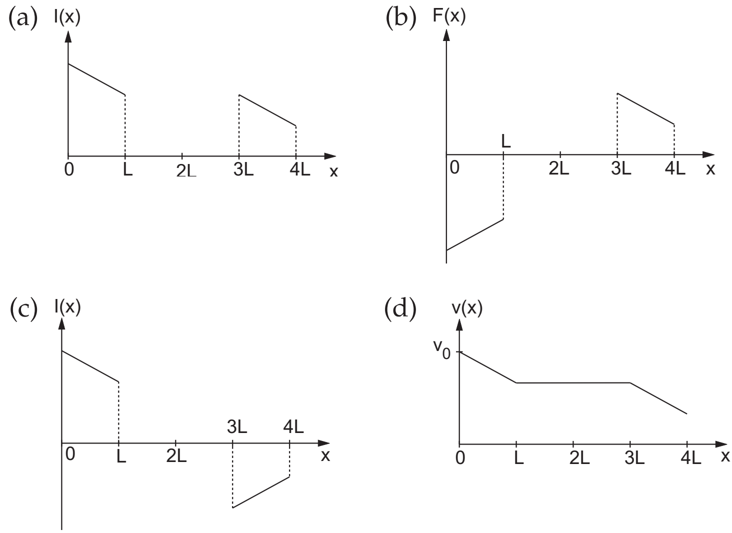
(c) For a fixed magnetic field B, particles carrying the same charge Q and moving with the same velocity v, the distance between the point P_1 and the point of re-entry into Region I is inversely proportional to the mass of the particle.

(d) When the particle re-enters Region I through the longest possible path in Region II, the magnitude of the change in its linear momentum between the point P_1 and the farthest point from the y-axis is $\frac{p}{\sqrt{2}}$.

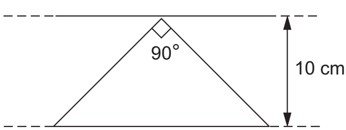
14.



A rigid square-shaped wire loop of side L and resistance R is moving along the x-axis with a constant velocity v_0 in the plane of the paper. At $t = 0$, the right edge of the loop enters the region of length $3L$ where there is a uniform magnetic field B_0 directed into the plane of the paper, as shown in the figure. For a sufficiently large v_0, the loop eventually crosses the region. Let x be the location of the right edge of the loop. Let $v(x)$, $I(x)$ and $F(x)$ represent the velocity of the loop, current in the loop and force on the loop respectively, as a function of x. Counterclockwise current is taken as positive. Which of the following schematic plots is/are correct? (Ignore gravity.)



15. A conducting loop in the shape of a right-angled isosceles triangle of height 10 cm is kept such that the 90° vertex is very close to an infinitely long conducting wire as shown in the figure. The wire is electrically insulated from the loop. The hypotenuse of the triangle is parallel to the wire. The current in the triangular loop is in counterclockwise direction and increased at a constant rate of 10 A s^{-1}. Which of the following statements is/are true?



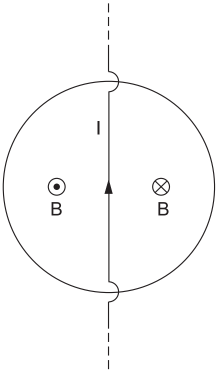
(a) The induced current in the wire is in the opposite direction to the current along the hypotenuse.

(b) There is a repulsive force between the wire and the loop.

(c) The magnitude of the induced emf in the wire is $\left(\frac{\mu_0}{\pi}\right)$ volt.

(d) If the loop is rotated at a constant angular speed about the wire, an additional emf of $\left(\frac{\mu_0}{\pi}\right)$ volt is induced in the wire.

16. A current-carrying, infinitely long wire is kept along the diameter of a circular wire loop, without touching it. Which of the statements is/are correct?



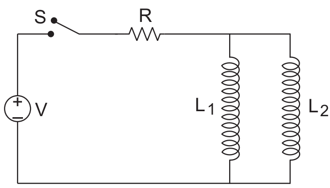
(a) The emf induced in the loop is zero if the current is constant.

(b) The emf induced in the loop is finite if the current is constant.

(c) The emf induced in the loop is zero if the current decreases at a steady state.

(d) The emf induced in the loop is infinite if the current decreases at a steady state.

17. A source of constant voltage V is connected to a resistance R and two ideal inductors L_1 and L_2 through a switch S as shown. There is no mutual inductance between the two inductors. The switch is initially open. At t = 0, the switch is closed and current begins to flow. Which of the following options is/are correct?



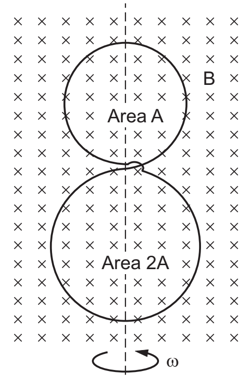
(a) At $t = 0$, the current through the resistance R is $\frac{V}{R}$.

(b) After a long time, the current through L_2 will be $\frac{V}{R}\frac{L_1}{L_1+L_2}$.

(c) After a long time, the current through L_1 will be $\frac{V}{R}\frac{L_2}{L_1+L_2}$.

(d) The ratio of the current through L_1 and L_2 is fixed at all times ($t > 0$).

18. A circular insulated copper wire loop is twisted to form two loops of areas A and $2A$ as shown in the figure. At the point of crossing, the wires remain electrically insulated from each other. The entire loop lies in the plane (of the paper). A uniform magnetic field $\vec{B}$ points towards the plane of the paper. At t = 0, the loop starts rotating about the common diameter as axis with a constant angular velocity ω in the magnetic field. Which of the following options is/are correct?



(a) The rate of change of the magnetic flux is maximum when the plane of the loops is perpendicular to the plane of the paper.

(b) The net emf induced due to both the loops is proportional to cos ωt.

(c) The emf induced in the loop is proportional to the sum of the areas of the two loops.

(d) The amplitude of the maximum net emf induced due to both the loops is equal to the amplitude of the maximum emf induced in the smaller loop alone.

19. The instantaneous voltages at three terminals marked X, Y and Z are given by

$$V_X = V_0 \sin \omega t,$$

$$V_Y = V_0 \sin\left(\omega t + \frac{2\pi}{3}\right) \text{ and}$$

$$V_Z = V_0 \sin\left(\omega t + \frac{4\pi}{3}\right).$$

An ideal voltmeter is configured to read the rms value of the potential difference between its terminals. It is connected between the points X and Y and then between Y and Z. The readings of the voltmeter will be

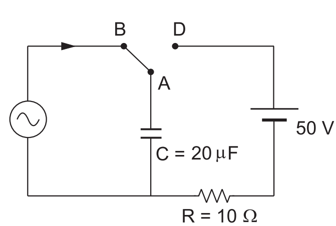
(a) $V_{YZ}^{rms} = V_0\sqrt{\frac{3}{2}}$

(b) $V_{XY}^{rms} = V_0$

(c) $V_{YZ}^{rms} = V_0\sqrt{\frac{1}{2}}$

(d) independent of the choice of the two terminals

20. At time $t = 0$, terminal A in the circuit shown in the figure is connected to B by a key and an alternating current $I(t) = I_0 \cos \omega t$ (where $I_0 = 1$ A and $\omega = 500$ rad s^{-1}) starts flowing in it with the initial direction as shown



in the figure. At $t = \frac{7\pi}{6\omega}$, the key is shifted from B to D. Now onwards, only A and D are connected. A total charge Q flows from the battery to charge the capacitor fully. If $C = 20\ \mu F$, $R = 10\ \Omega$ and battery is ideal with emf of 50 V, identify the correct statement(s).

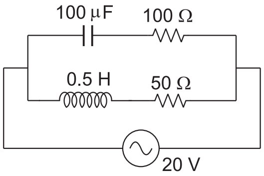
(a) The magnitude of maximum charge on the capacitor before $t = \frac{7\pi}{\omega}$ is 1×10^{-3} C.

(b) The current in the left part of the circuit just before $t = \frac{7\pi}{6\omega}$ is clockwise.

(c) Immediately after A is connected to D, the current in R is 10 A.

(d) $Q = 2 \times 10^{-3}$ C.

21. In the given circuit, the AC source has $\omega = 100$ rad s^{-1}. Considering the inductor and the capacitor to be ideal, which of the following statements is/are correct?



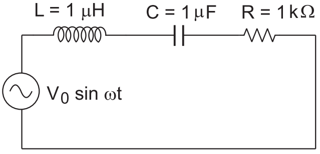
(a) The current through the circuit is 0.3 A.

(b) The current through the circuit is $0.3\sqrt{2}$ A.

(c) The voltage across 100 Ω resistor is $10\sqrt{2}$ V.

(d) The voltage across 50 Ω resistor is 10 V.

22. In the circuit shown, $L = 1\ \mu H$, $C = 1\ \mu F$ and $R = 1\ k\Omega$. They are connected in series with an AC source $V = V_0 \sin \omega t$ as shown. Which of the following options is/are correct?



(a) The frequency at which the current will be in phase with the voltage is independent of R.

(b) At $\omega \rightarrow 0$, the current flowing through the circuit becomes nearly zero.

(c) At $\omega >> 10^6$ rad s^{-1}, the circuit behaves like a capacitor.

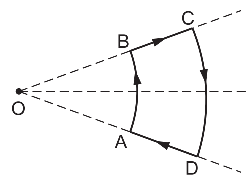
(d) The current will be in phase with the voltage if $\omega = 10^4$ rad s^{-1}.

23. A series RC circuit is connected to an AC voltage source. Consider two cases: (A) when C is without a dielectric medium, and (B) when C is filled with a medium of dielectric constant 4. The current I_R through the resistor and the voltage V_C across the capacitor are compared in the two cases. Which of the following is/are true?

(a) $I_R^A > I_R^B$ (b) $I_R^A < I_R^B$

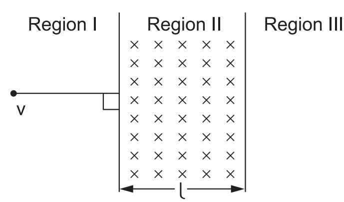
(c) $V_C^A > V_C^B$ (d) $V_C^A < V_C^B$

24. An infinitely long current-carrying wire passes through point O and is perpendicular to the plane containing a current-carrying loop ABCD as shown in the figure. Choose the correct option(s).



(a) The net force on the loop is zero.
(b) The net torque on the loop is zero.
(c) As seen from O, the loop rotates clockwise.
(d) As seen from O, the loop rotates anticlockwise.

25. A particle of mass m and charge q, moving with a velocity v enters Region II normal to the boundary as shown in the figure. Region II has a uniform magnetic field B perpendicular to the plane of the paper. The length of Region II is l. Choose the correct choice(s).



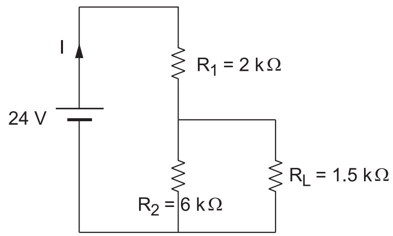
(a) The particle enters Region III only if its velocity $v > \frac{qlB}{m}$.

(b) The particle enters Region III only if its velocity $v < \frac{qlB}{m}$.

(c) Path length of the particle in Region II is maximum when velocity $v = \frac{qlB}{m}$.

(d) Time spent in Region II is same for any velocity v as long as the particle returns to Region I.

26. For the circuit shown in the figure,



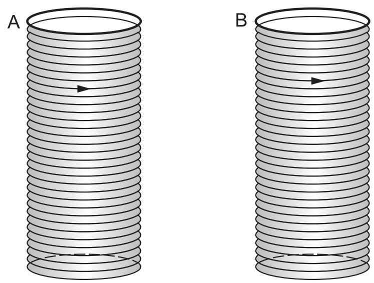
(a) the current I through the battery is 7.5 mA

(b) the potential difference across R_L is 18 V

(c) the ratio of powers dissipated in R_1 and R_2 is 3

(d) if R_1 and R_2 are interchanged, the magnitude of power dissipated in R_L will decrease by a factor of 9.

27. Two metallic rings A and B, identical in shape and size but having different resistivities ρ_A and ρ_B, are kept on top of two identical solenoids as shown in the figure. When current I is switched on in both the solenoids in identical manner, the rings A and B jump to heights h_A and h_B respectively, with $h_A > h_B$. The possible relation(s) between their resistivities and their masses m_A and m_B is/are



(a) $\rho_A > \rho_B$ and $m_A = m_B$

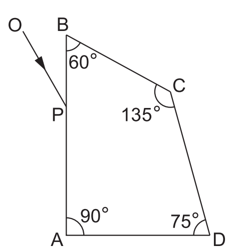
(b) $\rho_A < \rho_B$ and $m_A = m_B$

(c) $\rho_A > \rho_B$ and $m_A > m_B$

(d) $\rho_A < \rho_B$ and $m_A < m_B$

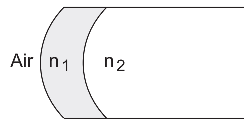
2.6 Ray Optics and Wave Optics

1. A ray OP of monochromatic light is incident on the face AB of prism ABCD near vertex B at an incident angle of 60° (see figure). If the refractive index of the material of the prism is $\sqrt{3}$, which of the following is/are correct?



(a) The ray gets totally internally reflected at face CD.
(b) The ray comes out through face AD.
(c) The angle between the incident ray and the emergent ray is 90°.
(d) The angle between the incident ray and the emergent ray is 120°.

2. A transparent thin film of uniform thickness and refractive index $n_1 = 1.4$ is coated on the convex spherical surface of radius R at one end of a long solid glass cylinder of refractive index $n_2 = 1.5$, as shown in the figure. Rays of light parallel to the axis of the cylinder traversing through the film from air to glass get focused at a distance f_1 from the film, while rays of light traversing from glass to air get focused at a distance f_2 from the film. Then

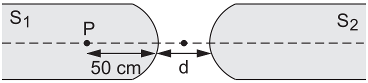


(a) $|f_1| = 3R$ (b) $|f_1| = 2.8R$
(c) $|f_2| = 2R$ (d) $|f_2| = 1.4R$

3. For an isosceles prism of angle A and refractive index μ, it is found that the angle of minimum deviation $\delta_m = A$. Which of the following options is/are correct?

(a) At minimum deviation, the incident angle i_1 and the refracting angle r_1 at the first refracting surface are related by $r_1 = \dfrac{i_1}{2}$.
(b) For the prism, the refractive index μ and the angle of prism A are related as $A = \dfrac{1}{2}\cos^{-1}\left(\dfrac{\mu}{2}\right)$.
(c) For the prism, the emergent ray at the second surface will be tangential to the surface when the angle of incidence at the first surface is $i_1 = \sin^{-1}\left[\sin A\sqrt{4\cos^2\dfrac{A}{2} - 1} - \cos A\right]$.
(d) For the angle of incidence $i_1 = A$, the ray inside the prism is parallel to the base of the prism.

4. Two identical glass rods S_1 and S_2 (refractive index = 1.5) have one convex end of radius of curvature 10 cm. They are placed with curved surfaces at a distance d as shown in the figure with their axes (shown by the dashed line) aligned. When a point source of light P is placed inside the rod S_1 on its axis at a distance of 50 cm from the curved face, the light rays emanating from it are found to be parallel to the axis inside S_2. The distance d is

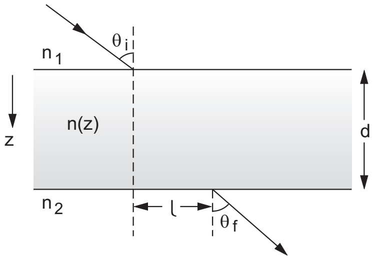


(a) 60 cm (b) 70 cm
(c) 80 cm (d) 90 cm

5. A plano-convex lens is made of a material of refractive index n. When a small object is placed 3 cm away in front of the curved face of the lens, an image double the size of the object is produced. Due to reflection from the convex surface of the lens, another faint image is observed at a distance of 10 cm away from the lens. Which of the following statements is/are true?

(a) The refractive index of the lens is 2.5.
(b) The radius of curvature of the convex surface is 45 cm.
(c) The faint image is erect and real.
(d) The focal length of the lens is 20 cm.

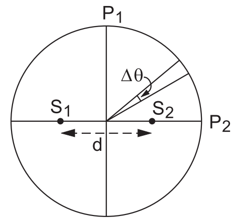
6. A transparent slab of thickness d has a refractive index $n(z)$ that increases with z. Here z is the vertical distance measured from the top. The slab is placed between two media with uniform refractive indices n_1 and n_2 ($n_2 > n_1$) as shown in the figure. A ray of light is incident with angle θ_i from medium 1 and emerges into medium 2 with refraction angle θ_f with lateral displacement l. Which of the following statements is/are correct?



(a) $n_1 \sin\theta_i = n_2 \sin\theta_f$. (b) $n_1 \sin\theta_i = (n_2 - n_1)\sin\theta_f$.

(c) l is independent of n_2. (d) l is dependent on $n(z)$.

7. Two coherent point sources S_1 and S_2 of wavelength $\lambda = 600$ nm are placed symmetrically on either side of the centre of a circle as shown. The sources are separated by a distance $d = 1.8$ mm. This arrangement produces interference fringes visible as alternate bright and dark spots on the circumference of the circle. The angular separation between two consecutive bright spots is $\Delta\theta$. Which of the following options is/are correct?



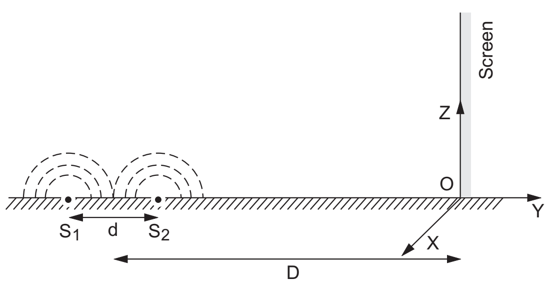
(a) The total number of fringes produced between P_1 and P_2 in the first quadrant is close to 3000.

(b) A dark spot will be formed at the point P_2.

(c) At P_2, the order of the fringe will be maximum.

(d) The angular separation between two consecutive bright spots decreases as we move from P_1 to P_2 along the first quadrant.

8. While conducting the Young's double slit experiment, a student replaced the two slits with a large opaque plate in the xy-plane containing two small holes S_1 and S_2 that act as two coherent point sources emitting light of wavelength 600 nm. The student mistakenly placed the screen parallel to the xz-plane (for $z > 0$) at a distance $D = 3$ m from the mid-point of S_1S_2 as shown schematically in the figure. The distance between the sources is $d = 0.6003$ mm. The origin O is at the intersection of the screen and the line joining S_1S_2. Which of the following is/are true of the intensity pattern on the screen?



(a) Hyperbolic bright and dark bands with foci symmetrically placed about O in the x-direction

(b) Semicircular bright and dark bands centred at point O

(c) Straight bright and dark bands parallel to the x-axis

(d) The region very close to the point O will be dark

9. Using the expression $2d \sin \theta = \lambda$, one calculates the value of d by measuring the corresponding angles θ in the range from 0 to 90°. The wavelength λ is exactly known and the error in θ is constant for all values of θ. As θ increases from 0,

(a) the absolute error in d remains constant
(b) the absolute error in d increases
(c) the fractional error in d remains constant
(d) the fractional error in d decreases

10. A light source which emits two wavelengths $\lambda_1 = 400$ nm and $\lambda_2 = 600$ nm is used in a Young's double slit experiment. If the recorded fringe widths for λ_1 and λ_2 are β_1 and β_2, and the number of fringes for them within a distance y on one side of the central maximum are m_1 and m_2 respectively then

(a) $\beta_2 > \beta_1$
(b) $m_1 > m_2$
(c) from the central maximum, the 3rd maximum of λ_2 overlaps with 5th minimum of λ_1
(d) the angular separation of the fringes for λ_1 is greater than λ_2

11. In a Young's double slit experiment, the separation between the two slits is d and the wavelength of the light is λ. The intensity of light falling on slit 1 is four times the intensity of light falling on slit 2. Choose the correct choice(s).

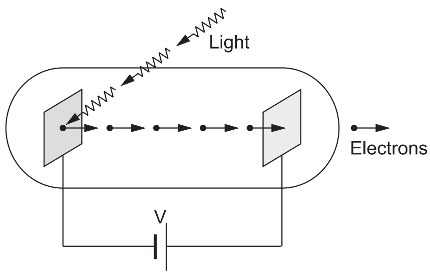
(a) If $d = \lambda$, the screen will contain only one maximum.
(b) If $\lambda < d < 2\lambda$, at least one more maximum (besides the central maximum) will be observed on the screen.
(c) If the intensity of light falling on slit 1 is reduced so that it becomes equal to that of slit 2, the intensities of the observed dark and bright fringes will increase.
(d) If the intensity of light falling on slit 2 is increased so that it becomes equal to that of slit 1, the intensities of the observed dark and bright fringes will increase.

12. A student performed the experiment to determine the focal length of a concave mirror by u–v method using an optical bench of length 1.5 m. The focal length of the mirror used is 24 cm. The maximum error in the location of the image can be 0.2 cm. The 5 sets of (u, v) values recorded by the student (in cm) are: (42, 56), (48, 48), (60, 40), (66, 33), (78, 39). The data set(s) that cannot come from the experiment and is/are incorrectly recorded, is/are

(a) (42, 56) (b) (48, 48)
(c) (66, 33) (d) (78, 39)

2.7 Modern Physics

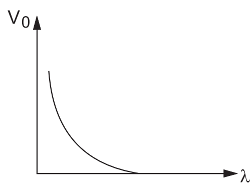
1. Light of wavelength λ_{Ph} falls on a cathode plate inside a vacuum tube as shown in the figure. The work function of the cathode surface is ϕ and the anode is a wire mesh of a conducting material kept at a distance d from the cathode. A potential difference V is maintained between the electrodes. If the minimum de Broglie wavelength of the electrons passing through the anode is λ_e, which of the following statements is/are true?



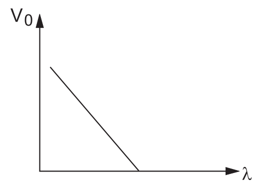
(a) For a large potential difference ($V >> \phi/e$), λ_e is approximately halved if V is made four times.
(b) λ_e increases at the same rate as λ_{Ph} does for $\lambda_{Ph} < hc < \phi$.
(c) λ_e decreases with increase in ϕ and λ_{Ph}.
(d) λ_e is approximately halved if d is doubled.

2. For photoelectric effect with incident photons of wavelength λ, the stopping potential is V_0. Identify the correct variations of V_0 with λ and $1/\lambda$.

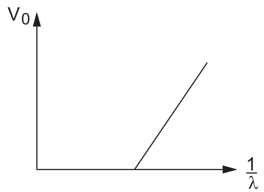
(a)



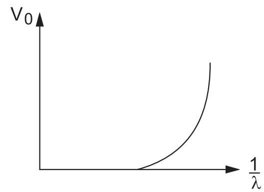
(b)



(c)



(d)



3. The highly excited states ($n >> 1$) for hydrogen-like atoms (also called Rydberg states) with nuclear charge Ze are defined by their principal quantum number n. Which of the following statements is/are true?

(a) The relative change in the radii of two consecutive orbitals does not depend on Z.

(b) The relative change in the radii of two consecutive orbitals varies as $1/n$.

(c) The relative change in the energy of two consecutive orbitals varies as $1/n^3$.

(d) The relative change in the angular momentum of two consecutive orbitals varies as $1/n$.

4. The radius of the orbit of an electron in a hydrogen-like atom is $4.5a_0$, where a_0 is the Bohr radius. Its orbital angular momentum is $3h/2\pi$. It is given that h is Planck constant and R is Rydberg constant. The possible wavelength(s) when the atom de-excites, is/are:

(a) $\frac{9}{32R}$ (b) $\frac{9}{16R}$

(c) $\frac{9}{5R}$ (d) $\frac{4}{3R}$

5. A fission reaction is given by ${}^{236}_{92}\text{U} \rightarrow {}^{140}_{54}\text{Xe} + {}^{94}_{38}\text{Sr} + \text{X} + \text{Y}$, where X and Y are two particles. Considering ${}^{236}_{92}\text{U}$ to be at rest, the kinetic energies of the products are denoted by K_{Xe}, K_{Sr}, K_{X} (2 MeV) and K_{Y} (2 MeV) respectively. Let the binding energies per nucleon of ${}^{236}_{92}\text{U}$, ${}^{140}_{54}\text{Xe}$ and ${}^{94}_{38}\text{Sr}$ be 7.5 MeV, 8.5 MeV and 8.5 MeV respectively. Considering the different conservation laws, the correct options is/are

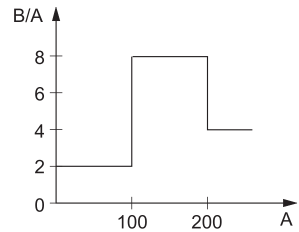
(a) X = n, Y = n, K_{Sr} = 129 MeV, K_{Xe} = 86 MeV

(b) X = p, Y = e^-, K_{Sr} = 129 MeV, K_{Xe} = 86 MeV

(c) X = p, Y = n, K_{Sr} = 129 MeV, K_{Xe} = 86 MeV

(d) X = n, Y = n, K_{Sr} = 86 MeV, K_{Xe} = 129 MeV

6. Assume that the nuclear binding energy per nucleon (B/A) versus mass number (A) is as shown in the figure. Use this plot to choose the correct choice(s) given below.



(a) Fusion of two nuclei with mass numbers lying in the range of $1 < A < 50$ will release energy.

(b) Fusion of two nuclei with mass numbers lying in the range of $51 < A < 100$ will release energy.

(c) Fission of a nucleus lying in the mass range of $100 < A < 200$ will release energy when broken into two equal fragments.

(d) Fission of a nucleus lying in the mass range of $200 < A < 260$ will release energy when broken into two equal fragments.

Answers

2.1 General Physics

1. b, c	**2.** a, c	**3.** a, b, d	**4.** a, c, d	**5.** b, d	**6.** a, c
7. a, c	**8.** a, b, d	**9.** c, d	**10.** a, d	**11.** a, c	**12.** a, b
13. a, b, d	**14.** d	**15.** c, d	**16.** b, c	**17.** a, d	**18.** b, d
19. a, d	**20.** c	**21.** b, d	**22.** b, c	**23.** a, b	**24.** b, d
25. a, b, d	**26.** c, d	**27.** a, b, d	**28.** a	**29.** b, c	**30.** a, d
31. a, d	**32.** a, b, d				

2.2 Heat and Thermodynamics

1. a, b, d	**2.** a, b, c	**3.** a, b, c, d	**4.** a, b, c	**5.** a, b, c, d	**6.** a, b, c, d
7. b, d	**8.** b, d				

2.3 Sound Waves

1. b, c	**2.** a, c, d	**3.** a, b	**4.** a, b, d	**5.** a, c, d	**6.** d
7. b, d	**8.** a, b, d	**9.** a, d			

2.4 Electrostatics

1. a, d	**2.** a, b, c, d	**3.** c, d	**4.** a, c, d	**5.** b, d	**6.** a, b, c
7. c	**8.** c	**9.** d	**10.** a, d	**11.** b, d	**12.** a, d
13. d	**14.** a, c	**15.** a, b, c, d	**16.** a		

2.5 Current Electricity and Magnetism

1. c, d	**2.** a, b, c, d	**3.** b	**4.** b, d	**5.** a, b, c, d	**6.** a, b, d
7. b, c	**8.** a, d	**9.** a, b, c	**10.** c, d	**11.** b, d	**12.** a, c
13. a, b	**14.** c, d	**15.** b, c	**16.** a, c	**17.** b, c, d	**18.** a, d
19. a, d	**20.** c, d	**21.** a, c	**22.** a, b	**23.** b, c	**24.** a, c
25. a, c, d	**26.** a, c, d	**27.** b, d			

2.6 Ray Optics and Wave Optics

1. a, b, c	**2.** a, c	**3.** a, c, d	**4.** b	**5.** a, d	**6.** a, c, d
7. a, c	**8.** b, d	**9.** d	**10.** a, b, c	**11.** a, b	**12.** c, d

2.7 Modern Physics

1. a	**2.** a, c	**3.** a, b, d	**4.** a, c	**5.** a	**6.** b, d

Hints and Solutions

2.1 General Physics

1. For the vernier callipers, 1 ms = $\frac{1}{8}$ cm. 5 vs = 4 ms

$\Rightarrow$ 1 vs $= \frac{4}{5}$ ms $= \frac{4}{5} \times \frac{1}{8}$ cm $= \frac{1}{10}$ cm.

$\therefore$ LC of the vernier callipers = 1 ms – 1 vs = $\frac{1}{40}$ cm = 0.025 cm.

Pitch of the screw gauge = 2 × 0.025 cm = 0.05 cm.

LC of the screw gauge = $\frac{0.05}{100}$ cm = 0.005 mm.

LC of the linear scale of the screw gauge = 0.05 cm.

Pitch = 0.05 × 2 cm = 0.1 cm.

$\therefore$ LC of the screw gauge = $\frac{0.1}{100}$ cm = 0.01 mm.

Correct options are (b) and (c).

2. Error in measuring $T = \frac{2\text{ s}}{40} = 0.05\text{ s} = \Delta T$.

$$T \propto \frac{1}{\sqrt{g}} \Rightarrow \frac{\Delta g}{g} = \frac{2\Delta T}{T} = \frac{1}{20} \Rightarrow \frac{\Delta g}{g} \times 100\% = 5\%.$$

3. Measured value of r is (10 + 1) mm.

Thus % error in $r = \frac{\Delta r}{r} \times 100 = \frac{1}{10} \times 100 = 10\%$.

$(T)_{\text{av}} = \frac{1}{5}\Sigma T_{\text{i}} = \frac{2.78\text{ s}}{5} = 0.56$ s.

Mean absolute error $= \frac{1}{5}\Sigma|T - T_{\text{av}}| = 0.02$ S.

$\therefore$ % error in $T = \frac{0.02}{0.56} \times 100\% = 3.57\%$.

Now, $g \propto \frac{R-r}{T^2}$, so $\frac{dg}{g} = 2\frac{dT}{T} + \frac{dR+dr}{R-r}$

$$\Rightarrow \frac{dg}{g} = 2(3.57\%) + \frac{1+1}{60-10} \times 100\% = 7.14\% + 4\% = 11.14\% \approx 11\%.$$

4. $[h] = ML^2T^{-1}$, $[c] = LT^{-1}$, $G = M^{-1}L^3T^{-2}$

$\Rightarrow\ M \propto \sqrt{\dfrac{hc}{G}}, L \propto \sqrt{\dfrac{hG}{c^3}}.$

5. Energy $\equiv FL \equiv k_B T$, where F = force, L = length, k_B = Boltzmann constant and T = temperature.

Permittivity $\varepsilon \equiv \dfrac{Q^2}{FL^2}$, where Q = charge and mole $(n) \equiv L^{-3}$.

With these expressions, correct options are (b) and (d).

6. Magnetic energy in the inductor $= \dfrac{1}{2}LI^2 = \dfrac{1}{2}\dfrac{\mu_0 N^2 A}{l}I^2$.

Electrical energy with the capacitor $= \dfrac{1}{2}CV^2 = \dfrac{1}{2}\dfrac{\varepsilon_0 A}{d}V^2$.

$\Rightarrow \dfrac{\mu_0 A I^2}{l}$ and $\dfrac{\varepsilon_0 A V^2}{d}$ have the same dimension.

$\therefore$ $\mu_0 I^2$ and $\varepsilon_0 V^2$ have the same dimension.

For a capacitor, $Q = CV$

or $\dfrac{Q}{t} = \dfrac{CV}{t} = \dfrac{\varepsilon_0 A}{d}\cdot\dfrac{V}{t}$.

or $I = \varepsilon_0 \dfrac{A}{td} V$.

$\because$ $\dfrac{A}{td}$ has the dimension of speed,

$\therefore$ $I = \varepsilon_0 CV$.

7. $f = 0$, if $\sin\theta = \cos\theta \Rightarrow \theta = 45°$.

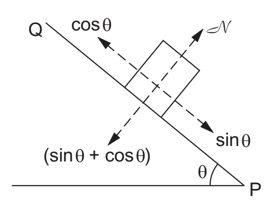
f directed towards Q, $\sin\theta > \cos\theta$

$\Rightarrow\ Q > 45°$.

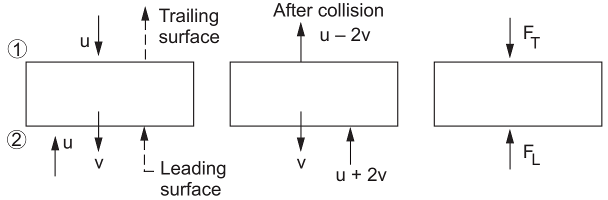
f directed towards P,

$\sin\theta < \cos\theta \Rightarrow \theta < 45°$.

Hence, options (a) and (c) are correct.



8.



The change in momentum at leading surface,

$$\Delta P = 2m\,(u+v).$$

$$F_L = \frac{\Delta P}{\Delta t} = \frac{2m(u+v)}{(d/u)}$$

(assuming that the particles return after a distance d, when $\Delta t = d/u$).

Similarly, for the trailing surface:

$$\Rightarrow \qquad F_L = \frac{2m(u+v)u}{d}.$$

Similarly, for the trailing surface:

$$F_T = \frac{2m(u-v)u}{d}.$$

$$\text{Pressure difference} = \frac{2m(u+v)u}{Ad} - \frac{2m(u-v)u}{Ad}$$

$$\Rightarrow \qquad \Delta P = \frac{2muv}{Ad}.$$

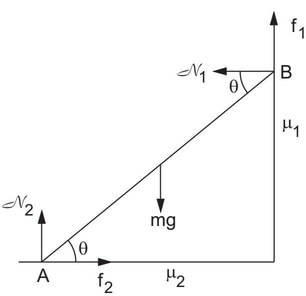
9. Since the ladder is about to slip, both f_1 and f_2 will be limiting.

$$\Rightarrow \qquad f_1 = \mu_1\mathcal{N}_1 \text{ and } f_2 = \mu_2\mathcal{N}_2.$$

For translational equilibrium

$$\mathcal{N}_1 = \mu_2\mathcal{N}_2, \text{ and}$$

$$\mathcal{N}_2 + \mu_1\mathcal{N}_1 = mg.$$



$$\text{Solving,} \quad \mathcal{N}_1 = \frac{\mu_2 mg}{1+\mu_1\mu_2} \qquad \text{...(1)}$$

$$\text{and} \quad \mathcal{N}_2 = \frac{mg}{1+\mu_1\mu_2}. \qquad \text{...(2)}$$

Taking torque about A,

$$mg\frac{l}{2}\cos\theta = \mathcal{N}_1 l\sin\theta + \mu_1\mathcal{N}_1 l\cos\theta$$

$$\text{or} \quad \frac{mg}{2}\cos\theta = \frac{\mu_2 mg}{1+\mu_1\mu_2}\sin\theta + \mu_1\mathcal{N}_1\sin\theta. \qquad \text{...(3)}$$

Taking $\mu_1 = 0$ and $\mu_2 \neq 0$, we get $\mathcal{N}_1 = \mu_2\, mg$, hence

from equation (3), $\frac{mg}{2}\cos\theta = \mathcal{N}_2$ mg $\sin\theta = \mathcal{N}_1 \sin\theta$

$$\Rightarrow \qquad \mathcal{N}_1 \tan\theta = \frac{mg}{2}.$$

Taking, $\mu_1 \neq 0$, $\mu_2 \neq 0$, from (2) we get

$$\mathcal{N}_2 = \frac{mg}{1+\mu_1\mu_2}.$$

Taking torque about B, we can conclude that options (a) and (b) are incorrect.

10. Initially, at $t = 0$, $v = u_0$. Thus $v = u_0 \cos \omega t$. At the time of collision,

$$\frac{u_0}{2} = u_0 \cos \omega t_1 \Rightarrow t_1 = \frac{\pi}{3\omega}.$$

After collision the motion is reversed and at time $t_2 = 2t_1 = \frac{2\pi}{3\omega}$, it acquires the same speed u_0.

If t_3 be the time at which the particle passes through the equilibrium position for the second time then

$$t_3 = \frac{T}{2} + 2t_1 = \frac{\pi}{\omega} + \frac{2\pi}{3\omega} = \frac{5\pi}{3\omega} = \frac{5\pi}{3}\sqrt{\frac{m}{k}}.$$

Hence, the correct options are (a) and (d).

11. Conserving linear momentum, $mv = MV$

$$\Rightarrow \qquad V = \frac{mv}{M}.$$

Conserving energy, $\frac{1}{2}mv^2 + \frac{1}{2}M\left(\frac{mv}{M}\right)^2 = mgR$

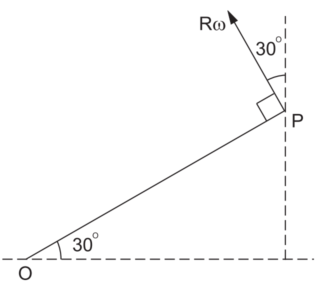
$$\Rightarrow \qquad v = \sqrt{\frac{2gR}{1 + \frac{m}{M}}}.$$

In absence of external force, $\Delta x_{cm} = 0$,

$$m\Delta x_m = -M\Delta x_M, \; m(R - x) = Mx$$

$$\Rightarrow \qquad x = \frac{mR}{(M + m)} \text{ (leftward)}.$$

12. $\vec{V}_0 - (3R)\omega\hat{i} = 0.$



$\therefore \; \vec{V}_0 = 3R\omega\hat{i}.$

Now, $\vec{V}_{P,O} = -\frac{R\omega}{4}\hat{i} + \frac{R\omega\sqrt{3}}{4}\hat{j}$

$$\Rightarrow \qquad \vec{V}_P = \vec{V}_{P,O} + \vec{V}_0$$

$$= -\frac{R\omega}{4}\hat{i} + \frac{R\omega\sqrt{3}}{4}\hat{j} + 3R\omega\hat{i}$$

$$= \frac{11}{4}R\omega\hat{i} + R\omega\frac{\sqrt{3}}{4}\hat{j}.$$

Hence, the correct options are (a) and (b).

13. $$\vec{r}(t) = \alpha t^3\hat{i} + \beta t^2\hat{j}$$

$$\vec{v}(t) = \frac{d\vec{r}}{dt} = 3\alpha t^2\hat{i} + 2\beta t\hat{j} = 10t^2\hat{i} + 10t\hat{j}$$

$$\vec{a} = (20t\hat{i} + 10\hat{j}) \text{ m s}^{-2}.$$

At $t = 1\text{s}, \vec{v} = 10(\hat{i} + \hat{j}) \text{ m s}^{-1}$.

Angular momentum

$$\vec{L}_0 = \vec{r} \times m\vec{v}.$$

At $t = 1 \text{ s}, \vec{r} = \left(\frac{10}{3}\hat{i} + 5\hat{j}\right) \text{m}.$

$$\therefore \quad \vec{L}_0 = \left(\frac{10}{3}\hat{i} + 5\hat{j}\right) \text{m } (0.1 \text{ kg}) \times (10\hat{i} + 10\hat{j}) \text{ m s}^{-1}$$

$$= \left(-\frac{5}{3}\hat{k}\right) \text{N m s}.$$

$$\vec{F} = m\vec{a} = (0.1 \text{ kg})\left(20\hat{i} + 10\hat{j}\right) \text{m s}^{-2}$$

$$= (2\hat{i} + \hat{j}) \text{ N}.$$

Torque about the origin,

$$\vec{\tau}_0 = \vec{r} \times \vec{F} = \left(\frac{10}{3}\hat{i} + 5\hat{j}\right) \text{m} \times (2\hat{i} + \hat{j}) \text{ N} = \left(-\frac{20}{3} \text{ N m}\right)\hat{k}.$$

Hence, the correct options are (a), (b) and (d).

14. Acceleration of a body rolling down the plane,

$$a = \frac{mg\sin\theta}{m + \frac{I}{R^2}}.$$

For P, $$a_P = \frac{mg\sin\theta}{m + \frac{mR^2}{R^2}} = \frac{g\sin\theta}{2}.$$

For Q, $$a_Q = \frac{mg\sin\theta}{m} = g\sin\theta, \text{ since } I_Q \approx 0.$$

$$\omega_P = \frac{\sqrt{2\cdot\frac{g}{2}\cdot l}}{R} \text{ and } \omega_Q = \frac{\sqrt{2gl}}{R}.$$

$$\therefore \quad \omega_Q > \omega_P.$$

Hence, the correct option is (d).

15. In the absence of external horizontal forces on the bar, its CM will fall vertically downwards.

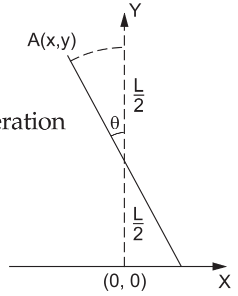
For translational motion:

$$mg - N = ma_{CM}. \quad ...(i)$$

For rotational motion:

$$N \cdot \frac{L}{2} \sin\theta = I_{CM}\alpha \quad ...(ii)$$

and $$a_{CM} = \alpha \cdot \frac{L}{2} \sin\theta. \quad ...(iii)$$



Solving equations (i), (ii) and (iii), the angular acceleration

$$\alpha = \frac{mg\frac{L}{2}\sin\theta}{I_{CM} + \frac{mL^2}{4}\sin^2\theta}.$$

Torque about the point of contact $= I\alpha = \frac{mL^2}{3}\alpha$.

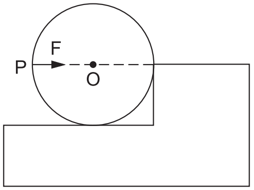
Displacement of the mid-point $= \frac{L}{2}(1 - \cos\theta)$.

For the point A, $x = -\frac{L}{2}\sin\theta$ and $y = L\cos\theta$

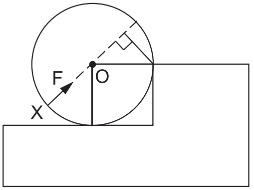
$\Rightarrow$ $$\frac{y^2}{L^2} + \frac{4x^2}{L^2} = 1,$$ which represents an ellipse.

Hence, the correct options are (c) and (d).

16. When the force is applied normal to the circumference at point P, the line of action always passes through O, so the torque is always zero.

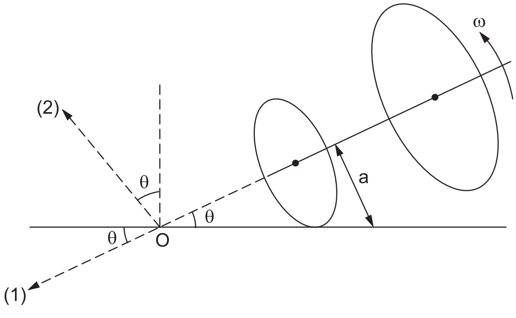


When the force is applied normal to the circumference at point X, the force and its perpendicular distance from the line of action remain constant, so torque τ = constant.



Hence, the correct options are (b) and (c).

17.



For angular momentum about the CM, we find L about the axis denoted by (1):

$$L_1 = \frac{ma^2}{2}\omega + \frac{4m(2a)^2}{2}\omega = \frac{17ma^2}{2}\omega.$$

$$V_{CM} = \frac{1}{5m}(m\cdot\omega a + 4m\cdot\omega 2a) = \frac{9\omega a}{5},$$

$$l_{CM} = \frac{1}{5m}(ml + 4m\cdot 2l) = \frac{9}{5}l.$$

Angular velocity (ω) about the axis (2): $\dfrac{V_{CM}}{l_{CM}} = \dfrac{\omega a}{l}$.

Component of ω along the z-axis,

$$\omega\cos\theta = \frac{\omega a}{l}\frac{l}{5a} = \frac{\omega}{5}.$$

Hence, the correct options are (a) and (d).

18. By simple calculation, if mass of P is M, and its radius is R then

for Q: mass = $8M$ and radius = $2R$,

for R: mass = $9M$ and radius = $9^{1/3}R$.

Since escape velocity $v_e = \sqrt{2GM/R}$,

$$(v_e)_P = \sqrt{2G\frac{M}{R}},\ (v_e)_Q = \sqrt{2G.\frac{8M}{2R}} = 2(v_e)_P, \text{ and}$$

$$(v_e)_R = \sqrt{2G\times\frac{9M}{9^{1/3}R}} = 9^{1/3}(v_e)_P.$$

$$\Rightarrow \quad (v_e)_R > (v_e)_Q > (v_e)_P.$$

Now, $(v_e)_Q/(v_e)_P = 2$.

Hence, the correct options are (b) and (d).

19. In case A:

$$mg\frac{L}{2}\sin\theta + MgL\sin\theta = \left(\frac{1}{3}mL^2 + \frac{1}{2}MR^2 + ML^2\right)\alpha_A.$$

In case B:

$$mg\frac{L}{2}\sin\theta + MgL\sin\theta = \left(\frac{1}{3}mL^2 + ML^2\right)\alpha_B.$$

$$\Rightarrow \quad \tau_A = \tau_B;\ \omega_A < \omega_B.$$

Hence, the correct options are (a) and (d).

20. During collision, friction is impulsive and soon after collision the ring has clockwise angular velocity, so friction acts towards left.

Hence, the correct option is (c).

21. $\frac{1}{2}mv^2 + \left(-\frac{2GMm}{L}\right) = 0$

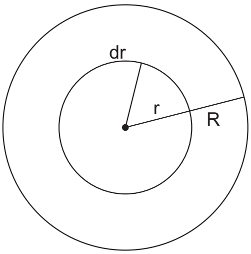
$\Rightarrow \quad v = 2\sqrt{\frac{GM}{L}}.$

Gravitational field is conservative, in which total mechanical energy (KE + PE) remains conserved. Hence, the kinetic energy imparted to the mass m is gradually reduced and gets converted into its potential energy, so that at every point of its flight the total mechanical energy remains constant.

Hence, the correct options are (b) and (d).

22. Gravitational field E at a distance r from the centre of the solid sphere:

$E = G \cdot \frac{4}{3}\pi r^3 \rho / r^2 = \frac{4}{3}\pi G \rho r.$



$\therefore$ force on the concentric shell of radius r and thickness dr is

$dF = E \cdot 4\pi r^2 dr\rho$

$\Rightarrow$ pressure $dp = \frac{dF}{\Delta A} = \frac{dF}{4\pi r^2} = E\rho\, dr.$

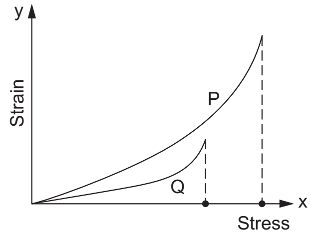
Integrating, $-p = -\int_0^p dp = \int_R^r \left(\frac{4}{3}\pi G\rho r\right)(\rho dr) = K\int_R^r rdr$

$\Rightarrow \quad p = \frac{K}{2}(R^2 - r^2) = A\left(1 - \frac{r^2}{R^2}\right).$

Let us calculate p for different values of $\frac{r}{R}$:

$\frac{r}{R}$	p
0	A
1/2	$3A/4$
1/3	$8A/9$
2/3	$5A/9$
2/5	$21A/25$
3/5	$16A/25$
3/4	$7A/16$

With these values, options (b) and (c) are correct.



23. From the stress–strain graph, breaking stress of P is more than that of Q.
So, P is more ductile then Q.
Strain $= \frac{1}{Y} \cdot$ stress, so $Y_P < Y_Q$.
In addition, P has greater tensile strength.
Hence, the correct options are (a) and (b).

24. First oscillator:

$$b = ma\omega_1.$$

$$\therefore \quad \frac{a}{b} = \frac{1}{m\omega_1} = n^2.$$

$$E_1 = \frac{1}{2}m\omega_1^2 a^2.$$

Second oscillator:

$$\frac{1}{m\omega_2} = 1.$$

$$\therefore \quad \frac{\omega_2}{\omega_1} = n^2.$$

$$E_2 = \frac{1}{2}m\omega_2^2 R^2.$$

$$\frac{E_1}{E_2} = \left(\frac{\omega_1}{\omega_2}\right)^2 \cdot n^2 = \left(\frac{\omega_1}{\omega_2}\right)^2 \frac{\omega_2}{\omega_1} = \frac{\omega_1}{\omega_2}.$$

$$\Rightarrow \quad \frac{E_1}{\omega_1} = \frac{E_2}{\omega_2}.$$

Hence, the correct options are (b) and (d).

25. $\omega_{v_1} = \sqrt{\frac{K}{M}}, \quad \omega' = \sqrt{\frac{K}{M+m}}.$

Case (i): When m is placed at mean position—
Let v' = velocity of system $(M + m)$ just after placing m.
Conserving linear momentum, $Mv_0 = (M + m)v'$

$\Rightarrow \quad M\omega A = (M+m)\omega' A'$

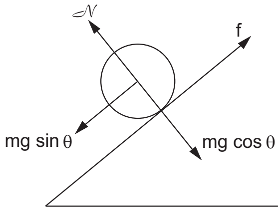
or $\frac{A'}{A} = \left(\frac{M}{M+m}\right)\frac{\omega}{\omega'} = \sqrt{\frac{M}{M+m}}.$

Case (ii): When m is placed at the extreme position—
v_M before placing is zero $\Rightarrow v_{M+m} = 0$ after placing m
$\Rightarrow$ extreme position and mean position remain unchanged, so $A = A'$.
$T' = \frac{2\pi}{\omega'}$, which is same in both the cases.
Energy decreases in case (i) but not in case (ii).
Velocity at the mean position $= \omega' A'$, which decreases in both the cases.
Hence, the correct options are (a), (b) and (d).

26. Friction acts upwards, so it supports rotation and opposes translation.

Acceleration of a rolling body,

$$a_{CM} = \frac{g\sin\theta}{1+K^2/R^2} = \frac{2}{3}g\sin\theta.$$



Frictional force,

$$f = \frac{mg\sin\theta}{1+R^2/K^2} = \frac{1}{3}mg\sin\theta.$$

$\Rightarrow$ f decreases with decrease in θ.

For pure rolling, there is no energy dissipation due to absence of slipping.

Hence, the correct options are (c) and (d).

27. Since mechanical energy is conserved, so

$$E_A = E_B = E_C.$$

$$\Rightarrow \quad K_A + mgh_A = K_B = K_C + mgh_C.$$

$$\therefore \quad K_B > K_A \ ; \quad K_B > K_C.$$

Now, $E_A - E_C = 0$, so $mg(h_A - h_C) = (K_C - K_A)$.

For, $h_A > h_C; K_C > K_A$.

For kinetic energy K_B, only the translational parts gets converted into potential energy at point C.

28. $$\Sigma\vec{F}_{ext} = \frac{d\vec{p}}{dt}.$$

When $\Sigma\vec{F}_{ext} = 0$·, linear momentum $\vec{p}$ is conserved.

Hence, the correct option is (a).

29. $$\vec{v}_C = \vec{v}_{CM} + \vec{r\omega} = 2\vec{v}_{CM};$$

$$\vec{v}_B = \vec{v}_{CM} \text{ and } \vec{v}_A = 0.$$

$$\Rightarrow \quad \vec{v}_C - \vec{v}_B = \vec{v}_{CM}; \ \vec{v}_B - \vec{v}_A = \vec{v}_{CM}$$

$$\vec{v}_C - \vec{v}_A = 2\vec{v}_{CM}; \ \vec{v}_B - \vec{v}_C = -\vec{v}_{CM}$$

Hence, the correct options are (b) and (c).

30. For the system to float,

$\downarrow (\rho_1 + \rho_2)V = (\sigma_1 + \sigma_2)V \uparrow \ \Rightarrow \ \rho_1 + \rho_2 = \sigma_1 + \sigma_2.$

Since the string is taut,

$$\rho_1 < \sigma_1 \text{ and } \rho_2 > \sigma_2.$$

Now, $v_P = \frac{2}{9}\frac{r^2(\sigma_2 - \rho_1)g}{\eta_2}$, and $v_Q = \frac{2}{9}\frac{r^2(\sigma_1 - \rho_2)g}{\eta_1}$.

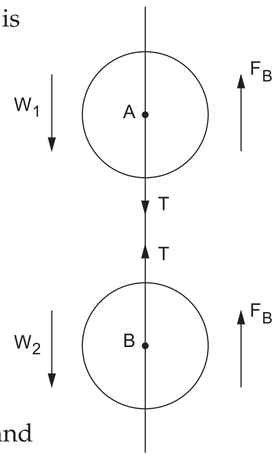
Since, $\sigma_2 - \rho_1 = -(\sigma_1 - \rho_2)$; $\left|\frac{\vec{v}_P}{\vec{v}_Q}\right| = \frac{\eta_1}{\eta_2}$

$\vec{v}_P \cdot \vec{v}_Q < 0$, as they are in opposite directions.

Hence, the correct options are (a) and (d).

31. The buoyant force on each of the two spheres is

$$F_B = \frac{4}{3}\pi R^3 2\rho g.$$



The weights of the spheres are

$$W_1 = \frac{4}{3}\pi R^3 \rho g \quad \text{and} \quad W_2 = \frac{4}{3}\pi R^3 3\rho g.$$

Let T = tension in the spring.

For equilibrium of A:

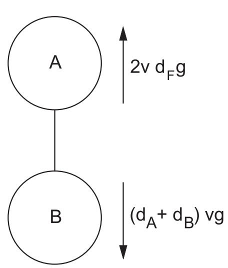
$$\frac{4}{3}\pi R^3 2\rho g = \frac{4}{3}\pi R^3 \rho g + T$$

$$\Rightarrow \quad T = \frac{4}{3}\pi R^3 \rho g = k\Delta l \Rightarrow \Delta l = \frac{4\pi R^3 \rho g}{3k}.$$

For the whole system, T is the internal force, and $W_1 + W_2 = 2F_B$ is true when both the spheres are completely submerged.

Hence, the correct options are (a) and (d).

32. For equilibrium, forces acting on the system are shown in the adjoining figure.



Now, it is clear from the figure that

$$d_A < d_F,\ d_B > d_F$$

and $d_A + d_B = 2d_F$.

Hence, the correct options are (a), (b) and (d).

2.2 Heat and Thermodynamics

1. Total internal energy of the system,

$$U = U_1 + U_2 = (nC_V T)_1 + (nC_V T)_2 = \frac{5}{2}RT + \frac{3}{2}RT = 4RT.$$

Average energy per mole $= \frac{4RT}{2} = 2RT$.

$\because$ the mixture has two moles with $U = 4RT$,

$\therefore$ $2(C_V)_{\text{mix}}\, T = 4RT \;\Rightarrow\; (C_V)_{\text{mix}} = 2R$.

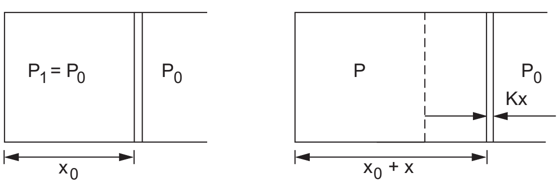
The speed of sound, c, in a gas $= \sqrt{\dfrac{\gamma RT}{M}}$.

$$\Rightarrow \frac{c_{\text{mix}}}{c_{\text{He}}} = \sqrt{\left(\frac{\gamma_{\text{mix}}}{\gamma_{\text{He}}}\right)\left(\frac{M_{\text{He}}}{M_{\text{max}}}\right)} = \sqrt{\frac{3}{2}\times\frac{3}{5}\times\frac{4}{3}} = \sqrt{\frac{6}{5}}.$$

Now, $\dfrac{(v_{\text{rms}})_{\text{He}}}{(v_{\text{rms}})_{H_2}} = \sqrt{\dfrac{M_{H_2}}{M_{\text{He}}}} = \dfrac{1}{\sqrt{2}}$.

Hence, the correct options are (a), (b) and (d).

2.



$$p = p_1 + \frac{Kx}{A}.$$

$$p_2 = \frac{3}{2}p_1 \;\Rightarrow\; x = \frac{V_1}{A}.$$

$$\frac{3}{2}p_1 = p_1 + \frac{Kx}{A} \quad \text{or} \quad Kx = \frac{p_1 A}{2}.$$

Energy stored in the spring $= \dfrac{1}{2}Kx^2 = \dfrac{1}{2}(Kx)x = \dfrac{1}{4}p_1V_1$.

Now, $\quad \Delta U = \dfrac{f}{2}(p_2V_2 - p_1V_1) = 3p_1V_1$.

In option (c):

$\because \qquad p_2 = \dfrac{4}{3}p_1,\; Kx = \dfrac{p_1}{3}A,\;$ so $\; x = \dfrac{2V_1}{A}$.

Now, $\quad W_{\text{gas}} = -\,(W_{\text{atm}} + W_{\text{spring}}) = p_1Ax + \dfrac{1}{2}Kx^2$

$$= \left[p_1 A\cdot\frac{2V_1}{A} + \frac{1}{2}\cdot\frac{p_1 A}{3}\cdot\frac{2V_1}{A}\right] = \frac{7}{3}p_1V_1,$$

Heat supplied to the system,

$$\Delta Q = W + \Delta U = \frac{7}{3}p_1V_1 + \frac{3}{2}(p_2V_2 - p_1V_1) = \frac{41}{6}p_1V_1.$$

which is incorrect.

Hence, the correct options are (a), (b) and (c).

3. The graph between 0 and 100 K is linear, so statement (a) is correct.
The area under the given curve is proportional to the heat absorbed, so statement (b) is correct.
The given graph is parallel to T axis and has constant C, so statement (c) is correct.
The value of C increases with temperature in the range 200–300 K, so statement (d) is correct.

4. Net amount of heat radiated per second

$$\begin{aligned} Q &= \sigma A(T^4 - T_0^4) = \sigma A[(T_0 + \Delta T_0)^4 - T_0^4] \\ &= \sigma A\, T_0^4\left[\left(1+\frac{\Delta T_0}{T_0}\right)^4 - 1\right] = \sigma A T_0^4\left[\left(1+4\frac{\Delta T_0}{T_0}\right) - 1\right] \\ &= \sigma A\cdot 4\frac{\Delta T_0}{T_0}\cdot T_0^4 = A(\sigma T_0^4)\left(\frac{4\Delta T_0}{T_0}\right) \\ &= (1\ \text{m}^2)\,(460\ \text{W m}^{-2})\left(\frac{4\times 10}{300}\right) = 61.3\ \text{J} \approx 60\ \text{J}. \end{aligned}$$

In the first case, $Q_1 = \sigma[(T_0 + \Delta T)^4 - T_0^4]$.

In the second case, $Q_2 = \sigma[(T_0 + \Delta T)^4 - (T_0 - \Delta T_0)^4]$

$\Rightarrow$ more heat energy required to be radiated per second to maintain the body temperature,

$$\begin{aligned} Q_2 - Q_1 &= \sigma[T_0^4 - (T_0 - \Delta T_0)^4] \quad [\because \Delta T << T_0] \\ &= \sigma T_0^4\left[1 - \left(1 - \frac{\Delta T_0}{T_0}\right)^4\right] \approx 4\sigma T_0^3 \Delta T_0. \end{aligned}$$

Net heat radiated by a human body in 1 s,

$$Q = \sigma A(T^4 - T_0^4).$$

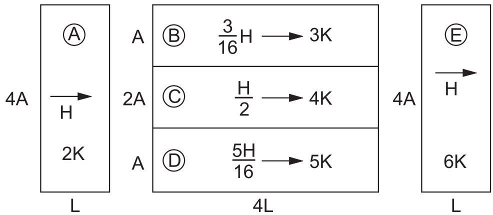
If the exposed area is reduced, the rate of heat loss Q is also reduced, so that the body temperature is maintained the same.

From Wien's law, $\lambda_m T$ = constant.

Thus the increase in body temperature corresponds to the shift of the peak of wavelength λ_m of the spectrum of electromagnetic radiation towards *smaller* wavelength side.

Hence, the correct options are (a), (b) and (c).

5.



The given system comprises three components shown separately in the figure, whose thermal resistances are

$$R_A = \frac{L}{8KA}, R_B = \frac{4L}{3KA},$$

$$R_C = \frac{4L}{8KA}, R_D = \frac{4L}{5KA}$$

and $$R_E = \frac{L}{24KA}.$$

Since the three are in series, thermal current through A and E will be the same.

Temperature difference: $\Delta\theta_A = HR_A = \dfrac{HL}{8KA}$,

$$\Delta\theta_B = \frac{3H}{16}R_B = \frac{HL}{4KA},\ \Delta\theta_C = \frac{HR_C}{2} = \frac{HL}{4KA},$$

$$\Delta\theta_D = \frac{5H}{16}R_D = \frac{HL}{4KA} \text{ and } \Delta\theta_E = HR_E = \frac{HL}{24KA}.$$

$\Rightarrow \Delta\theta_E$ is the smallest.

Now, $$I_C = \frac{\Delta\theta_C}{R_C} = \frac{H}{2}, I_B = \frac{\Delta\theta_B}{R_B} = \frac{3}{16}H$$

and $$I_D = \frac{\Delta\theta_D}{R_D} = \frac{5H}{16}$$

$\Rightarrow$ $I_C = I_B + I_D$.

Hence, the correct options are (a), (c) and (d).

6. Sunrays falling on the black body leads to more absorption of radiation. Since the temperature remains constant, it emits more radiation.

Sunlight is white light, which comprises the visible spectrum (red to violet), hence it radiates more energy in the visible spectrum.

Hence, the correct options are (b), (c) and (d).

7.

Gas	C_V	C_P	$C_P - C_V$	$C_P + C_V$	$\frac{C_p}{C_V}$	$C_P \times C_V$
Monatomic	$\frac{3}{2}R$	$\frac{5}{2}R$	R	$4R$	$\frac{5}{3}$	$\frac{15}{4}R^2$
Diatomic	$\frac{5}{2}R$	$\frac{7}{2}R$	R	$6R$	$\frac{7}{5}$	$\frac{35}{4}R^2$

So, options (b) and (d) are true.

8. The process A $\rightarrow$ B is a part of the semicircle, so it cannot be isothermal. For a process to be isothermal, the path should be a rectangular hyperbola.

In the process B $\rightarrow$ C $\rightarrow$ D, the gas undergoes compression. The work done by the gas is negative ($\Delta W < 0$), hence heat is expelled (flows out).

The work done during the process A $\rightarrow$ B $\rightarrow$ C is the area enclosed by the semicircle ($\int p\mathrm{d}V$), which is not zero.

For the clockwise process A $\rightarrow$ B $\rightarrow$ C $\rightarrow$ D $\rightarrow$ A, the work done by the gas is positive.

Hence, the correct options are (b) and (d).

2.3 Sound Waves

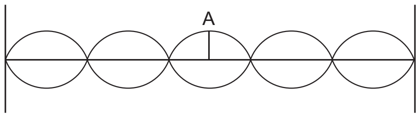
1. Comparing the given equation with the equation of a standing wave,

$$y(x, t) = A \sin \frac{2\pi}{\lambda} x \cos(\omega t),$$

we have, $A = 0.01$ m.

$$\frac{2\pi}{\lambda} = 62.8 \text{ m}^{-1}$$

$$= 20\,\pi \text{ m}^{-1}$$



or $$\frac{\lambda}{2} = \frac{1}{20} \text{ m} = 5 \text{ cm}.$$

$\therefore$ length of the string, $L = \frac{5\lambda}{2} = 25$ cm $= 0.25$ m.

Angular frequency of this mode, $\omega = 628 \text{ s}^{-1}$, $2\pi\nu = 200\pi \text{ s}^{-1}$.

$\therefore$ fundamental frequency $= \frac{\nu}{5} = \frac{100}{5}$ Hz $= 20$ Hz.

Hence, the correct options are (b) and (c).

2. Beat frequency:

(i) When the observer is at P,

$$\nu_P = (121 - 118)\frac{(V + V_0\cos\alpha)}{V}\text{Hz} = 3\left(\frac{V + V_0\cos\alpha}{V}\right)\text{Hz.}$$

(ii) When the observer is at Q,

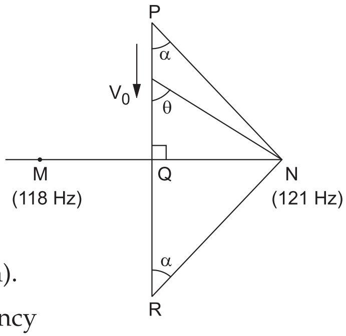
$$\nu_Q = 121 - 118 = 3\text{ Hz.}$$

(iii) When the observer is at R,

$$\nu_R = (121 - 118)\frac{(V - V_0\cos\alpha)}{V}\text{Hz} = 3\left(\frac{V - V_0\cos\alpha}{V}\right)\text{Hz}$$

$$\Rightarrow \nu_P + \nu_R = 2\nu_Q.$$

At any position (θ), the beat frequency



$$\nu(\theta) = 3\left(\frac{V + V_0\cos\theta}{V}\right)\text{Hz}$$

$$\Rightarrow \quad \frac{d\nu}{d\theta} = -3\frac{V_0}{V}\sin\theta.$$

At Q, $\sin\theta = \sin 90° = 1$ (maximum).

$\therefore$ the rate of change of beat frequency with time is maximum at Q.

Hence, the correct options are (a), (c) and (d).

3. Expressions for apparent frequency are

$f_2 = \left(\frac{v + w + u}{v + w - u}\right)f_1$, for wind blowing from the source to the observer.

$f_2 = \left(\frac{v - w + u}{v - w - u}\right)f_1$, for wind blowing from the observer to the source.

$\Rightarrow$ $f_2 > f_1$ in both the cases.

Hence, the correct options are (a) and (b).

4. $v = \sqrt{\frac{F}{\mu}}\cdot$ Speed is same at any point for both the pulses, hence $T_{AO} = T_{OA}$.

$\because$ $v = f\lambda$, so $\lambda \propto v$. At A, tension F decreases, so v decreases

$\Rightarrow$ λ decreases, so option (c) is incorrect.

v depends on F and μ and is independent of f and λ.

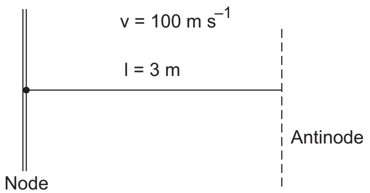
Hence, the correct options are (a), (b) and (d).

5. For possible modes of vibration in standing waves,

$$l = (2n+1)\frac{\lambda}{4} \Rightarrow \lambda = \frac{12}{(2n+1)} \text{ m.}$$

$$k = \frac{2\pi}{\lambda} = \frac{(2n+1)\pi}{6} \text{ and}$$

$$\omega = vk = \frac{(2n+1)50\pi}{3}.$$



For $n = 0$, $k = \frac{\pi}{6}$ and $\omega = \frac{50\pi}{3}$.

For $n = 2$, $k = \frac{5\pi}{6}$ and $\omega = \frac{250\pi}{3}$.

For $n = 7$, $k = \frac{5\pi}{2}$ and $\omega = 250\pi$.

Hence, the correct options are (a), (c) and (d).

6. $$v = \sqrt{\frac{\gamma p}{\rho}} = \sqrt{\frac{\gamma RT}{M}}, \text{ also } \nu = \frac{v}{\lambda} = \frac{v}{4l}$$

$$\Rightarrow l = \frac{1}{4\nu}\sqrt{\frac{\gamma RT}{M}}.$$

From the above equation the values of l for gases mentioned in the options (a), (b), (c) and (d) are found to be 0.459 m, 0.363 m, 0.340 m and 0.348 m respectively.

As $l = (0.350 \pm 0.005)$ m, the correct option is (d).

7. When reflection of a sound wave occurs at a rigid boundary (like the closed end of a pipe), the particles at the boundary are unable to vibrate. The reflected wave thus generated interferes with the incident wave to produce zero displacement (or node). At this displacement node exists the pressure antinode. Thus reflected pressure wave has the same phase as the incident wave and a high pressure compression pulse gets reflected as a compression pulse.

Similarly, for reflection from the open end of the pipe, the particles vibrate with increased amplitude (displacement antinode) and pressure remains at the average value (pressure node). The reflected pressure wave interferes destructively with the incident wave so that a phase change of π occurs from the open end.

Hence, a high pressure compression pulse reflects as a low pressure rarefaction pulse.

Thus, the correct options are (b) and (d).

8. $X = A\sin^2\alpha x + B\cos^2\alpha x + C\sin\alpha x\cos\alpha x$

$$= \frac{A}{2}(1-\cos 2\alpha x) + \frac{B}{2}(1+\cos 2\alpha x) + \frac{C}{2}\sin 2\alpha x$$

$$= \frac{1}{2}(A+B) - \frac{1}{2}(A\cos 2\alpha x - B\cos 2\alpha x) + \frac{C}{2}\sin 2\alpha x.$$

For $A = -B$, $C = 2B$, $X = B\cos 2\alpha x + B\sin 2\alpha x$.

This represents SHM with amplitude $\sqrt{2}B$.

For $A = B$, $C = 0$, we get

$X = A$. The motion is not SHM.

For $A = B$, $C = 2B$,

$X = B + B\sin 2\alpha x$.

This represents SHM with displaced origin and amplitude $= B$.

For any value of A, B and C (except $C = 0$),

$$X = \frac{1}{2}(A+B) + \frac{1}{2}(B-A)\cos 2\alpha x + \frac{C}{2}\sin 2\alpha x.$$

This represents SHM.

Hence, the correct options are (a), (b) and (d).

9. The first resonance is more intense than the second resonance.

The prongs are kept vertically (not horizontally) to produce longitudinal vibration in the air column.

For the first resonance, $l_1 + E = \frac{\lambda}{4}$, where E = end correction. So l_1 is slightly shorter than $\frac{\lambda}{4}$.

Hence, the correct options are (a) and (d).

2.4 Electrostatics

1. Q_1 is positive, Q_2 is negative. Also $|Q_1| > |Q_2|$.

Lines of force start from a positive charge and end on a negative charge, and are denser near a larger charge.

Hence, the correct options are (a) and (d).

2. $k\frac{Q_A}{R_A} = k\frac{Q_B}{R_B} = k\frac{2Q}{R_A + R_B}$ = common potential V

$$\Rightarrow \frac{Q_A}{Q_B} = \frac{R_A}{R_B} > 1.$$

$$\therefore \quad Q_A > Q_B.$$

$$\frac{\sigma_A R_A^2}{\sigma_B R_B^2} = \frac{R_A}{R_B} \Rightarrow \frac{\sigma_A}{\sigma_B} = \frac{R_B}{R_A}.$$

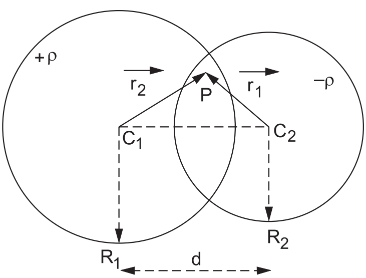
The field in the cavity of the metallic shell is zero.

On the surface, $\dfrac{E_A}{E_B} = \dfrac{Q_A}{Q_B}\dfrac{R_B^2}{R_A^2} = \dfrac{R_A}{R_B}\cdot\dfrac{R_B^2}{R_A^2} = \dfrac{R_B}{R_A} < 1$

$$\Rightarrow \qquad E_A < E_B.$$

Hence, the correct options are (a), (b), (c) and (d).

3. In $\triangle PC_1C_2$, $\vec{r}_2 = \vec{d} + \vec{r}_1$.



The electrostatic field in the overlapped region at any point P is

$$\vec{E} = \frac{k\frac{4}{3}\pi r_2^3 \rho}{r_2^2}\hat{r}_2 + \frac{k\frac{4}{3}\pi r_1^3(-\rho)}{r_1^2}\hat{r}_1$$

$$= k\cdot\frac{4}{3}\pi\rho(r_2\hat{r}_2 - r_1\hat{r}_1)$$

$$= k\frac{4}{3}\pi\rho(\vec{r}_2 - \vec{r}_1)$$

$$= k\frac{4}{3}\pi\rho\vec{d} = \frac{1}{4\pi\varepsilon_0}\frac{4\pi}{3}\rho\vec{d}$$

$$= \frac{\rho\vec{d}}{3\varepsilon_0} = \text{constant and parallel.}$$

Hence, the correct options are (c) and (d).

4. The position of all the charges are symmetric about the planes $x = \pm\frac{a}{2}$, so the net electric flux through them will be the same.

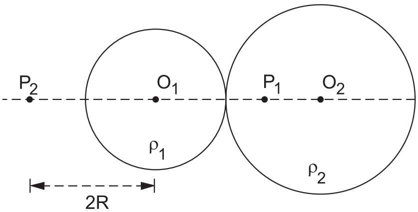
The same is true for the planes $y = \pm\frac{a}{2}$.

Hence $\phi = \dfrac{1}{\varepsilon_0}\Sigma q_{\text{inside}} = \dfrac{3q - q - q}{\varepsilon_0} = \dfrac{q}{\varepsilon_0}$.

By symmetry, flux through $z = +\frac{a}{2}$ is equal to flux through $x = +\frac{a}{2}$.

Hence, the correct options are (a), (c) and (d).

5. At point P_1,



$$\frac{1}{4\pi\varepsilon_0}\frac{\rho_1\left(\frac{4}{3}\pi R^3\right)}{4R^2} = \frac{\rho_2 R}{3\varepsilon_0}$$

$\Rightarrow \quad \dfrac{\rho_1}{\rho_2} = 4.$

At point P_2,

$$\frac{\rho_1\left(\frac{4}{3}\pi R^3\right)}{(2R)^2} + \frac{\rho_2\left(\frac{4}{3}\pi\times 8R^3\right)}{(5R)^2} = 0$$

$\Rightarrow \quad \dfrac{\rho_1}{\rho_2} = -\dfrac{32}{25}.$

Hence, the correct options are (b) and (d).

6. Field at O is $6K$, along OD.

Line PR is the perpendicular bisector of all the dipoles, so $V_0 = 0$.

Hence, the correct options are (a), (b) and (c).

7. The displacement of $+q$ towards $+x$ direction produces a net restoring force towards $-x$ direction and vice versa.

Restoring force,

$$F = q\left[\frac{2K\lambda}{d-x} - \frac{2K\lambda}{d+x}\right] \approx \frac{4K\lambda q}{d^2}x$$

$\Rightarrow$ motion is SHM with

$$T = 2\pi\sqrt{\frac{md^2}{4K\lambda q}}.$$

In the second case, $-q$ will move along the direction of displacement.

Hence, the correct option is (c).

8. $$\frac{Q}{4\pi\varepsilon_0 r_0^2} = \frac{\lambda}{2\pi\varepsilon_0 r_0} = \frac{\sigma}{2\varepsilon_0}$$

$\Rightarrow \quad Q = 2\pi\sigma r_0^2.$

$$r_0 = \frac{\lambda}{\pi\sigma}.$$

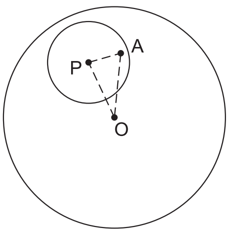
$$E_1\left(\frac{r_0}{2}\right) = \frac{4E_1(r_0)}{1}.$$

$$E_1\left(\frac{r_0}{2}\right) = 2E_2\left(\frac{r_0}{2}\right).$$

$$E_3\left(\frac{r_0}{2}\right) = E_3(r_0) = E_2(r_0).$$

Hence, the correct option is (c).

9. The electric field at A ($\overrightarrow{OA} = \vec{r}$) due to the solid sphere,



$$\vec{E}_1 = \frac{1}{4\pi\varepsilon_0}\left(\frac{4}{3}\pi r^3 \rho\right)\frac{\hat{r}}{r^2} = \frac{\rho\,\vec{r}}{3\varepsilon_0}.$$

Similarly, the field at A due to cavity,

$$\vec{E}_2 = \frac{\rho}{3\varepsilon_0}(-\overrightarrow{PA})$$

(–ve sign due to absence of charge)

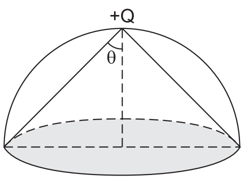
$$= \frac{\rho}{3\varepsilon_0}\overrightarrow{AP}.$$

$\Rightarrow$ net field at A

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \frac{\rho}{3\varepsilon_0}(\overrightarrow{OA} + \overrightarrow{AP}) = \frac{\rho}{3\varepsilon_0}\,\overrightarrow{OP} = \frac{\rho}{3\varepsilon_0}\vec{a}.$$

Hence, the correct option is (d).

10. The electric flux linked with the hemisphere will be contained within the solid angle subtended by the flat surface at the position of the charge,



which is $\Omega = 2\pi(1 - \cos 45°) = 2\pi\left(1 - \frac{1}{\sqrt{2}}\right)$.

$\Rightarrow$ flux through the curved surface,

$$\psi = -\frac{Q}{\varepsilon_0}\frac{2\pi}{4\pi}\left(1 - \frac{1}{\sqrt{2}}\right) = -\frac{Q}{2\varepsilon_0}\left(1 - \frac{1}{\sqrt{2}}\right).$$

All points at the circumference are at the same distance ($\sqrt{2}R$) from the charge, so it is equipotential.

Hence, the correct options are (a) and (d).

11. After closing the switch S_1, C_1 is charged by $2CV_0$. When S_2 is closed, this charge is equally shared between C_1 and C_2 and both of their upper plates have charge CV_0.

When S_3 is closed, the upper plate of C_2 is charged by $-CV_0$ and the lower plate, by $+CV_0$.

Hence, the correct options are (b) and (d).

12. Since both the parts have a common potential difference (V) and the same separation d, the ratio $\frac{E_1}{E_2} = 1$ $\left[\because\ E = \frac{V}{d}\right]$.

Let C_0 = capacitance of the whole air-capacitor (without dielectric):

$$K\frac{C_0}{3} + 2\frac{C_0}{3} = C \quad \text{or} \quad C = \left(\frac{K+2}{3}\right)C_0.$$

For the upper portion, $C_1 = K\dfrac{C_0}{3}$.

$\therefore \qquad \dfrac{C}{C_1} = \dfrac{2+K}{K}$.

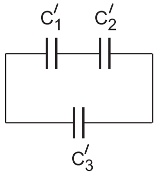
Hence, the correct options are (a) and (d).

13. The system comprises three capacitors whose capacitances are

$$C_1' = \frac{\varepsilon_1\varepsilon_0 A/2}{d/2} = 4\frac{\varepsilon_0 A}{d} = 4C_1,$$

$$C_2' = \frac{\varepsilon_2\varepsilon_0 A/2}{d/2} = \frac{2\varepsilon_0 A}{d} = 2C_1,$$

and $$C_3' = \frac{\varepsilon_1\varepsilon_0 A/2}{d} = \frac{2\varepsilon_0 A}{2d} = C_1.$$



Here C_1', C_2' are in series with C_3' in parallel with the combination.

$\therefore$ equivalent capacitance, $C_2 = \dfrac{C_1' \times C_2'}{C_1' + C_2'} + C_3' = \left(\dfrac{4}{3}+1\right)C_1 = \dfrac{7}{3}C_1$.

$\therefore$ ratio $\dfrac{C_2}{C_1} = \dfrac{7}{3}$.

Hence, the correct option is (d).

14. Electrostatic and gravitational static fields are both conservative and they never form closed loops.

Hence, the correct options are (a) and (c).

15. The nature of the graph showing the variation of the potential ϕ_r with distance r corresponds to that due to a uniformly charged spherical conducting shell of radius R_0.

Inside the shell ($r < R_0$), electric field $E = 0$, so the total electrostatic energy stored is zero.

Total charge q will be on the surface of the shell ($r = R_0$) and nowhere else.

Within the concentric spherical shell of radius $r = 2R_0$, total charge contained is q.

Electric field within the shell is zero and at the surface ($r = R_0$), it suddenly changes to $\dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{R_0^2}$, so there is discontinuity at $r = R_0$.

Hence, the correct options are (a), (b), (c) and (d).

16. Coulomb force between Q and $-q$ is radial,

so the torque $\vec{\tau} = \vec{r} \times \vec{F} = rF\sin 180° = 0$.

$\therefore$ angular momentum $(\vec{r} \times \vec{p})$ is conserved.

The linear speed, linear momentum and angular velocity change with time.

Hence, the correct option is (a).

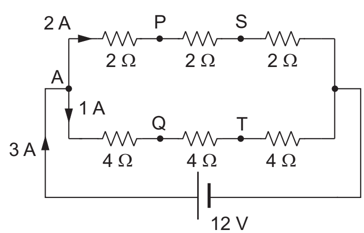
2.5 Current Electricity and Magnetism

1. At the time of breaking up of the filament, the temperature becomes higher and according to Wien's law $\lambda_{max} \propto \frac{1}{T}$, so $\nu_m \propto T$, the filament emits more light at higher band frequencies.

   As the supply voltage remains constant, so consumed power $P = \frac{V^2}{R}$. Since the resistance R increases with the rise in temperature, so the filament consumes less power towards the end of the life of the bulb.

   Hence, the correct options are (c) and (d).

2. By symmetry, P and Q as well as S and T are at the same potential, so resistance across them are ineffective because potential difference across them is zero. The simplified circuit will be as shown in the figure.



$$R_{eq} = \frac{6 \times 12}{18} = 4\ \Omega.$$

$$I_1 = \frac{12}{4} = 3\text{ A}.$$

$$I_2 = \frac{12}{6+12} \times 3 = 2\text{ A}.$$

$$V_A - V_S = (2\text{ A})(4\ \Omega) = 8\text{ V}.$$

$$V_A - V_T = (1\text{ A})(8\ \Omega) = 8\text{ V}.$$

$$V_P = V_Q \Rightarrow \text{current through PQ} = 0.$$

$$V_P = V_Q \Rightarrow V_Q > V_S.$$

$$I_2 = 3\text{ A} - 1\text{ A} = 2\text{ A}.$$

Hence, the correct options are (a), (b), (c) and (d).

3. $$R_{iron} = \rho\frac{l}{A} = \frac{(1.0 \times 10^{-7}\ \Omega\text{ m})(50 \times 10^{-3}\text{ m})}{4 \times 10^{-6}\text{ m}^2} = 1250\ \Omega.$$

$$R_{aluminium} = \rho\frac{l}{A} = \frac{(2.7 \times 10^{-8}\ \Omega\text{ m})(50 \times 10^{-3}\text{ m})}{(49-4) \times 10^{-6}\text{ m}^2} = 30\ \Omega.$$

$$\Rightarrow \text{equivalent resistance} = \frac{R_1 \times R_2}{R_1 + R_2} = \frac{1250 \times 30}{1280} \times 10^{-6}\ \Omega = \frac{1875}{64}\ \mu\ \Omega.$$

Hence, the correct option is (b).

4. $R_1 = \rho\dfrac{L}{A} = \rho\dfrac{L}{\pi r^2}$; $R_2 = \rho\dfrac{L}{\pi 4r^2} = \dfrac{R_1}{4}$.

Initially, $H = \dfrac{V^2}{R_1}\cdot 4.$

When both the wires of resistance R_2 are in series,

$$H = \frac{V^2 t_1}{2R_2} = \frac{V^2 t_1}{2(R_1/4)} = \frac{2V^2 t_1}{R_1}.$$

$\therefore \qquad 2t_1 = 4 \text{ min}, \; t_1 = 2 \text{ min}.$

When both the wires of resistance R_2 are in parallel, the equivalent resistance

$$= \frac{R_2}{2} = \frac{R_1}{8}$$

$$\Rightarrow \qquad H = \frac{V^2}{R_1/8}t_2 = 8\frac{V^2}{R_1}t_2.$$

$\therefore \qquad 8t_2 = 4 \text{ min} \Rightarrow t_2 = 0.5 \text{ min}.$

Hence, the correct options are (b) and (d).

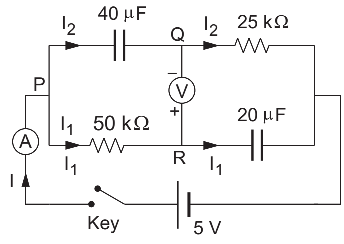
5. When the key is closed at $t = 0$, voltage across each capacitor is zero, so reading of the voltmeter is –5 V.

At $t = \infty$, the capacitors are fully charged, so the voltmeter reads +5 V.

During the transient state, $I_1 = \dfrac{5}{50}e^{-t/\tau}$ mA, $I_2 = \dfrac{5}{25}e^{-t/\tau}$ mA

and $I = I_1 + I_2$, where $\tau = RC = 1$ s.

After 1 s, current through the ammeter is $\dfrac{1}{e}$ times the initial value, and at $t = \infty$, it becomes zero.



Reading of the voltmeter at any instant is

$$\begin{aligned} V_Q - V_R &= (V_P - V_R) - (V_P - V_Q) \\ &= \Delta V_{50\,k\Omega} - \Delta V_{40\,\mu F} \\ &= 5\,e^{-t/\tau} - 5(1 - e^{-t/\tau}) = 5(2e^{-t/\tau} - 1). \end{aligned}$$

But $t = \ln 2$ s and $\tau = 1$ s, so $e^{-t/\tau} = e^{-\ln 2} = \dfrac{1}{2}$.

$\therefore$ voltmeter reads 0 V at $t = \ln 2$ s.

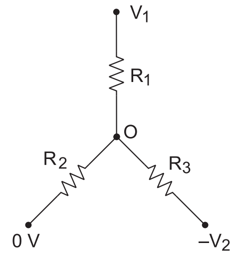
Hence, the correct options are (a), (b), (c) and (d).

6. Let V_0 = potential at O.

According to junction rule:

$$\frac{V_1 - V_0}{R_1} = \frac{V_0 - 0}{R_2} + \frac{V_0 + V_2}{R_3}$$

$$\Rightarrow \qquad V_0 = \frac{\dfrac{V_1}{R_1} - \dfrac{V_2}{R_3}}{\dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}}.$$



Current through R_2 will be zero, if

$$V_0 = 0 \quad \text{so} \quad \frac{V_1}{V_2} = \frac{R_1}{R_3}.$$

This condition is satisfied by the options given (a), (b) and (d).

7. Consider the situation shown in Figure (i), in which all the elements are connected in series.

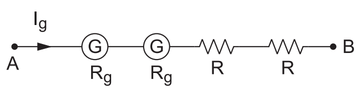


Fig. (i)

If I_g be the current through galvanometer for full-scale deflection (max range), the potential difference across AB is

$$V_1 = I_g(2R_g + 2R) = 2I_g(R + R_g).$$

Again, consider the situation shown in Figure (ii).

I_g flows through each galvanometer for full-scale deflection.

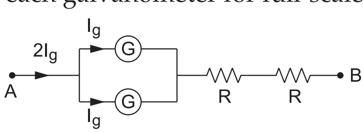


Fig. (ii)

$\therefore$ voltage range, $V_2 = V_A - V_B = I_gR_g + 2I_g \cdot 2R = I_g(R_g + 4R)$.

If $2R_g < R$ then $V_2 > V_1$.

For maximum current range, all the components are to be connected in parallel.

Hence, the correct options are (b) and (c).

8. The magnetic field is nonzero in the region $0 < r < R$ as well as in the region $r > 2R$.

Hence, the correct options are (a) and (d).

9. Effective length of the wire, $\vec{l} = 2(L + R)\hat{i}$ and magnetic force, $\vec{F} = I(\vec{l} \times \vec{B}) = 2I(L + R)\hat{i} \times \vec{B}$.

If $\vec{B} = B\hat{k}$, $\vec{F} = 2I(L+R)B\,\hat{i} \times \hat{k}$

$= 2I(L+R)B(-\hat{j})$

$\Rightarrow F \propto (L+R)$.

If $\vec{B} = B\hat{i}$, $\vec{F} = 2I(L+R)B\,(\hat{i} \times \hat{i}) = 0$.

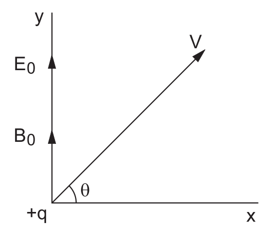
If $\vec{B} = B\hat{j}$, $\vec{F} = 2I(L+R)B\,(\hat{i} \times \hat{j}) = 2B(L \times R)\hat{k}$

$\Rightarrow F \propto (L+R)$.

If $\vec{B} = B\hat{k}$, $F \neq 0$, so option (d) is incorrect.

Hence, the correct options are (a), (b) and (c).

10. For $\theta = 0$, path of the charge is circular due to $\vec{B}$ field. In addition, it experiences force due to the electric field ($qE_0\hat{j}$); it has an acceleration along the y-axis. So the resulting path will be helical with a variable pitch.



With similar arguments, for $\theta = 10°$, the path will be helical with the pitch increasing with time.

For $\theta = 90°$, $\vec{F}_B = q(\vec{v} \times \vec{B})$ is zero but $F_{el} = qE_0$: the motion is linear, accelerated along the y-axis.

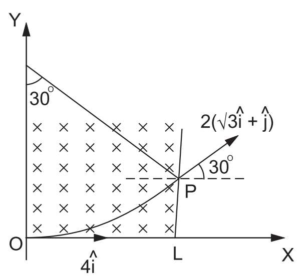
Hence, the correct options are (c) and (d).

11. $qvB = \dfrac{mv^2}{r}$, $r = \dfrac{mv}{qB}\cdot$ So $r \propto m$.

The particle will describe semicircular paths (parallel) with radii $\propto$ mass. Time $T = \dfrac{2\pi m}{qB}$, so $T \propto m$.

Hence, the correct options are (b) and (d).

12. The magnetic field exists along the negative z-direction as shown.



The radius of the circular path $r = \dfrac{Mv}{QB}\cdot$

The velocity at the point of emergence P is $\vec{v} = 2(\sqrt{3}\,\hat{i} + \hat{j})$ m s^{-1}.

So it makes an angle θ with the x-axis, when $\theta = \tan^{-1}\dfrac{1}{\sqrt{3}} = 30° = \dfrac{\pi}{6}\cdot$

$\therefore$ path OP $= \dfrac{\pi}{6}\cdot r = \dfrac{\pi}{6}\dfrac{Mv}{QB} = \dfrac{\pi}{6}\dfrac{M\,4}{QB} = \dfrac{2}{3}\dfrac{\pi M}{QB}.$

$$\therefore \text{ time } t = 10 \text{ m s} = 10\times10^{-3}\text{ s} = \frac{\text{Path OP}}{v} = \frac{2}{3}\frac{\pi M}{QB\times 4}$$

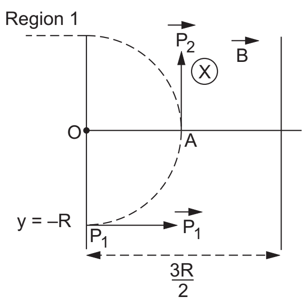
$$\Rightarrow B = \frac{50\,\pi M}{3Q}.$$

Hence, the correct options are (a) and (c).

13. Since $r = \dfrac{mv}{QB}$, the distance of re-entry into Region 1 from P_1 is $2r$

$$\Rightarrow d = \frac{2\,mv}{QB}$$

$$\Rightarrow d \propto m.$$



Given, $\quad B = \dfrac{8}{13}\dfrac{p}{QR}$,

so the radius, $r = \dfrac{p}{QB} = \dfrac{13}{8}R = 1.6R$,

which is greater than $\dfrac{3R}{2}$.

Hence the particle will enter Region 3.

For $B > \dfrac{2}{3}\dfrac{p}{QR}$, $r' < \dfrac{3}{2}R$, so the particle will re-enter Region 1.

Linear momentum has a constant magnitude in a magnetic field.

$\therefore$ at P_1, $\vec{p}_i = p\hat{i}$ and at the farthest point from the y-axis (at A), $\vec{p}_f = p\hat{j}$.

$\therefore$ the magnitude of the change in momentum,

$$|\Delta\vec{p}| = |\vec{p}_f - \vec{p}_i| = p|\hat{j} - \hat{i}| = \sqrt{2}p.$$

Hence, the correct options are (a) and (b).

14. Induced emf $0 \to L$ (region) and $3L \to 4L$ (region).

Motional emf $\mathcal{E} = -BLv$, $i = \dfrac{\mathcal{E}}{R} = \dfrac{BLv}{R}$.

Force on a current-carrying conductor in a $\vec{B}$ field,

$$F = iLB.$$

$$\text{Now, } F = -m\frac{dv}{dt} = -m\frac{dv}{dx}\cdot\frac{dx}{dt} = -mv\frac{dv}{dx} = iLB = \frac{B^2L^2v}{R}$$

$$\Rightarrow \quad dv = -\frac{B^2L^2}{mR}dx.$$

Integrating, $\int_{v_0}^{v} dv = -\frac{B^2L^2}{mR}\int_0^x dx.$

$\therefore$ velocity as a function of x,

$$v(x) - v_0 = -\frac{B^2L^2}{mR}\cdot x \Rightarrow v(x) = v_0 - \frac{B^2L^2}{mR}\cdot x.$$

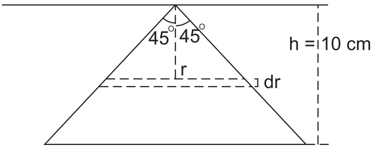
Now, current at any position (x),

$$i(x) = \frac{BL}{R}v = \frac{BL}{R}\left[v_0 - \frac{B^2L^2}{mR}x\right],$$

and the force $F(x) = LB\cdot i = \frac{L^2B^2}{R}\left[v_0 - \frac{B^2L^2}{mR}x\right].$

Considering Lenz's law with these results, the graphs represented in options (c) and (d) are correct.

15. The magnetic flux linked with the loop due to current (assumed) through the wire,



$$\phi_{l,w} = \int_0^h B\,dA$$

$$= \int_0^h \frac{\mu_0 I}{2\pi r}\cdot 2r\,dr = \frac{\mu_0 Ih}{\pi}$$

$\Rightarrow$ mutual inductance $M = \frac{\phi_{l,w}}{I} = \frac{\mu_0 h}{\pi}.$

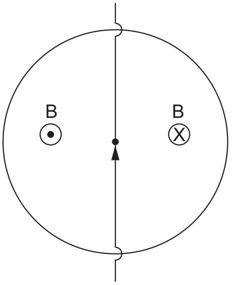
The magnitude of induced emf in the wire

$$\mathcal{E}_w = \left|-M\frac{dI}{dt}\right| = \frac{\mu_0 h}{\pi}\cdot\frac{dI}{dt} = \frac{\mu_0}{\pi}(0.1\text{ m})(10\text{ A s}^{-1}) = \left(\frac{\mu_0}{\pi}\right)\text{V}.$$

According to Lenz's law, the loop should move away, so there will be a repulsive force between the wire and the loop.

Hence, the correct options are (b) and (c).

16. The flux of the magnetic field through the wire loop is as much positive (outward) as negative (inward).



Hence, ϕ = zero and

$$\frac{d\phi}{dt} = \text{zero.}$$

Hence, the correct options are (a) and (c).

17. After a very long time, magnetic flux linked with each coil is the same, hence

$$L_1 i_1 = L_2 i_2 \qquad \left(\because -L_1\frac{dI_1}{dt} = -L_2\frac{dI_2}{dt}\right)$$

$$\Rightarrow i_1 = \frac{L_2}{L_1+L_2}i = \frac{L_2}{L_1+L_2}\frac{V}{R} \text{ and}$$

$$i_2 = \frac{L_1}{L_1+L_2}i = \frac{L_1}{L_1+L_2}\frac{V}{R}.$$

At $t = 0$, the inductors offer a large obstruction to the current through R. So, $i = 0$.

Now, $\dfrac{i_1}{i_2} = \dfrac{L_2}{L_1}$ = constant.

Hence, the correct options are (b), (c) and (d).

18. The net magnetic flux through the loop at time t is

$$\phi = B(2A - A)\cos\omega t = BA\cos\omega t. \qquad \text{...(i)}$$

$$\Rightarrow \left|\frac{d\phi}{dt}\right| = BA\omega\sin\omega t, \text{ which is maximum when } \omega t = \frac{\pi}{2}.$$

In the smaller loop,

$$\mathcal{E} = -\frac{d\phi}{dt} = AB\omega\sin\omega t,$$

and the net emf in the system due to both the loops (see Equation i),

$$\mathcal{E}' = -\frac{d\phi}{dt} = AB\omega\sin\omega t$$

$\Rightarrow$ amplitudes ($AB\omega$) in both the cases is the same.

Hence, the correct options are (a) and (d).

19. The amplitude of the voltage across the terminals X and Y,

$$V_{XY} = \sqrt{V_0^2 + V_0^2 - 2V_0^2\cos\frac{2\pi}{3}} = \sqrt{3}\,V_0,$$

Similarly, $V_{YZ} = V_{XZ} = \sqrt{3}\,V_0$.

The rms value $= \dfrac{\text{peak value}}{\sqrt{2}}$

$$\Rightarrow \qquad V_{XY}^{rms} = \sqrt{\frac{3}{2}}\,V_0$$

and $\qquad V_{YZ}^{rms} = \sqrt{\dfrac{3}{2}}\,V_0.$

Voltmeter reading (V^{rms}) is the same for any two terminals.
Hence, the correct options are (a) and (d).

20. In a capacitor, current leads the voltage by $\frac{\pi}{2}$, hence initially

$$V = V_0 \sin \omega t.$$

$\therefore$ charge $Q = CV = CV_0 \sin \omega t = Q_0 \sin \omega t,$

where $Q_0 = CV_0 = CI_0X_C = \frac{I_0}{\omega} = 2 \times 10^{-3}$ C.

At $t = \frac{7\pi}{6\omega}$; $\cos \omega t = -\frac{\sqrt{3}}{2}$, hence

$$I = -\frac{\sqrt{3}}{2} I_0, \text{ which is anticlockwise.}$$

Immediately after $t = \frac{7\pi}{6\omega}$, the current

$$i = \frac{V_c + 50}{R} = 10 \text{ A.}$$

Charge flow $= Q_{\text{final}} - Q_{\left(\text{at } \frac{7\pi}{6\omega}\right)} = 2 \times 10^{-3}$ C.

Hence, the correct options are (c) and (d).

21. $X_C = \frac{1}{\omega C} = \frac{1}{(100)(100 \times 10^{-6})} = 100\ \Omega.$

$X_L = \omega L = (100)(0.5) = 50\ \Omega.$

For the upper branch (1), $z_1 = \sqrt{R^2 + X_C^2} = 100\sqrt{2}\ \Omega.$

For the lower branch (2), $z_2 = \sqrt{R^2 + X_L^2} = 50\sqrt{2}\ \Omega.$

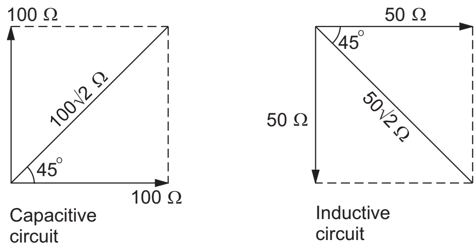
Source voltage $V = V_0 \sin \omega t = 20\sqrt{2} \sin \omega t.$

$$\therefore \quad I_1 = \frac{20\sqrt{2}}{100\sqrt{2}} \sin\left(\omega t + \frac{\pi}{4}\right),$$

and $I_2 = \frac{20\sqrt{2}}{50\sqrt{2}} \sin\left(\omega t - \frac{\pi}{4}\right)$.

Taking the rms values,

$$\Rightarrow \quad I = \sqrt{I_1^2 + I_2^2} = \sqrt{\frac{1}{50} + \frac{4}{50}} = \sqrt{\frac{1}{10}} \approx 0.3 \text{ A.}$$



Voltage across the resistor,

$$V_{R_{100}} = (I_1)_{\text{rms}} \times 100 = \frac{0.2}{\sqrt{2}} \times 100 = 10\sqrt{2}\ \text{V}.$$

Similarly, $V_{R_{50}} = \dfrac{0.4}{\sqrt{2}} \times 50 = 10\sqrt{2}\ \text{V}.$

Hence, the correct options are (a) and (c).

22. Peak current $I_0 = \dfrac{V_0}{z} = \dfrac{V_0}{\sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}}.$

At resonance,

(i) the phase difference between current and voltage is zero, and

(ii) the circuit is resistive.

$\therefore \quad X_L = X_C \Rightarrow \omega_r = \sqrt{\dfrac{1}{LC}}$

$\Rightarrow \quad \omega_r = 10^6\ \text{rad s}^{-1}.$

The resonant frequency $\dfrac{1}{\sqrt{LC}}$ does not depend on R.

At $\omega\ (>> \omega_r = 10^6)$, $X_L >> X_C$, the circuit acts as inductive.

If $\omega \to 0,\ X_C = \dfrac{1}{\omega C} \to \infty,\ z \to \infty,$ so $I \to 0.$

Hence, the correct options are (a) and (b).

23. $I = \dfrac{V}{Z},\ V^2 = V_R^2 + V_C^2 = (IR)^2 + \left(\dfrac{I}{\omega C}\right)^2$ and $Z^2 = R^2 + \left(\dfrac{1}{\omega C}\right)^2.$

$\therefore$ current $I = \dfrac{V}{\sqrt{R^2 + \dfrac{1}{\omega^2 C^2}}}$ and $V_C = I\left(\dfrac{1}{\omega C}\right).$

In case (A), C_A is less and in case (B), C_B has larger value.

$\therefore\ (X_C)_A > (X_C)_B \Rightarrow Z_A > Z_B \Rightarrow I_R^A < I_R^B.$

Similarly, $V_C = \frac{V}{Z} X_C = \frac{V}{\sqrt{1+R^2\omega^2C^2}}$

$\Rightarrow \; V_C^A > V_C^B.$

Hence, the correct options are (b) and (c).

24. The forces on BC and DA are equal and opposite in direction. Hence F_{net} on the loop is zero.

Torque $\vec{\tau} = \vec{m} \times \vec{B} = I\vec{A} \times \vec{B}.$

$\because \; \vec{A}$ is $\perp \vec{B}, |\vec{\tau}| = IAB \neq 0.$

As seen from O, the loop rotates clockwise.

Hence, the correct options are (a) and (c).

25. For a charge q in a magnetic field B,

$$F = qvB, \frac{mv^2}{r} = qvB, \; r = \frac{mv}{qB}.$$

For the particle to enter Region III, $r > l \Rightarrow \frac{mv}{qB} > l \;\; \Rightarrow \; v > \frac{qlB}{m}.$

For the path length to be maximum, $l = r$

$$\Rightarrow \; v = \frac{qBl}{m}.$$

For $r < l$, the particle describes a semicircle for which time $t = \frac{T}{2} = \frac{\pi m}{qB}$, which is independent of velocity.

Hence, the correct options are (a), (c) and (d).

26. Equivalent resistance of the circuit,

$$R \; = \; 2\text{ k}\Omega + \frac{(6\text{ k}\Omega)\,(1.5\text{ k}\Omega)}{(7.5\text{ k}\Omega)} = \frac{16}{5}\text{ k}\Omega.$$

$\therefore$ current through the battery,

$$I \; = \; \frac{V}{R} = \frac{24\text{ V}}{\frac{16}{5}\text{ k}\Omega} = 7.5\text{ mA}.$$

The potential difference across R_1 is IR_1 = (7.5 mA) (2 kΩ) = 15 V.

$\therefore$ the potential difference across the parallel combination of 6 kΩ and 1.5 kΩ will be 24 V – 15 V = 9 V.

The ratio of power dissipated in R_1 and R_2 is

$$\frac{P_1}{P_2} \; = \; \frac{I^2R_1}{V_2^2/R_2} = \frac{(7.5\text{ mA})^2 \times (2\text{ k}\Omega)}{(15\text{ V})^2/6\text{ k}\Omega} = 3:1.$$

Interchanging R_1 and R_2, equivalent resistance

$$R' = 6\text{ k}\Omega + \frac{(2\text{ k}\Omega)(1.5\text{ k}\Omega)}{3.5\text{ k}\Omega} = \frac{48}{7}\ \Omega.$$

$\therefore$ current $I' = \dfrac{24}{48/7}\text{ A} = \dfrac{7}{2}\text{ A}.$

The PD across R_L, $V' = 24\text{ V} - \left(\dfrac{7}{2}\times 6\right)\text{V} = 3\text{ V}.$

The power dissipated in R_L is $\dfrac{V'^2}{R_L} = \dfrac{(3\text{ V})^2}{R_L}.$

Previous power dissipation $= \dfrac{V^2}{R_L} = \dfrac{(9\text{ V})^2}{R_L}.$

$\therefore$ the power dissipation is decreased by a factor of 9.

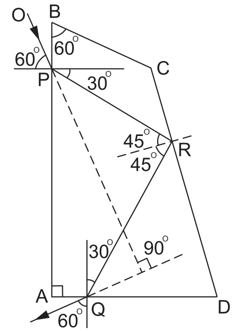
Hence, the correct options are (a), (c) and (d).

27. As the induced emf $-\dfrac{d\phi}{dt}$ is same in both the rings, the strength of the induced current will depend upon the resistance of the ring.

Larger the resistivity, smaller is the current. Thus for $h_A > h_B$, the correct options are (b) and (d).

2.6 Ray Optics and Wave Optics

1. At P, $\sin 60° = \sqrt{3}\sin\gamma$. $\therefore\ \gamma = 30°$.



$\sin C = \dfrac{1}{\sqrt{3}}$, $C < 45°$.

Internal reflection occurs at R. Angle between the initial and final rays = angle of deviation = 90°.

Hence, the correct options are (a), (b) and (c).

2. For air to glass,

$$\frac{1.5}{f_1} = \frac{1.4-1}{R} + \frac{1.5-1.4}{R} \Rightarrow f_1 = 3R.$$

For glass to air,

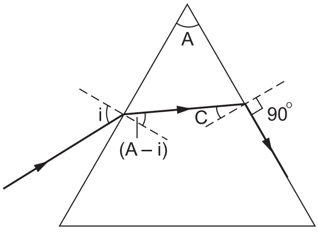
$$\frac{1}{f_2} = \frac{1.4-1.5}{-R} + \frac{1-1.4}{-R} \Rightarrow f_2 = 2R.$$

Hence, the correct options are (a) and (c).

3. $$\mu = \frac{\sin\frac{A+\delta_m}{2}}{\sin\frac{A}{2}} = \frac{\sin A}{\sin\frac{A}{2}} = 2\cos\frac{A}{2}. \quad ...(i)$$

At minimum deviation, $i + i = A + \delta_m = 2A$

$\Rightarrow \quad i_1 = A, \; r_1 = \dfrac{A}{2} = \dfrac{i_1}{2}.$



From equation (i), $A = 2\cos^{-1}\left(\dfrac{\mu}{2}\right).$

$\sin C = \dfrac{1}{\mu}, \; \cos C = \sqrt{1-\dfrac{1}{\mu^2}}.$

$$\begin{aligned}
\text{Now,}\quad \sin i &= \mu\sin(A-C)\\
&= \mu(\sin A\cos C - \cos A\sin C)\\
&= \mu\left(\sin A\sqrt{1-\frac{1}{\mu^2}} - \frac{\cos A}{\mu}\right)\\
&= \left(\sin A\sqrt{\mu^2-1} - \cos A\right)\\
\Rightarrow \quad \sin i &= \left(\sin A\sqrt{4\cos^2\frac{A}{2}-1} - \cos A\right).
\end{aligned}$$

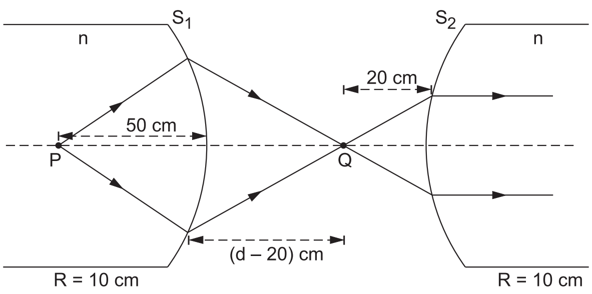
At $i_1 = A$, the ray undergoes minimum deviation through the prism and passes symmetrically so that the ray inside the prism is parallel to its base.

Hence, the correct options are (a), (c) and (d).

4. For refraction through S_1:

$$\frac{1}{v} - \frac{1.5}{-50} = \frac{1-1.5}{-10}$$

$\Rightarrow \quad v = 50$ cm (at Q).



For refraction through S_2:

$$u = -(d - 50) \text{ cm},$$

$$v = \infty \ (\because \text{rays are parallel}).$$

$$\therefore \quad \frac{1.5}{\infty} - \frac{1}{-(d-50)} = \frac{1.5-1}{+10} \Rightarrow d = 70 \text{ cm}.$$

Hence, the correct option is (b).

5. From lens formula, $\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{60} - \frac{1}{(-30)} = \frac{1}{20}$

$\Rightarrow f = 20 \text{ cm} = \frac{R}{n-1}$.

For reflection from the convex surface (using mirror formula),

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v} = \frac{1}{(-30)} + \frac{1}{10} = \frac{1}{15}.$$

$$\therefore \quad R = 2f = 30 \text{ cm}.$$

But $\quad n - 1 = \frac{R}{20} = 1.5. \quad \therefore\ n = 2.5.$

The faint image is erect and virtual.

Hence, the correct options are (a) and (d).

6. From Snell's law,

$n_1 \sin\theta_i = n_2 \sin\theta_2 = n_2 \sin\theta_f$.

The deviation of ray in the slab will depend on n(z). Hence, l will depend on n(z) and is independent of n_2.

Hence, the correct options are (a), (c) and (d).

7. The path difference at any angular position P(θ) is

$$\Delta = d\cos\theta.$$

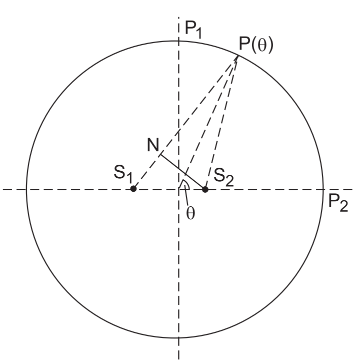
Path difference at P_1:

$$\Delta = d\cos 90° = 0 \text{ (max)}.$$

Path difference at P_2:

$$\Delta = d\cos 0° = d = 1.8 \text{ mm}$$

$$\Rightarrow \quad \frac{\Delta}{\lambda} = \frac{1.8 \text{ mm}}{600 \text{ nm}} = 3000.$$



$\therefore$ 3000th-order bright fringe will form at P_2, and the total number of fringes between P_1 and P_2 is close to 3000.

Angular separation between two consecutive fringes

$$= \left|\frac{d\theta}{dn}\right| = \frac{\lambda}{d\sin\theta}.$$

As we move from P_1 to P_2, the angular width increases.

Hence, the correct options are (a) and (c).

8. Since the line joining S_1 and S_2 is perpendicular to the screen, the fringe pattern will consist of semicircles.

At O, phase difference

$$= \frac{2\pi}{\lambda}(S_1O - S_2O) = 2\pi\frac{(0.6003\times10^{-3})\text{ m}}{(600\times10^{-9})\text{ m}} = 2001\,\pi.$$

Thus the region very close to O is dark.

Hence, the correct options are (b) and (d).

9. $d = \dfrac{\lambda}{2\sin\theta}$, $\ln d = \ln\left(\dfrac{\lambda}{2}\right) - \ln(\sin\theta)$.

Differentiating, $\dfrac{\Delta d}{d} = 0 - \dfrac{\cos\theta\,\Delta\theta}{\sin\theta}$.

$$\therefore \left(\frac{\Delta d}{d}\right)_{\max} = \cot\theta\cdot\Delta\theta \quad \text{or} \quad (\Delta d)_{\max} = d\cot\theta\cdot\Delta\theta$$

$$= \frac{\lambda}{2\sin\theta}\cdot\cot\theta\,\Delta\theta = \frac{\lambda\cos\theta}{2\sin^2\theta}\Delta\theta.$$

As θ increases, $\cot\theta$ decreases and $\dfrac{\cos\theta}{\sin^2\theta}$ also decreases.

Hence fractional error in d decreases.

Hence, the correct option is (d).

10. Fringe width $\beta = \dfrac{D\lambda}{d}$. $\because \lambda_2 > \lambda_1 \Rightarrow \beta_2 > \beta_1$.

Number of fringes in a given distance $= m = \dfrac{y}{\beta} \Rightarrow m_2 < m_1$.

3rd maximum of $\lambda_2 = 3\beta_2 = \dfrac{3D\lambda_2}{d} = \dfrac{1800D}{d}$,

and 5th minimum of $\lambda_1 = \dfrac{9\lambda_1 D}{2d} = \dfrac{1800D}{d}$.

Angular separation $\theta = \dfrac{\beta}{D} = \dfrac{\lambda}{d}$.

Hence, the correct options are (a), (b) and (c).

11. For at least one maxima, $\sin\theta = \dfrac{\lambda}{d}$.

If $d = \lambda$, $\sin\theta = 1$, $\theta = \frac{\pi}{2}\cdot$ So $y \to \infty$.

If $\lambda < d < 2\lambda$, $\sin\theta$ exists and y is finite.

$$I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2}.$$
$$I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2}.$$

Initially, $I_{max} = 9I$ and $I_{min} = I$.

If $I_1 = I_2$ then $I'_{max} = 4I$, $I'_{min} = 0$.

Thus intensities of both the dark and bright fringes will decrease.

Hence, the correct options are (a) and (b).

12. Using mirror formula $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$, we get:

For (a): (42, 56); $f = -24$ cm

For (b): (48, 48); $f = -24$ cm

For (c): (66, 33); $f = -22$ cm

For (d): (78, 39); $f = -26$ cm

Thus options (c) and (d) are incorrectly recorded.

2.7 Modern Physics

1. KE_{max} of an electron just after emission,

$$KE_i = \frac{hc}{\lambda_{Ph}} - \phi,$$

and on reaching the anode,

$$KE_f = \left(\frac{hc}{\lambda_{Ph}} - \phi\right) + eV.$$

For $V >> \frac{\phi}{e}$, $KE_f \approx eV$, and $\lambda_e = \frac{h}{\sqrt{2meV}} \Rightarrow \lambda_e \propto \frac{1}{\sqrt{V}}\cdot$

If ϕ and λ_{Ph} increase, KE_f decreases and λ_e increases. So option (c) is incorrect.

$$\because \qquad \lambda_e = \frac{h}{\sqrt{2m\,(KE_f)}},$$

$$\therefore \qquad \frac{d\lambda_e}{dt} \neq \frac{d\lambda_{Ph}}{dt}$$ ... option (b) is incorrect.

Hence, the correct option is (a).

2. $\frac{hc}{\lambda} = eV_0 + \phi$ or $V_0 = \frac{hc}{e}\left(\frac{1}{\lambda}\right) - \frac{\phi}{e}\cdot$

The plot of V_0 vs $\frac{1}{\lambda}$ is linear with negative intercept on the V_0 axis.

The plot of V_0 vs λ is nonlinear.

Hence, the correct options are (a) and (c).

3. Orbital radius, $r = r_0 \frac{n^2}{Z} \Rightarrow \frac{\Delta r}{r} = \frac{2\Delta n}{n}$.

For $\Delta n = 1, \frac{\Delta r}{r} \propto \frac{1}{n}$.

Angular momentum, $L = \frac{nh}{2\pi}, \frac{\Delta L}{L} = \frac{\Delta n}{n}$.

$\therefore$ $\frac{\Delta L}{L} \propto \frac{1}{n}$.

Total energy, $E_n = -\frac{13.6\, Z^2}{n^2}$.

$\therefore$ $\frac{\Delta E_n}{E_n} = -\frac{2\Delta n}{n} \Rightarrow \frac{\Delta E}{E} \propto \frac{1}{n}$. ... option (c) incorrect.

Hence, the correct options are (a), (b) and (d).

4. Given: $4.5a_0 = a_0 \frac{n^2}{Z}$ and $\frac{nh}{2\pi} = \frac{3h}{2\pi}$.

Solving, $n = 3$ and $Z = 2$.

Possible wavelengths correspond to the transitions $3 \rightarrow 2$, $2 \rightarrow 1$ and $3 \rightarrow 1$ and are given by

$$\frac{1}{\lambda_1} = RZ^2\left(\frac{1}{1^2} - \frac{1}{3^2}\right) \Rightarrow \lambda_1 = \frac{9}{32R}$$

$$\frac{1}{\lambda_2} = RZ^2\left(\frac{1}{1^2} - \frac{1}{2^2}\right) \Rightarrow \lambda_2 = \frac{1}{3R}$$

$$\frac{1}{\lambda_3} = RZ^2\left(\frac{1}{2^2} - \frac{1}{3^2}\right) \Rightarrow \lambda_3 = \frac{9}{5R}$$

Hence, the correct options are (a) and (c).

5. ${}^{236}_{92}\mathrm{U} \rightarrow {}^{140}_{54}\mathrm{Xe} + {}^{94}_{38}\mathrm{Sr} + {}^{1}_{0}\mathrm{X} + {}^{1}_{0}\mathrm{Y}$,

where X = Y = n (neutron).

Q-value of reaction,

$$Q = 236 \times 7.5 - (140 + 94) \times 8.5$$
$$= 1170 - 1989 = 219 \text{ MeV}.$$

In options (a) and (d), the energy and charge conservation laws are followed, so

$$\begin{aligned} Q &= K_{Xe} + K_{Sr} + K_X + K_Y \\ &= 129 + 86 + 4 = 219. \end{aligned}$$

In (d), $p_{Xe} > p_{Sr} + p_X + p_Y$,

so, the conservation of linear momentum will not hold and only option (a) will hold.

6. Energy is released during nuclear fusion and fission when the final binding energy per nucleon is greater than the initial binding energy so that the nucleus gets more stable. This will occur in fusion [option (b)] and fission [option (d)].

❏

3

Integer-Answer-Type Questions

*In this chapter the answer to each question is a **single-digit integer**, ranging from 0 to 9 (both inclusive). For each question, darken the bubble corresponding to the correct integer in the ORS.*

3.1 General Physics

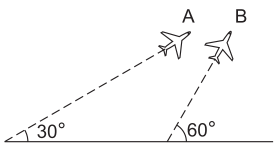
1. During Searle's experiment, the zero of a vernier scale lies between 3.20×10^{-2} m and 3.25×10^{-2} m of the main scale divisions. The 20th division of the vernier scale exactly coincides with one of the main scale divisions. When an additional load of 2 kg is applied to the wire, the zero of the vernier scale still lies between 3.20×10^{-2} m and 3.25×10^{-2} m of the main scale but now the 45th division of the vernier scale coincides with one of the main scale divisions. The length of the thin metallic wire is 2 m and its cross-sectional area is 8.0×10^{-7} m^2. The least count of the vernier scale is 1.0×10^{-5} m. The maximum percentage error in the Young modulus of the wire is

2. To find the distance d over which a signal can be seen clearly in foggy conditions, a railway engineer uses dimensional analysis and assumes that the distance depends on the mass density ρ of the fog, intensity (power/area) S of the light from the signal and its frequency f. The engineer finds that d is proportional to $S^{1/n}$. The value of n is

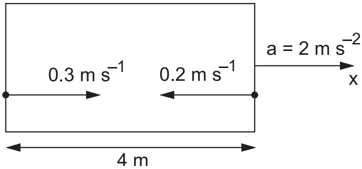
3. The energy of a system as a function of time t is given as $E(t) = A^2 \exp(-\alpha t)$, where $\alpha = 0.2$ s^{-1}. The measurement of A has an error of 1.25%. If the error in the measurement of time is 1.50%, find the percentage error in the value of $E(t)$ at $t = 5$ s.

4. A train is moving along a straight line with a constant acceleration a. A boy standing inside the train throws a ball forward with a speed of 10 m s^{-1}, at an angle of 60° to the horizontal. The boy has to move forward by 1.15 m inside the train to catch the ball back at the initial height. The acceleration of the train, in m s^{-2}, is

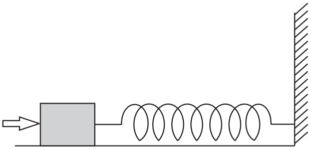
5. Aeroplanes A and B are flying with constant velocities in the same vertical plane at angles 30° and 60° with respect to the horizontal respectively as shown in the figure. The speed of A is $100\sqrt{3}$ m s^{-1}. At time $t = 0$ s, an observer in A finds B at a distance of 500 m. This observer sees B moving with a constant velocity perpendicular to the line of motion of A. If at $t = t_0$, A just escapes being hit by B then t_0 in seconds is



6. A rocket is moving in a gravity-free space with a constant acceleration of 2 m s^{-2} along the $+x$ direction (see figure). The length of a chamber inside the rocket is 4 m. A ball is thrown from the left end of the chamber in the $+x$ direction with a speed of 0.3 m s^{-1} relative to the rocket. At the same time, another ball is thrown in the $-x$ direction with a speed of 0.2 m s^{-1} from the right end relative to the rocket. The time in seconds when the two balls hit each other is

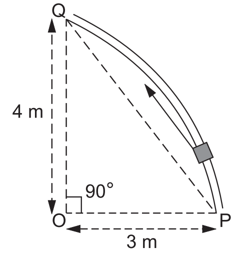


7. A block of mass 0.18 kg is attached to a spring of force constant 2 N m^{-1}. The coefficient of friction between the block and the floor is 0.1. Initially the block is at rest and the spring is unstretched. An impulse is given to the block as shown in the figure. The block slides a distance of 0.06 m and comes to rest for the first time. The initial velocity of the block in m s^{-1} is $v = N/10$. Then N is



8. A particle of mass 0.2 kg is moving in one dimension under a force that delivers a constant power of 0.5 W to the particle. If the initial speed (in m s^{-1}) of the particle is zero, the speed (in m s^{-1}) after 5 s is

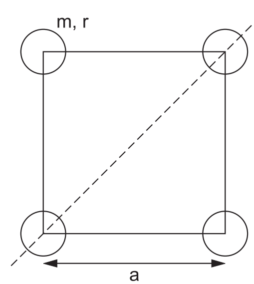
9. Consider an elliptical-shaped rail PQ in the vertical plane with OP = 3 m and OQ = 4 m. A block of mass 1 kg is pulled along the rail from P to Q with a force of 18 N, which is always parallel to line PQ (see the figure). Assuming no frictional losses, the kinetic energy of the block when it reaches Q is ($n \times 10$) joules. The value of n is (take $g = 10$ m s^{-2})



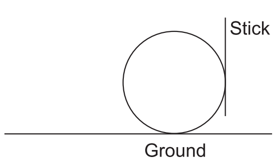
10. A block is moving on an inclined plane making an angle 45° with the horizontal. The coefficient of friction is μ. The force required to just push it up the inclined plane is 3 times the force required to just prevent it from sliding down. If we define $N = 10\mu$ then N is

11. A bob of mass m, suspended by a string of length l_1, is given a minimum speed required to complete a full circle in the vertical plane. At the highest point, it collides elastically with another bob of mass m, suspended by a string of length l_2, which is initially at rest. Both the strings are massless and inextensible. If the second bob, after the collision acquires a minimum speed required to complete a full circle in the vertical plane, the ratio l_1/l_2 is

12. Four solid spheres, each of diameter $\sqrt{5}$ cm and mass 0.5 kg, are placed with their centres at the corners of a square of side 4 cm. If the moment of inertia of the system about a diagonal of the square is $N \times 10^{-4}$ kg m^2 then N is



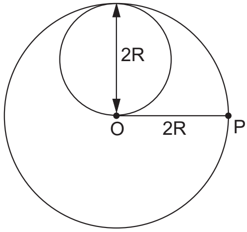
13. A boy is pushing a ring of mass 2 kg and radius 0.5 m with a stick as shown in the figure. The stick applies a force of 2 N on the ring and rolls it without slipping with an acceleration of 0.3 m s^{-2}. The coefficient of friction between the ground and the ring is large enough so that rolling always occurs. The coefficient of friction between the stick and the ring is $P/10$. The value of P is



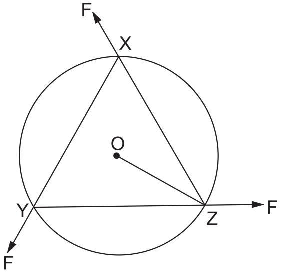
14. A binary star consists of two stars A (mass 2.2 M_s) and B (mass 11 M_s), where M_s is the mass of the sun. They are separated by a distance d and are rotating about their centre of mass, which is stationary. The ratio of the total angular momentum of the binary star to the angular momentum of star B about the centre of mass is

15. A uniform circular disc of mass 50 kg and radius 0.4 m is rotating with an angular velocity of 10 rad s^{-1} about its own axis, which is vertical. Two uniform rings, each of mass 6.25 kg and radius 0.2 m, are gently placed symmetrically on the disc in such a manner that they are touching each other along the axis of the disc and are horizontal. Assume that the friction is large enough such that the rings are at rest relative to the disc and the system rotates about the original axis. The new angular velocity (in rad s^{-1}) of the system is

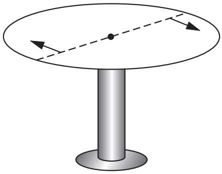
16. A lamina is made by removing a small disc of diameter $2R$, as shown in the figure. The moment of inertia of this lamina about the axes passing through O and P are I_O and I_P respectively. Both these axes are perpendicular to the plane of the lamina. The ratio $\frac{I_P}{I_O}$ to the nearest integer is



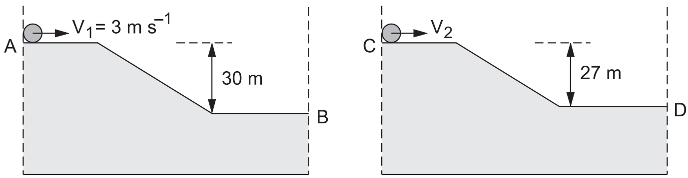
17. A uniform circular disc of mass 1.5 kg and radius 0.5 m is initially at rest on a horizontal frictionless surface. Three forces of equal magnitude F = 0.5 N are applied simultaneously along the three sides of an equilateral triangle XYZ with its vertices on the perimeter of the disc (see figure). One second after applying the forces, the angular speed of the disc is



18. A horizontal circular platform of radius 0.5 m and mass 0.45 kg is free to rotate about its axis. Two massless spring toy guns, each carrying a steel ball of mass 0.05 kg, are attached to the platform at a distance of 0.25 m from the centre on its either side along its diameter (see figure). Each gun simultaneously fires the balls horizontally and perpendicular to the diameter in opposite directions. After leaving the platform, the balls have horizontal speed of 9 m s^{-1} with respect to the ground. The rotational speed of the platform in rad s^{-1} after the balls leave the platform is



19. Two identical uniform discs roll without slipping on two different surfaces AB and CD (see figure), starting at A and C with linear speeds v_1 and v_2 respectively and always remaining in contact with the surfaces. If they reach B and D with the same linear speed and $v_1 = 3$ m s^{-1} then v_2 in m s^{-1} is ($g = 10$ m s^{-2})

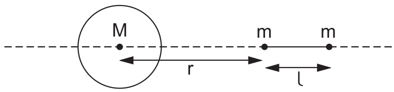


20. The densities of two solid spheres A and B of the same radii R vary with radial distance r as $\rho_A(r) = k\left(\frac{r}{R}\right)$ and $\rho_B(r) = k\left(\frac{r}{R}\right)^5$ respectively, where k is a constant. The moment of inertia of the individual spheres about the axes passing through their centres are I_A and I_B respectively. If $\frac{I_B}{I_A} = \frac{n}{10}$, find the value of n.

21. The gravitational acceleration on the surface of a planet is $\frac{\sqrt{6}}{11}g$, where g is the gravitational acceleration on the surface of the earth. The average mass density of the planet is 2/3 times that of the earth. If the escape velocity on the surface of the earth is taken to be 11 km s^{-1}, the escape velocity on the surface of the planet in km s^{-1} will be

22. A bullet is fired vertically upwards with a velocity v from the surface of a spherical planet. When the bullet reaches its maximum height, its acceleration due to the planet's gravity is 1/4th of its value at the surface of the planet. If the escape velocity from the planet is $v_{esc} = v\sqrt{N}$ then N is (ignore the energy loss due to atmosphere)

23. A large spherical mass M is fixed at one position and two identical point masses m are kept on a line passing through the centre of M as shown in the figure. The point masses are connected by a rigid massless rod of length l and this assembly is free to move along the line connecting them. All three masses interact only through their mutual gravitational interaction. When the point mass nearer to M is at a distance $r = 3l$ from M, the tension in the rod is zero for $m = K\left(\frac{M}{288}\right)$. Find the value of K.

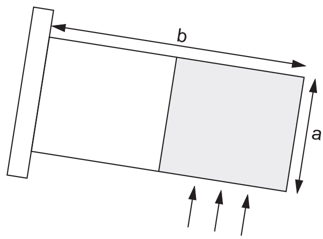


24. A 0.1-kg mass is suspended from a wire of negligible mass. The length of the wire is 1 m and its cross-sectional area is 4.9×10^{-7} m^2. If the mass is pulled a little in the vertically downward direction and released, it performs simple harmonic motion of angular frequency 140 rad s^{-1}. If the Young modulus of the material of the wire is $n \times 10^9$ N m^{-2}, the value of n is

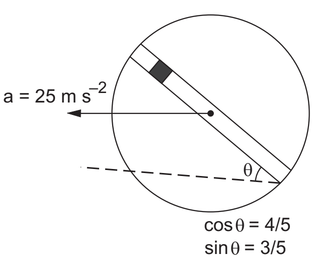
25. Consider two solid spheres P and Q each of density 8 g cm^{-3} and diameter 1 cm and 0.5 cm respectively. The sphere P is dropped into a liquid of density 0.8 g cm^{-3} and viscosity $\eta = 3$ poiseuilles. The sphere Q is dropped into a liquid of density 1.6 g cm^{-3} and viscosity $\eta = 2$ poiseuilles. Find the ratio of the terminal velocities of P and Q.

26. A steel wire of length L at 40 °C is suspended from the ceiling and then a mass m is hung from its free end. The wire is cooled down from 40 °C to 30 °C to regain its original length L. The coefficient of linear thermal expansion of the steel is 10^{-5} °C^{-1}, Young modulus of steel is 10^{11} N m^{-2} and the radius of the wire is 1 mm. Assume that $L >>$ diameter of the wire. Then, the value of m in kilogram is nearly

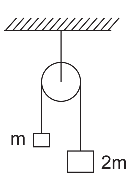
27. A rectangular plate of mass M and dimensions ($a \times b$) is held in a horizontal position by striking n small balls each of mass m per unit area per unit time. These are striking in the shaded half-region of the plate. The balls are colliding elastically with a velocity v. Find v. (Given: $n = 100$, $M = 3$ kg, $m = 0.01$ kg, $b = 2$ m, $a = 1$ m, $g = 10$ m s^{-2})



28. A circular disc with a groove along its diameter is placed horizontally. A block of mass 1 kg is placed as shown. The coefficient of friction between the block and all the surfaces of the groove in contact is $\mu = 0.4$. The disc has an acceleration of 25 m s^{-2}. Find the acceleration of the block with respect to the disc.

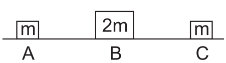


29. A light inextensible string that goes over a smooth fixed pulley as shown in the figure connects two blocks of masses 0.36 kg and 0.72 kg. Taking $g = 10$ m s^{-2}, find the work done (in joules) by the string on the block of 0.36 kg during the first second after the system is released from rest.



30. Two soap bubbles A and B are kept in a closed chamber where the air is maintained at 8 N m^{-2}. The radii of bubbles A and B are 2 cm and 4 cm respectively. The surface tension of the soap solution to form bubbles is 0.04 N m^{-1}. Find the ratio n_B/n_A, where n_A and n_B are the numbers of moles of air in bubbles A and B respectively. (Neglect the effect of gravity.)

31. Three blocks A, B and C are kept along a straight line on a frictionless horizontal surface. These have masses m, $2m$ and m respectively. The block A moves towards B with a speed of 9 m s^{-1} and makes an elastic collision with it. Thereafter, B makes a completely inelastic collision with C. All motions occur along the same straight line. Find the final speed of block C in m s^{-1}.



32. A cylindrical vessel of height 500 mm has an orifice at its bottom. The orifice is initially closed and water is filled in it up to a height H. Now the top is completely sealed with a cap and the orifice at the bottom is opened. Some water comes out through the orifice and the water level in the vessel becomes steady with the height of water column being 200 mm. Find the fall in height (in mm) of the water level due to the opening of the orifice.

(Take atmospheric pressure = 1.0×10^5 N m^{-2}, density of water = 1000 kg m^{-3} and g = 10 m s^{-2}. Neglect any effect of surface tension.)

33. A drop of liquid of radius $R = 10^{-2}$ m having the surface tension $S = \dfrac{0.1}{4\pi}$ N m^{-1} divides itself into K identical drops. In this process, the total change in the surface energy ΔU is 10^{-3} J. If $K = 10^{\alpha}$ then the value of α is

3.2 Heat and Thermodynamics

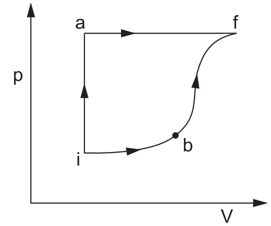
1. A piece of ice (specific heat capacity = 2100 J kg^{-1} °C^{-1} and latent heat = 3.36×10^5 J kg^{-1}) of mass m g is at –5 °C at atmospheric pressure. It is given 420 J of heat so that the ice starts melting. Finally, when the ice–water mixture is in equilibrium, it is found that 1 g of ice has melted. Assuming there is no other heat exchange in the process, the value of m is

2. In an insulated vessel, 0.05 kg of steam at 373 K and 0.45 kg of ice at 253 K are mixed. What would be the final temperature of the mixture in degree Celsius?

3. A metal rod AB of length $10x$ has its one end A in ice at 0 °C and the other end B in water at 100 °C. If a point P on the rod is maintained at 400 °C then it is found that equal amounts of water and ice evaporate and melt per unit time. The latent heat of evaporation of water is 540 cal g^{-1} and the latent heat of melting of ice is 80 cal g^{-1}. If the point P is at a distance of λx from the ice end A, find the value of λ. (Neglect any heat loss to the surroundings.)

4. A diatomic ideal gas is compressed adiabatically to 1/32 of its initial volume. If the initial temperature of the gas is T_i (in Kelvin) and the final temperature is aT_i, the value of a is

5. A thermodynamic system is taken from an initial state i with internal energy U_i = 100 J to the final state f along two different paths iaf and ibf, as schematically shown in the figure. The work done by the system along the paths af, ib and bf are W_{af} = 200 J, W_{ib} = 50 J and W_{bf} = 100 J respectively. The heat supplied to the system along the paths iaf, ib and bf are Q_{iaf}, Q_{ib} and Q_{bf} respectively. If the internal energy of the system in the state b is U_b = 200 J and Q_{iaf} = 500 J, the ratio $\dfrac{Q_{bf}}{Q_{ib}}$ is



6. Two spherical bodies A (radius 6 cm) and B (radius 18 cm) are at temperatures T_1 and T_2 respectively. The maximum intensity in the emission spectrum of A is at 500 nm and that of B is at 1500 nm. Considering them to be black bodies, what will be the ratio of the rate of total energy radiated by A to that of B?

7. Two spherical stars A and B emit black-body radiations. The radius of A is 400 times that of B, and A emits 10^4 times the power emitted by B. The ratio of their wavelengths λ_A to λ_B (i.e., λ_A/λ_B) at which the peaks occur in their respective radiation curves is

8. A metal is heated in a furnace where a sensor is kept above the metal surface to read the power radiated (P) by the metal. The sensor has a scale that displays $\log_2(P/P_0)$, where P_0 is a constant. When the metal surface is at a temperature of 487 °C, the sensor shows a value 1. Assume that the emissivity of the metallic surface remains constant. What is the value displayed by the sensor when the temperature of the metal surface is raised to 2767 °C?

3.3 Sound Waves

1. When two progressive waves $y_1 = 4\sin(2x - 6t)$ and $y_2 = 3\sin\left(2x - 6t - \frac{\pi}{2}\right)$ are superimposed, the amplitude of the resultant wave is

2. Four harmonic waves of equal frequencies and equal intensities (I_0) have phase angles 0, $\frac{\pi}{3}$, $\frac{2\pi}{3}$ and π. When they are superposed, the intensity of the resulting wave is nI_0. The value of n is

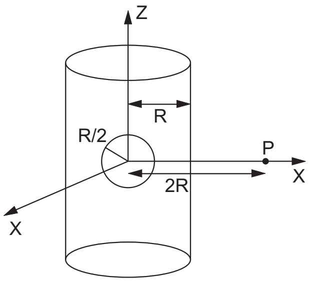
3. A stationary source is emitting sound at a fixed frequency f_0, which is reflected by two cars approaching the source. The difference between the frequencies of sound reflected from the cars is 1.2% of f_0. What is the difference in the speeds of the cars (in km per hour) to the nearest integer? The cars are moving at constant speeds much lower than the speed of sound which is 330 m s^{-1}.

4. A 20-cm-long string, having a mass of 1.0 g, is fixed at both the ends. The tension in the string is 0.5 N. The string is set into vibrations using an external vibrator of frequency 100 Hz. Find the separation (in cm) between the successive nodes on the string.

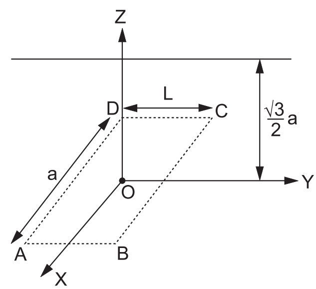
5. A stationary source emits sound of frequency f_0 = 492 Hz. The sound is reflected by a large car approaching the source with a speed of 2 m s^{-1}. The reflected signal is received by the source and superposed with the original. What will be the beat frequency of the resulting signal in Hz? (Given that the speed of sound in air is 330 m s^{-1} and the car reflects the sound at the frequency it has received.)

3.4 Electrostatics

1. An infinitely long solid cylinder of radius R has a uniform volume charge density ρ. It has a spherical cavity of radius $R/2$ with its centre on the axis of the cylinder as shown in the figure. The magnitude of the electric field at the point P, which is at a distance 2R from the axis of the cylinder, is given by the expression $\frac{23\rho R}{16K\varepsilon_0}$. The value of K is



2. An infinitely long uniform line charge distribution of charge per unit length λ lies parallel to the y-axis in the yz-plane at $z = \frac{\sqrt{3}}{2}a$, as shown in the figure. If the magnitude of the flux of electric field through the rectangular surface ABCD lying in the xy-plane with its centre at the origin is $\frac{\lambda L}{n\varepsilon_0}$ (ε_0 = permittivity of free space) then the value of n is



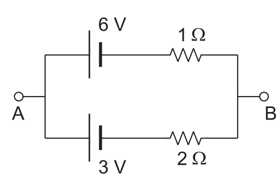
3. Four point charges, each of $+q$, are rigidly fixed at the four corners of a square planar soap film of side a. The surface tension of the soap film is S. The system of charges and planar film are in equilibrium, and $a = K\left[\frac{q^2}{S}\right]^{1/N}$, where K is a constant. Then N is

4. A solid sphere of radius R has a charge Q distributed in its volume with a charge density $\rho = Kr^n$, where K and n are constants and r is the distance of a point from its centre. If the electric field at the distance $r = R/2$ is $1/8$ times that at $r = R$, find the value of n.

3.5 Current Electricity and Magnetism

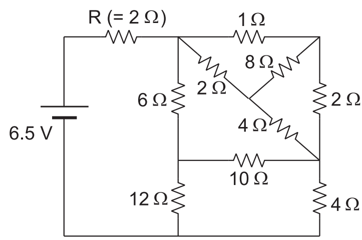
1. When two identical batteries of internal resistance 1 Ω each are connected in series across a resistor R, the rate of heat produced in R is J_1. When the same batteries are connected in parallel across R, the rate is J_2. If $J_1 = 2.25J_2$ then the value of R in Ω is

2. Two batteries of different emfs and different internal resistances are connected as shown. The voltage across AB in volt is



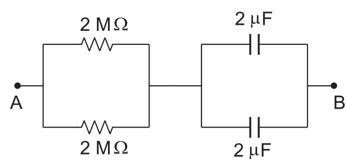
3. A galvanometer gives a full-scale deflection with 0.006-A current. By connecting it to a 4990 Ω resistance, it can be converted into a voltmeter of range 0–30 V. If connected to a $\frac{2n}{249}$-Ω resistance, it becomes an ammeter of range 0–1.5 A. The value of n is

4. In the given circuit, the current through the resistor R (= 2 Ω) is I amperes. The value of I is

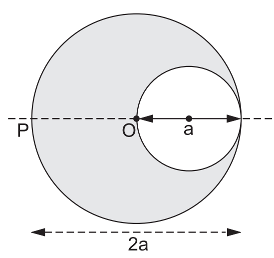


5. At time $t = 0$, a battery of 10 V is connected across points A and B in the given circuit. If the capacitors have no charge initially, at what time (in seconds) does the voltage across them become 4 V?

(Take: ln 5 = 1.6, ln 3 = 1.1.)

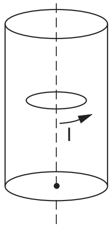


6. A cylindrical cavity of diameter a exists inside a cylinder of diameter $2a$ as shown in the figure. Both the cylinder and the cavity are infinitely long. A uniform current density J flows along the length. If the magnitude of the magnetic field at the point P is given by $\frac{N}{12}\mu_0 aJ$ then the value of N is

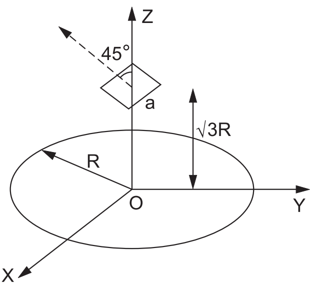


7. Two parallel wires in the plane of a paper are at a distance X_0 apart. A point charge is moving with a speed u between the wires in the same plane at a distance X_1 from one of the wires. When the wires carry current of magnitude I in the same direction, the radius of curvature of the path of the point charge is R_1. In contrast, if the currents of magnitude I in the two wires have directions opposite to each other, the curvature of the path is R_2. If $\frac{X_0}{X_1} = 3$, the value of $\frac{R_1}{R_2}$ is

8. A long circular tube of length 10 m and radius 0.3 m carries a current I along its curved surface as shown. A wire loop of resistance 0.005 Ω and radius 0.1 m is placed inside the tube with its axis coinciding with the axis of the tube. The current varies as $I = I_0 \cos 300\, t$, where I_0 is a constant. If the magnetic moment of the loop is $N\mu_0 I_0 \sin 300\, t$ then N is



9. A circular wire loop of radius R is placed in the xy-plane centred at the origin O. A square loop of side a ($a << R$) having two turns is placed with its centre at $\sqrt{3}R$ along the axis of the circular wire loop, as shown in the figure. The plane of the square loop makes an angle of 45° with respect to the z-axis. If the mutual inductance between the loops is given by $\dfrac{\mu_0 a^2}{2^{p/2} R}$ then the value of p is



10. Two inductors L_1 (inductance 1 mH, internal resistance 3 Ω) and L_2 (inductance 2 mH, internal resistance 4 Ω), and a resistor R (resistance 12 Ω) are all connected in parallel across a 5-V battery. The circuit is switched on at time t = 0. Find the ratio of the maximum to the minimum current (I_{max}/I_{min}) drawn from the battery.

11. A series RC combination is connected to an AC voltage of angular frequency ω = 500 rad s^{-1}. If the impedance of the RC circuit is $R\sqrt{1.25}$, the time constant (in millisecond) of the circuit is

12. A steady current I goes through a wire loop PQR having the shape of a right-angled triangle with PQ = $3x$, PR = $4x$ and QR = $5x$. If the magnitude of the magnetic field at P due to this loop is $K(\mu_0 I/48\pi x)$, find the value of K.

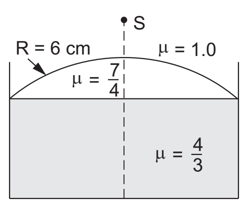
3.6 Ray Optics and Wave Optics

1. The image of an object that is approaching a convex mirror of radius of curvature 20 m along its optical axis is observed to move from 25/3 m to 50/7 m in 30 seconds. What is the speed of the object in km per hour?

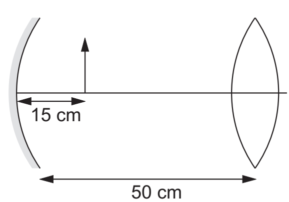
2. A large glass slab ($\mu = 5/3$) of thickness 8 cm is placed over a point source of light on a plane surface. It is seen that light emerges out of the top surface of the slab from a circular area of radius R cm. What is the value of R?

3. The focal length of a thin biconvex lens is 20 cm. When an object is moved from a distance of 25 cm in front of it to 50 cm, the magnification of its image changes from m_{25} to m_{50}. The ratio $\dfrac{m_{25}}{m_{50}}$ is

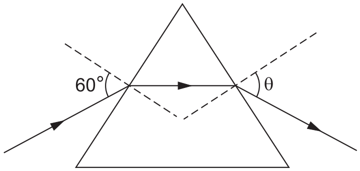
4. The water (refractive index 4/3) kept in a tank is 18 cm deep. A layer of oil of refractive index 7/4 lies on this water making a convex surface of radius of curvature $R = 6$ cm as shown in the figure. Consider the oil to act as a thin lens. An object S is placed 24 cm above the water surface. The location of its image is at X cm above the bottom of the tank. Then X is



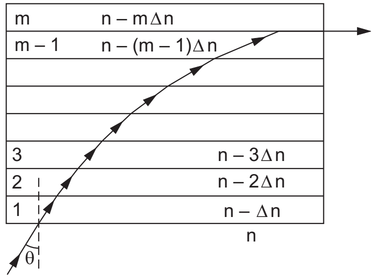
5. Consider a concave mirror and a convex lens (refractive index = 1.5) of focal length 10 cm each, separated by a distance of 50 cm in air as shown in the figure. An object is placed at a distance of 15 cm from the mirror. Its erect image formed by this combination has magnification M_1. When the set-up is kept in a medium of refractive index 7/6, the magnification becomes M_2. The magnitude $\left|\dfrac{M_2}{M_1}\right|$ is



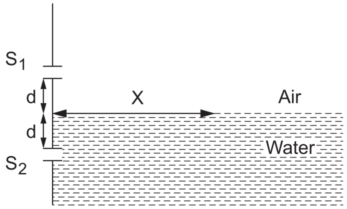
6. A monochromatic beam of light is incident at 60° on one face of an equilateral prism of refractive index n and emerges from the opposite face making an angle $\theta(n)$ with the normal. For $n = \sqrt{3}$, the value of θ is 60° and $\dfrac{d\theta}{dn} = m$. The value of m is



7. A monochromatic light is travelling in a medium of refractive index $n = 1.6$. It enters a stack of glass layers from the bottom side at an angle $\theta = 30°$. The interfaces of the glass layers are parallel to each other. The refractive indices of the different glass layers are monotonically decreasing as $n_s = n - m\Delta n$, where n_m is the refractive index of the mth slab and $\Delta n = 0.1$ (see the figure). The ray is refracted out parallel to the interface between the $(m-1)$th and mth slabs from the right side of the stack. What is the value of m?



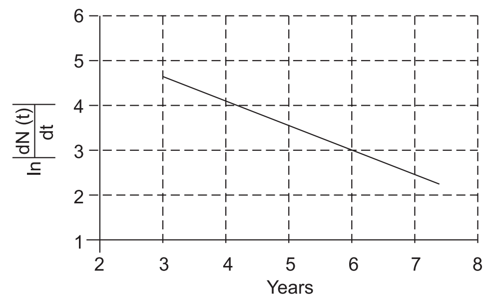
8. A Young's double-slit interference arrangement with slits S_1 and S_2 is immersed in water (refractive index = 4/3) as shown in the figure. The positions of the maxima on the surface of water are given by $x^2 = p^2m^2\lambda^2 - d^2$, when λ is the wavelength of light in air (refractive index = 1), $2d$ is the separation between the slits and m is an integer. The value of p is



3.7 Modern Physics

1. An α-particle and a proton are accelerated from rest by a potential difference of 100 V. After this, their de Broglie wavelengths are λ_α and λ_p respectively. The ratio $\dfrac{\lambda_p}{\lambda_\alpha}$, to the nearest integer, is

2. To determine the half-life of a radioactive element, a student plots a graph of $\ln\left|\frac{dN(t)}{dt}\right|$ versus t. Here $\frac{dN(t)}{dt}$ is the rate of radioactive decay at time t. If the number of radioactive nuclei of this element decreases by a factor of p after 4.16 years, the value of p is



3. A silver sphere of radius 1 cm and work function 4.7 eV is suspended from an insulating thread in a free space. It is under continuous illumination of a 200-nm-wavelength light. As photoelectrons are emitted, the sphere gets charged and acquires a potential. The maximum number of photoelectrons emitted from the sphere is $A \times 10^n$ (where $1 < A < 10$). The value of n is

4. The work function of silver and sodium are 4.6 eV and 2.3 eV respectively. The ratio of the slope of the stopping potential versus frequency plot for silver to that of sodium is

5. A proton is fired from very far away towards a nucleus with charge $Q = 120\,e$, where e is the electronic charge. It makes a closest approach of 10 fm to the nucleus. The de Broglie wavelength (in units of fm) of the proton at its start is
(proton-mass $= \frac{5}{3} \times 10^{-27}$ kg, $h/e = 4.2 \times 10^{-15}$ J s C^{-1})

6. Consider a hydrogen atom with its electron in the nth orbital. An electromagnetic radiation of wavelength 90 nm is used to ionise the atom. If the kinetic energy of the ejected electron is 10.4 eV then the value of n is ($hc = 1242$ eV nm)

7. An electron in an excited state of Li^{2+} ion has angular momentum $3h/2\pi$. The de Broglie wavelength of the electron in this state is $p\pi a_0$ (where a_0 is the Bohr radius). The value of p is

8. A hydrogen atom in its ground state is irradiated by a light of wavelength 970 Å. Taking $hc = 1.237 \times 10^{-6}$ eV m and the ground state energy of hydrogen atom as –13.6 eV, find the number of lines present in the emission spectrum.

9. An electron in a hydrogen atom undergoes a transition from an orbit with quantum number n_i to another with quantum number n_f. V_i and V_f are respectively the initial and final potential energies of the electron. If $\frac{V_\mathrm{i}}{V_\mathrm{f}} = 6.25$ then the smallest possible n_f is

10. A freshly prepared sample of radioisotope of half-life 1386 s has activity 10^3 disintegration per second. Given that $\ln 2 = 0.693$, the fraction of the initial number of nuclei (expressed in nearest integer percentage) that will decay in the first 80 s after the preparation of the sample is

11. A nuclear power plant supplying electrical power to a village uses a radioactive material of half-life T years as the fuel. The amount of fuel at the beginning is such that the total power requirement of the village is met by 12.5% of the electrical power available from the plant at that time. If the plant is able to meet the total power needs of the village for a maximum period of nT years then the value of n is

12. For a radioactive material, its activity A and the rate of change of its activity R are defined as $A = -\frac{dN}{dt}$ and $R = -\frac{dA}{dt}$, where $N(t)$ is the number of active nuclei at time t. Two radioactive sources P (mean life τ) and Q (mean life 2τ) have the same activity at $t = 0$. Their rates of change of activities at $t = 2\tau$ are R_P and R_Q respectively. If $R_\mathrm{P}/R_\mathrm{Q} = \frac{n}{e}$ then the value of n is

13. The isotope ${}^{12}_{5}\text{B}$ having a mass 12.014 u undergoes β-decay to ${}^{12}_{6}\text{C}$, ${}^{12}_{6}\text{C}$ has an excited state of the nucleus ${}^{12}_{6}\text{C}^*$ at 4.041 MeV above the ground state. If ${}^{12}_{5}\text{B}$ decays to ${}^{12}_{6}\text{C}^*$, find the maximum kinetic energy of the β-particle in units of MeV. (Take 1 u = 931.5 MeV c^{-2}, where c is the speed of light in vacuum.)

14. ${}^{131}\text{I}$ is an isotope of iodine that β decays to an isotope of xenon with a half-life of 8 days. A small amount of a serum labelled with ${}^{131}\text{I}$ is injected into the blood of a person. The activity of the amount of ${}^{131}\text{I}$ injected was 2.4×10^5 becquerel (Bq). It is known that the injected serum will get distributed uniformly in the blood serum in less than half an hour. After 11.5 hours, 2.5 mL of blood is drawn from the person's body and it gives an activity of 11.5 Bq. The total volume of blood in the person's body in litres, is approximately

15. The activity of a freshly prepared radioactive sample is 10^{10} disintegrations per second and whose mean life is 10^9 s. The mass of an atom of this radioisotope is 10^{-25} kg. The mass (in mg) of the radioactive sample is

Answers

3.1 General Physics

1. 4	**2.** 3	**3.** 4	**4.** 5	**5.** 5	**6.** 2
7. 4	**8.** 5	**9.** 5	**10.** 5	**11.** 5	**12.** 9
13. 4	**14.** 6	**15.** 8	**16.** 3	**17.** 2	**18.** 4
19. 7	**20.** 6	**21.** 2	**22.** 2	**23.** 7	**24.** 4
25. 3	**26.** 3	**27.** 10	**28.** 10	**29.** 8	**30.** 6
31. 4	**32.** 6	**33.** 6			

3.2 Heat and Thermodynamics

1. 8	**2.** 0	**3.** 9	**4.** 4	**5.** 2	**6.** 9
7. 2	**8.** 9				

3.3 Sound Waves

1. 5	**2.** 3	**3.** 7	**4.** 5	**5.** 6

3.4 Electrostatics

1. 6	**2.** 6	**3.** 3	**4.** 2

3.5 Current Electricity and Magnetism

1. 4	**2.** 5	**3.** 5	**4.** 1	**5.** 2	**6.** 5
7. 3	**8.** 6	**9.** 7	**10.** 8	**11.** 4	**12.** 7

3.6 Ray Optics and Wave Optics

1. 3	**2.** 6	**3.** 6	**4.** 2	**5.** 7	**6.** 2
7. 8	**8.** 3				

3.7 Modern Physics

1. 3	**2.** 8	**3.** 7	**4.** 1	**5.** 7	**6.** 2
7. 2	**8.** 6	**9.** 5	**10.** 4	**11.** 3	**12.** 2
13. 9	**14.** 5	**15.** 1			

Hints and Solutions

3.1 General Physics

1. $Y = \frac{FL}{Al}$.

Here measurement is for l only,

$\therefore \frac{\Delta Y}{Y} = \frac{\Delta l}{l}$.

From observation, l_1 = MS + 20(LC), and $l_2 = MS$ + 45(LC).

Change in length = $l_2 - l_1$ = 25 × LC, and the maximum permissible error in measurement of elongation is one LC.

$$\therefore \frac{\Delta Y}{Y} \times 100\% = \frac{1\,\text{LC}}{25\,(\text{LC})} \times 100\% = 4\%.$$

2. Distance $d \propto (\text{density})^a \left(\frac{\text{power}}{\text{area}}\right)^b (\text{frequency})^c$

$$\Rightarrow \quad M^0LT^0 = (ML^{-3})^a\,(MT^{-3})^b\,(\text{T}^{-1})^c$$
$$= M^{a+b}\,L^{-3a}\,T^{-3b-c}.$$

Thus, $a + b = 0, -3a = 1, -3b - c = 0$

or $\quad a = -\frac{1}{3}, b = \frac{1}{3}, c = -1.$

$\because \quad b = \frac{1}{n} = \frac{1}{3} \Rightarrow n = 3.$

3. $\quad E(t) = A^2\,e^{-\alpha t}.$

Given: $\alpha = 0.2\ \text{s}^{-1}, \frac{dA}{A} = 1.25\%, \frac{dt}{t} = 1.50\%.$

$\log E = 2 \log \text{A} - \alpha t$

$$\Rightarrow \quad \frac{dE}{E} = \pm 2\frac{dA}{A} \pm \alpha\frac{dt}{t} \cdot t$$
$$= \pm 2(1.25) \pm 0.2\,(1.5)(5)$$
$$= \pm 2.50 \pm 1.5 = \pm 4\%.$$

4. $0 = 10\frac{\sqrt{3}}{2}t - \frac{1}{2}(10)t^2. \quad \therefore\ t = \sqrt{3}$ s.

$\therefore\ 1.15 = 5\sqrt{3} - \frac{3}{2}a \quad \Rightarrow \quad a \approx 5\ \text{m s}^{-2}.$

5. The relative velocity of B with respect to A is perpendicular to the line of motion of A as shown in Figure (ii). From this vector triangle,

$$V_B \cos 30° = V_A = 100\sqrt{3}\ \text{m s}^{-1}.$$

$$\Rightarrow \quad V_B = 200\ \text{m s}^{-1}.$$

$$\therefore \text{ time } t_0 = \frac{\text{relative distance}}{\text{relative velocity}}$$

$$= \frac{500}{V_B \sin 30°}$$

$$= \frac{500\ \text{m s}^{-1}}{200\ \text{m} \times \frac{1}{2}}$$

$$= 5 \text{ seconds.}$$

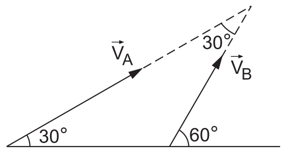


Fig. (i)

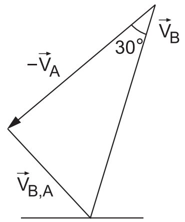
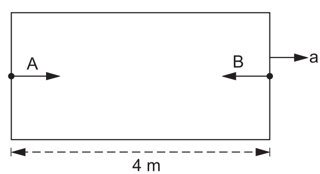


Fig. (ii)

6. Consider the motions of the balls A and B relative to the rocket. Maximum distance of the ball A from the left wall

$$= \frac{u^2}{2a} = \frac{(0.3)^2}{2 \times 2} \approx 0.02\ \text{m}.$$

The ball B will collide with the ball A when it is very close to the left wall. Thus, for B, $S = ut + \frac{1}{2}at^2$



$$\Rightarrow\ 4 = (0.2)t + \frac{1}{2} \cdot 2t^2.$$

Solving, $t \approx 1.9$ s

or nearest integer = 2 s.

7. From the work–energy principle for the given system,

$$-\frac{1}{2}kx^2 - \mu mgx = 0 - \frac{1}{2}mv^2$$

$$\Rightarrow \quad v = \frac{4}{10}, \quad \text{so } N = 4.$$

8. Power $= \dfrac{dW}{dt} = 0.5$ W

$$\Rightarrow \text{ work done } = \int_0^{5\,\text{s}} (0.5\ \text{W})\,dt = \frac{1}{2} \times 5\ \text{J} = 2.5\ \text{J}.$$

From the work–energy theorem,

$$W = \text{KE}_f - \text{KE}_i$$

$$\Rightarrow \frac{5}{2} = \frac{1}{2}\left(\frac{2}{10}\right)v^2, \text{ hence } v = 5\ \text{m s}^{-1}.$$

9. From the work–energy principle,

$$W_{\text{gravity}} + W_{\text{force}} = \Delta\text{KE} = (\text{KE})_f - (\text{KE})_i$$

$$\Rightarrow \quad -mgh + Fd = (\text{KE})_f$$

$$\Rightarrow \quad -1 \times 10 \times 4 + 18(5) = (\text{KE})_f = 50 = n \times 10.$$

10. $mg(\sin\theta + \mu\cos\theta) = 3mg(\sin\theta - \mu\cos\theta)$.

$\therefore \mu = 0.5$, so $N = 10\mu = 5$.

11. The initial speed of the first bob (suspended by an ideal string of length l_1) is $\sqrt{5gl_1}$ and its speed at the highest point will be $\sqrt{gl_1}$.

When this bob collides elastically with another identical bob, their speeds get interchanged, so the speed acquired by the second bob (to complete a full vertical circle), is

$$\sqrt{5gl_2} = \sqrt{gl_1} \Rightarrow \frac{l_1}{l_2} = 5.$$

12. $I = 2\left[\frac{2}{5}mR^2\right] + 2\left[\frac{2}{5}mR^2 + m\left(\frac{a}{\sqrt{2}}\right)^2\right]$.

Substituting, $I = 9 \times 10^{-4}$ kg m^2 = $N \times 10^{-4}$ kg m^2; $\therefore N = 9$.

13. For translation, $N - f_s = ma$,

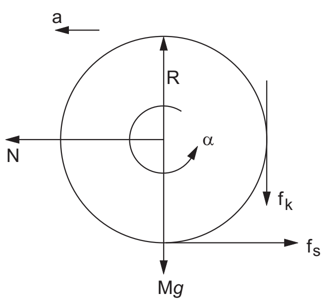
for rotation $(f_s - f_k)R = I\alpha$ and $a = R\alpha$.

Solving, $f_k = 0.8$ N; $f_s = 1.4$ N

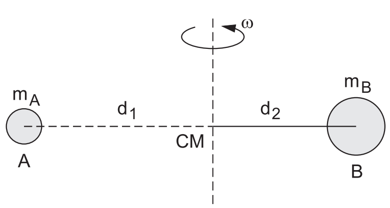
$$\Rightarrow \mu = \frac{P}{10} = \frac{f_k}{2N}$$

$$\Rightarrow \frac{P}{10} = \frac{0.8}{2}$$

$$\Rightarrow P = 4.$$



14. Distance of the centre of the mass of the system from A (mass $2.2M_s$) is $\frac{m_B d}{m_A + m_B} = \left(\frac{11}{2.2+11}\right)d = \frac{5d}{6}$ and that of B from the CM is $\left(d - \frac{5}{6}d\right) = \frac{d}{6}$. The system rotates about the axis through the CM of the system, so $I_A = m_A d_1^2$ and $I_B = m_B d_2^2$.



$\therefore$ angular momentum of A, $L_A = I_A\omega$ and that for B, $L_B = I_B\omega$

$$\Rightarrow \frac{\text{total angular momentum of the system}}{\text{angular momentum of star B}} = \frac{I_A\omega + I_B\omega}{I_B\omega} = \frac{I_A}{I_B} + 1$$

$$= \frac{(2.2M_s)\left(\frac{5}{6}d\right)^2}{(11M_s)\left(\frac{d}{6}\right)^2} + 1 = 6.$$

15. Mass of the disc = 50 kg = M.

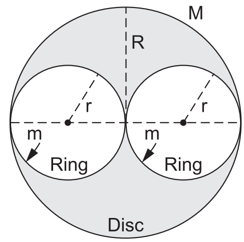
Mass of each ring = 6.25 kg = m.

$\therefore\ M = 8m$.

Radius of the disc = 0.4 m = R.

Radius of each ring = 0.2 m = r.

$\therefore\ R = 2r$.



Conserving angular momentum about the vertical axis of the disc,

$$\frac{1}{2}MR^2\cdot\omega = \left[\frac{1}{2}MR^2 + 2(2mr^2)\right]\omega'$$

$$\Rightarrow \quad \frac{1}{2}MR^2\cdot 10\ \text{rad s}^{-1} = \left[\frac{1}{2}MR^2 + 4\times\frac{M}{8}\times\frac{R^2}{4}\right]\omega'$$

$\therefore \quad \omega' = 8\ \text{rad s}^{-1}$.

16. If σ = mass per unit area then total mass $M = 4\sigma\pi R^2$ and the mass of cut-out disc $m = \sigma\pi R^2$.

$$\therefore \quad M = 4m.$$

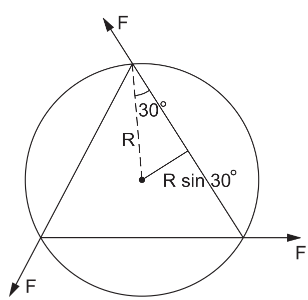
$$\text{Now,}\ I_O = \frac{1}{2}M\cdot 4R^2 - \left(\frac{1}{2}mR^2 + mR^2\right) = \frac{13}{2}mR^2,\ \text{and}$$

$$I_P = \frac{1}{2}M\cdot 4R^2 + M\cdot 4R^2 - \left(\frac{1}{2}mR^2 + m\cdot 5R^2\right) = \frac{37}{2}mR^2.$$

$$\Rightarrow \quad \frac{I_P}{I_O} = \frac{37}{2}\times\frac{2}{13} \approx 3.$$

17. Torque $\tau = 3F(R\sin 30°)$

$$= \frac{3}{2}FR.$$



Now, $\omega = \omega_0 + \alpha t$

$$= 0 + \frac{\tau}{I}t.$$

$$\omega = \frac{\frac{3}{2}FR}{M\frac{R^2}{2}}(1\ \text{s})$$

$$= \frac{3\left(\frac{1}{2}\text{N}\right)(1\ \text{s})}{(1.5\ \text{kg})(0.5\ \text{m})} = 2\ \text{rad s}^{-1}.$$

18. Since the net torque about the centre of rotation is zero, the angular momentum about the centre of the disc is conserved.

$$\therefore\ I\omega + 2mv\left(\frac{r}{2}\right) = 0.$$

In magnitude, $\frac{1}{2}Mr^2\cdot\omega = mvr$

$$\Rightarrow \frac{1}{2}(0.45)\left(\frac{1}{2}\right)^2 \omega = (0.05)(9)\left(\frac{1}{2}\right)$$

$$\Rightarrow \omega = 4 \text{ rad s}^{-1}.$$

19. The kinetic energy of a pure rolling disc moving with v_{CM} is

$$\frac{1}{2} m v_{\text{CM}}^2 \left(1 + \frac{K^2}{R^2}\right) = \frac{3}{4} m v_{\text{CM}}^2.$$

Conserving total mechanical energy for both:

$$\frac{3}{4} m(3)^2 + mg(30) = \frac{3}{4} m v_2^2 + mg27$$

$$\Rightarrow v_2 = 7 \text{ ms}^{-1}.$$

20. For the sphere A:

$$I_A = \int \frac{2}{3}\rho(r) 4\pi r^2 dr \cdot r^2$$

$$= \frac{2}{3}\frac{k}{R}\int_0^R r 4\pi r^4 dr = A\int r^5 dr = \frac{AR^6}{6}.$$

For the sphere B:

$$I_B = \int \frac{2}{3} k \frac{r^5}{R^5} 4\pi r^2 dr \cdot r^2 = \frac{A}{R^4}\int_0^R r^9 dr$$

$$= \frac{AR^{10}}{10R^4} = \frac{AR^6}{10}.$$

$$\therefore \frac{I_B}{I_A} = \frac{6}{10} = \frac{n}{10}, \; n = 6.$$

21. Escape velocity, $v = \sqrt{2gR}$.

But acceleration due to gravity, $g = \dfrac{GM}{R^2} = G \cdot \dfrac{4}{3}\pi R\rho$.

$$\therefore \quad R = \frac{3g}{4\pi G\rho}$$

$$\text{or} \quad v = \sqrt{\frac{2g \cdot 3g}{4\pi G\rho}} = K\sqrt{\frac{g^2}{\rho}} \Rightarrow v \propto \sqrt{\frac{g^2}{\rho}}.$$

$$\therefore \quad \frac{v_{\text{planet}}}{v_{\text{earth}}} = \sqrt{\left(\frac{g_p}{g_e}\right)^2\left(\frac{\rho_e}{\rho_p}\right)} = \sqrt{\left(\frac{\sqrt{6}}{11}\right)^2 \cdot \left(\frac{3}{2}\right)} = \frac{3}{11}.$$

$$\therefore \quad v_{\text{planet}} = \frac{3}{11} \times 11 \text{ km s}^{-1} = 3 \text{ km s}^{-1}.$$

22. At the height h, $g' = \dfrac{g}{4}$, so $\dfrac{gR^2}{(R+h)^2} = \dfrac{g}{4} \Rightarrow R = h.$

Conserving mechanical energy,

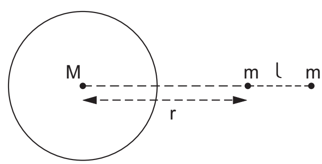
$$\frac{-GMm}{R} + \frac{1}{2}mv^2 = -\frac{GMm}{(R+h)} = -\frac{GMm}{2R}$$

$$\Rightarrow \frac{1}{2}mv^2 = \frac{GMm}{2R} \quad \Rightarrow v = \sqrt{\frac{GM}{R}} = \sqrt{\frac{2GM}{2R}}$$

$$\Rightarrow v = \frac{1}{\sqrt{2}}v_{esc} \text{ or } v_{esc} = v\sqrt{2} = v\sqrt{N}. \quad \therefore N = 2.$$

23. For m closer to M at $r = 3l$,

$$\frac{GMm}{9l^2} - \frac{Gm^2}{l^2} = ma. \qquad \text{...(i)}$$



For the other mass (m)

$$\frac{GMm}{16l^2} + \frac{Gm^2}{l^2} = ma. \qquad \text{...(ii)}$$

Equating (i) and (ii), $m = \dfrac{7M}{288} \quad \Rightarrow K\left(\dfrac{M}{288}\right) = 7\left(\dfrac{M}{288}\right)$

$$\Rightarrow K = 7.$$

24. For the wire–block system in equilibrium,

Young modulus, $Y = \dfrac{\text{stress}}{\text{strain}} = \dfrac{mg/A}{\Delta l/L} = \dfrac{mg}{A}\dfrac{L}{\Delta l}.$

$\therefore$ weight $mg = \dfrac{YA}{L}\Delta l = k\Delta l,$

where $\dfrac{YA}{L} = k$ = force constant.

The force equation for the block is,

$$F = -kx$$

$\Rightarrow \quad a = -\dfrac{k}{m}x \Rightarrow$ angular frequency, $\omega = \sqrt{\dfrac{k}{m}}$

or $\quad \omega = \sqrt{\dfrac{YA}{Lm}} = \sqrt{\dfrac{(n)(10^9 \text{ N m}^{-2})(4.9\times10^{-7} \text{ m}^2)}{(1 \text{ m})(0.1 \text{ kg})}} = (70 \text{ rad s}^{-1})\sqrt{n}.$

But $\omega = 140 \text{ rad s}^{-1}$

$\Rightarrow \quad n = 4.$

25. Terminal velocity $v_T = \dfrac{2}{9}\dfrac{r^2}{\eta}(\rho - \sigma)g,$

where r = radius of the sphere,

ρ = density of the solid sphere,
σ = density of the viscous liquid.

$$\frac{v_P}{v_Q} = \frac{(8-0.8)\times\left(\frac{1}{2}\right)^2 \times 2}{(8-1.6)\times\left(\frac{1}{4}\right)^2 \times 3} = 3.$$

26. Increase in the length of the wire due to weight mg, $\Delta L = \frac{mgL}{YA}$.

Decrease in length due to cooling $= |\Delta L| = L\alpha\Delta T$.

$$\therefore \quad \frac{mgL}{YA} = L\alpha\Delta T$$

$$\text{or} \quad m = \frac{YAL\alpha\Delta T}{gL} = \frac{(10^{11}\text{ N m}^{-2})(\pi 10^{-6}\text{ m}^2)(10^{-5}\text{ °C}^{-1})(10\text{ °C})}{(10\text{ m s}^{-2})}$$

$$= 3.1\text{ kg} \approx 3\text{ kg}.$$

27. The force exerted on the plate by the balls on the shaded area is

$$F = n(2mv)\left(\frac{ab}{2}\right) = mnv \cdot ab.$$

Torque of this force on the hinged end,

τ_1 = F × distance of the CM of the shaded part from the left end

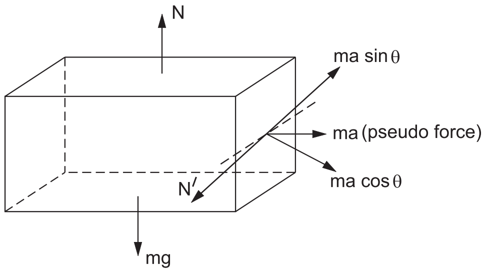
$$= mnvab \times \frac{3b}{4}.$$

This is balanced by the torque due to weight Mg

$$\Rightarrow \quad mnvab \cdot \frac{3}{4}b = Mg \cdot \frac{b}{2}.$$

$$\therefore \quad v = \frac{4Mg}{6mnab} = 10\text{ m s}^{-1}.$$

28. Consider the forces shown in the free-body diagram.



$N = mg$, $N' = ma\sin\theta$ and $ma\cos\theta - \mu(N + N') = ma_{\text{block}}$, where a_{block} is the acceleration of the block relative to the disc.

$$\Rightarrow \; m \cdot a \cdot \frac{2}{5} - \frac{2}{5}\left(mg + ma \times \frac{3}{5}\right) = ma_{\text{block}}.$$

Simplify to get $a_{\text{block}} = 10 \text{ m s}^{-2}$.

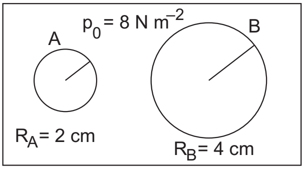
29. $2mg - T = 2ma$, $T - mg = ma$.

$\therefore$ acceleration of blocks, $a = \dfrac{g}{3}$ and tension in the string, $T = \dfrac{4mg}{3}$.

Work done by the string,

$$W = T \cdot S = T \cdot \frac{1}{2}at^2 = 8 \text{ J}.$$

30. The pressures inside soap bubbles A and B are



$$p_A = p_0 + \frac{4S}{R_A} = 16 \text{ N m}^{-2},$$

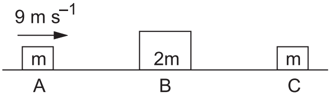
$$p_B = p_0 + \frac{4S}{R_B} = 12 \text{ N m}^{-2}.$$

$$p_A V_A = n_A RT.$$

$$p_B V_B = n_B RT.$$

$$\therefore \; \frac{n_A}{n_B} = \frac{p_A}{p_B} \times \left(\frac{R_A}{R_B}\right)^3 = \frac{1}{6} \Rightarrow \frac{n_B}{n_A} = 6.$$

31. After the first (elastic) collision between A and B,



$$m(9 \text{ m s}^{-1}) = mv'_A + 2mv'_B$$

$$\Rightarrow \quad v'_A + 2v'_B = 9 \text{ m s}^{-1}. \qquad \text{...(i)}$$

Coefficient of restitution, $e = 1$ (elastic collision).

Velocity of separation = e × velocity of approach.

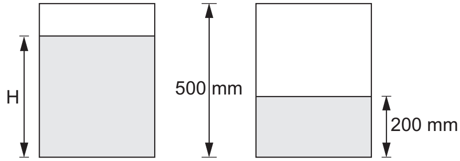
$$\therefore \quad v'_B - v'_A = 1 \times v_A. \qquad \text{...(ii)}$$

Solving (i) and (ii), $v'_B = 6 \text{ m s}^{-1}$.

After the second collision

$$2mv'_B = (2m + m)\, v_C \quad \Rightarrow \quad v_C = 4 \text{ m s}^{-1}.$$

32.



$p = p_0 - \rho g H = 98 \times 10^3 \text{ N m}^{-2}$.

$$p_0 V_0 = pV$$

$$\Rightarrow \quad 10^5\,[A(500 - H)] = 98 \times 10^3\,[A(500 - 200)]$$

$$\Rightarrow \quad H = 206 \text{ mm.}$$

Fall in water level = (206 – 200) mm = 6 mm.

33. Surface energy of the liquid drop, $U = 4\pi R^2 S$.

If r = radius of each droplet then

$$\frac{4}{3}\pi R^3 = K \cdot \frac{4}{3}\pi r^3 \Rightarrow r = \frac{R}{K^{\frac{1}{3}}}.$$

Increase in the surface energy

$$\Delta U = S(4\pi R^2)\left(K^{\frac{1}{3}} - 1\right)$$

$$\Rightarrow K^{\frac{1}{3}} = \frac{\Delta U}{4\pi R^2 \cdot S} + 1 = 101 \approx 10^2.$$

$\therefore K \approx 10^6 = 10^\alpha. \qquad \therefore \alpha = 6.$

3.2 Heat and Thermodynamics

1. Heat required to raise the temperature of the ice block,

$$Q_1 = mc\Delta\theta = m(2100 \text{ J kg}^{-1}\,°\text{C}^{-1})(5\,°\text{C})$$

Heat required to melt 1 g of ice $= mL = \left(\frac{1}{1000}\text{ kg}\right)(3.36 \times 10^5 \text{ J kg}^{-1})$.

$$\therefore \quad Q = Q_1 + Q,$$

or

$$420 \text{ J} = \left(\frac{m}{1000}\text{ kg}\right)(2100 \text{ J kg}^{-1}\,°\text{C}^{-1})(5\,°\text{C}) + 3.36 \times 100 \text{ J}$$

Simplifying, we get

$$m = \frac{(420 - 336)10}{105}\text{ g} = 8 \text{ g.}$$

2. The heat lost by the water during its condensation from steam (at 100 °C) to water (at 0 °C) is

$$\begin{aligned} Q_1 &= mL + mc_w\,\Delta\theta \\ &= (0.05 \text{ kg})(540 \text{ cal g}^{-1}) + (0.05 \text{ kg})(1 \text{ cal g}^{-1}\text{ K}^{-1})(100 \text{ K}) \\ &= 27 \text{ kcal} + 5 \text{ kcal} = 32 \text{ kcal.} \end{aligned}$$

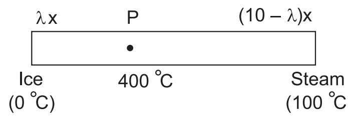
The heat required by the ice to reach 0 °C and then to melt (at 0 °C) is

$$\begin{aligned} Q_2 &= m's'\Delta\theta + m'L' \\ &= (0.45 \text{ kg})(0.5 \text{ cal g}^{-1}\text{ K}^{-1})(20 \text{ K}) + (0.45 \text{ kg})(80 \text{ cal g}^{-1}) \\ &= 4.5 \text{ kcal} + 36 \text{ kcal} = 40.5 \text{ kcal.} \end{aligned}$$

Since $Q_1 < Q_2$, the steam will not provide the required amount of heat to melt the entire ice. The temperature of the mixture remains at 0 °C.

3. Steady thermal current, $\dfrac{\Delta Q}{\Delta t} = kA\left(\dfrac{\Delta T}{\Delta x}\right)$

$$\Rightarrow \quad \frac{kA\,(400-0)\,°\mathrm{C}}{\lambda x \cdot L_{\mathrm{ice}}} = \frac{kA\,(400-100)\,°\mathrm{C}}{(10-\lambda)\,x \cdot L_{\mathrm{water}}}$$



Substituting the values, $\lambda = 9$.

4. For a diatomic gas undergoing adiabatic process,

$$TV^{\gamma-1} = \text{constant, where } \gamma = \frac{C_{\mathrm{p}}}{C_{\mathrm{V}}} = \frac{7}{5}.$$

$$\therefore \quad T_{\mathrm{i}}V^{2/5} = \alpha T_{\mathrm{i}}\left(\frac{V}{32}\right)^{2/5}$$

$$\Rightarrow \quad \alpha = (32)^{2/5} = (2^5)^{2/5} = 2^2 = 4.$$

5. $U_b = 200$ J, $U_i = 200$ J

For the process iaf:

Process	***W***	**Δ*U***	***Q***
ia		0	
af		200 J	
Net	300 J	200 J	500 J

$$\Rightarrow U_{\mathrm{f}} = 400 \text{ J}.$$

For the process ibf:

Process	***W***	**Δ*U***	***Q***
ib	100 J	50 J	150 J
bf	200 J	100 J	300 J
Net	300 J	150 J	450 J

$$\Rightarrow \frac{Q_{\mathrm{bf}}}{Q_{\mathrm{ib}}} = \frac{300 \text{ J}}{150 \text{ J}} = 2.$$

6. $\lambda_{\mathrm{m}} T = $ constant.

Rate of radiated energy, $Q = (4\pi R^2)(\sigma T^4)$.

$$\therefore \quad \frac{Q_1}{Q_2} = \left(\frac{R_1}{R_2}\right)^2\left(\frac{T_1}{T_2}\right)^4 = \left(\frac{R_1}{R_2}\right)^2\left(\frac{\lambda_2}{\lambda_1}\right)^4 = \left(\frac{6 \text{ cm}}{18 \text{ cm}}\right)^2\left(\frac{1500 \text{ nm}}{500 \text{ nm}}\right)^4$$

$$= \left(\frac{1}{3}\right)^2 (3)^4 = 9.$$

7. According to Wien's displacement law

$$\lambda_A T_A = \lambda_B T_B.$$

Ratio of power radiated

$$\frac{E_A}{E_B} = \frac{\sigma T_A^4 A_A}{\sigma T_B^4 A_B}$$

$$\Rightarrow \ 10^4 = \left(\frac{T_A}{T_B}\right)^4 \left(\frac{r_A}{r_B}\right)^2 = \left(\frac{\lambda_B}{\lambda_A}\right)^4 \left(\frac{r_A}{r_B}\right)^2 = \left(\frac{\lambda_B}{\lambda_A}\right)^4 \left(\frac{400}{1}\right)^2$$

$$\Rightarrow \ \frac{\lambda_A}{\lambda_B} = 2.$$

8. Given: $\log_2\left(\frac{P_1}{P_0}\right) = 1 \qquad \Rightarrow P_0 = \frac{P_1}{2}.$

According to Stefan's law,

$$\frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^4 = \frac{(2767+273)^4}{(487+273)^4} = 4^4.$$

$$\frac{P_2}{P_1} = \frac{P_2}{2P_0} = 4^4 \Rightarrow \frac{P_2}{P_0} = 2 \times 4^4 = 4^{9/2}.$$

$$\therefore \ \log_2\left(\frac{P_2}{P_0}\right) = \log_2(4)^{9/2} = \log_2 2^9 = 9.$$

3.3 Sound Waves

1. Phase difference $= \frac{\pi}{2}\cdot$ Hence resultant amplitude

$$A = \sqrt{a_1^2 + a_2^2} = \sqrt{4^2 + 3^2} = 5.$$

2. The first and the fourth waves have phase difference π, so they interfere destructively leading to zero intensity.

The second and the third waves differ in phase by $\left(\frac{2\pi}{3} - \frac{\pi}{3}\right) = \frac{\pi}{3}\cdot$ So the net intensity,

$$I_{net} = I_0 + I_0 + 2\sqrt{I_0}\sqrt{I_0}\cos\frac{\pi}{3} = 3I_0.$$

3. Apparent frequency of the sound wave received by the car (observer) approaching the source is

$$f' = \frac{v + v_0}{v} f_0,$$

where v_0 = velocity of the observer,
v = velocity of sound wave, and
f_0 = true frequency.

Frequency of the reflected wave,

$$f'' = \left(\frac{v}{v-v_0}\right) f' = \left(\frac{v+v_0}{v-v_0}\right) f_0.$$

$\therefore$ difference of the frequencies reflected from the two cars,

$$\Delta f = (1.2\%)f_0 = \left[\frac{v+v_1}{v-v_1} - \frac{v+v_2}{v-v_2}\right] f_0$$

$$\Rightarrow \qquad \frac{1.2}{100} = \left(1+\frac{v_1}{v}\right)\left(1-\frac{v_1}{v}\right)^{-1} - \left(1+\frac{v_2}{v}\right)\left(1-\frac{v_2}{v}\right)^{-1}$$

$$\approx \frac{2}{v}(v_1 - v_2) \qquad \text{(from binomial theorem).}$$

$\therefore$ difference in speeds of cars,

$$\Delta v = v_1 - v_2 = \frac{12}{1000}\,\frac{330\text{ m s}^{-1}}{2} = 1.98\text{ m s}^{-1} \approx 2\text{ m s}^{-1}$$

$$= 2 \times \frac{18}{5}\text{ km h}^{-1} = 7.2\text{ km h}^{-1} \approx 7\text{ km h}^{-1}.$$

4. $v = \sqrt{\dfrac{F}{\mu}} = \sqrt{\dfrac{(0.5\text{ N})(20\times 10^{-2}\text{ m})}{(1\times 10^{-3}\text{ kg})}} = 10\text{ m s}^{-1}.$

Wavelength, $\lambda = \dfrac{v}{f} = \dfrac{10\text{ m s}^{-1}}{100\text{ s}^{-1}} = 0.1\text{ m} = 10\text{ cm}.$

Separation between two successive nodes

$$= \frac{\lambda}{2} = 5\text{ cm}.$$

5. Frequency received by the approaching car (as observer)

$$f_1 = \left(\frac{v+v_0}{v}\right) f.$$

Frequency of the reflected wave as received by the source

$$f_2 = \left(\frac{v}{v-v_0}\right) f_1 = \left(\frac{v+v_0}{v-v_0}\right) f.$$

$\therefore$ beat frequency $= f_2 - f = \left(\dfrac{v+v_0}{v-v_0} - 1\right) f = \left(\dfrac{332}{328} - 1\right) 492\text{ s}^{-1} = 6\text{ s}^{-1}.$

3.4 Electrostatics

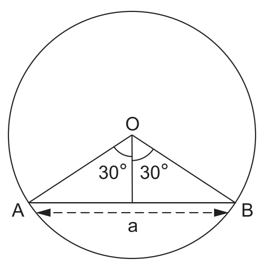
1. Due to cylinder: $E_1 = \dfrac{\rho R^2}{\varepsilon_0 \cdot 4R}$.

Due to sphere: $E_2 = \dfrac{1}{4\pi\varepsilon_0} \dfrac{\frac{4}{3}\pi \frac{R^3}{8}\rho}{(2R)^2}$.

Net field, $E_1 - E_2 = \dfrac{\rho R}{4\varepsilon_0} - \dfrac{\rho R}{4\varepsilon_0 \times 24} = \dfrac{23\,\rho R}{4 \times 24\varepsilon_0} = \dfrac{23\rho R}{16K\varepsilon_0}$

$\Rightarrow \quad K = 6.$

2. Flux from the total cylindrical surface of radius OA (= $a/2$) is equally distributed to the hexagonal shape formed by six given rectangular surface ABCD. Hence the required flux



$$\phi = \frac{Q_{\text{inside}}}{6\varepsilon_0} = \frac{\lambda L}{6\varepsilon_0} = \frac{\lambda L}{n\varepsilon_0} \Rightarrow n = 6.$$

3. $F_{\text{electrical}} \propto \dfrac{q^2}{a^2} \propto Sa.$

$\therefore\ a = K\left(\dfrac{q^2}{S}\right)^{1/3} \quad \Rightarrow N = 3.$

4. Total charge enclosed, $Q = \displaystyle\int_0^R (4\pi r^2 dr)\,Kr^n = 4\pi K \frac{R^{n+3}}{n+3}$.

Charge contained from $r = 0$ to $\dfrac{R}{2}$ is

$$Q' = \int_0^{R/2} 4\pi r^2 dr \cdot Kr^n = 4\pi K \frac{(R/2)^{n+3}}{n+3}.$$

$E(R) = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{R^2}$ and $E\left(\dfrac{R}{2}\right) = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q'}{(R/2)^2}$.

Given $\dfrac{1}{8}E(R) = E\left(\dfrac{R}{2}\right)$

$\Rightarrow\ 2^{n+3} = 2^5$

$\Rightarrow\ n = 2.$

3.5 Current Electricity and Magnetism

1. Rate of heat production, $J = I^2R = \left(\frac{\mathcal{E}}{R_{eq}}\right)^2 R.$

When in series, $J_1 = \left(\frac{2\mathcal{E}}{R+2r}\right)^2 R = \frac{4\mathcal{E}^2}{(R+2)^2}R \qquad [\because r = 1\,\Omega].$

When in parallel, $J_2 = \left(\frac{2\mathcal{E}}{2R+r}\right)^2 R = \frac{4\mathcal{E}^2}{(2R+1)^2}R.$

Given $J_1 = 2.25J_2$, so $\frac{1}{(R+2)^2} = \frac{2.25}{(2R+1)^2}$

$\Rightarrow \frac{2R+1}{R+2} = 1.5 \qquad \Rightarrow R = 4\,\Omega.$

2. The given circuit is a closed loop, so the current through the loop,

$$I = \frac{(6-3)\,\text{V}}{(2+1)\,\Omega} = 1\text{ A}.$$

From the loop rule, $V_A - 6 + 1 - V_B = 0 \Rightarrow V_A - V_B = 5$ V.

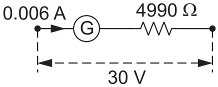
Alternative method: Equivalent emf of the given combination is

$$\mathcal{E} = \frac{(e_1r_2 + e_2r_1)}{(r_1+r_2)} = \frac{(6\times 2 + 3\times 1)}{(2+1)} = \frac{15}{3} = 5\text{ V}.$$

3. Let G be the resistance of the galvanometer coil.

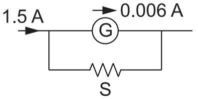
When used as a voltmeter:

$$0.006 = \frac{30}{4990+G} \Rightarrow G = 10\,\Omega.$$



When used as an ammeter:

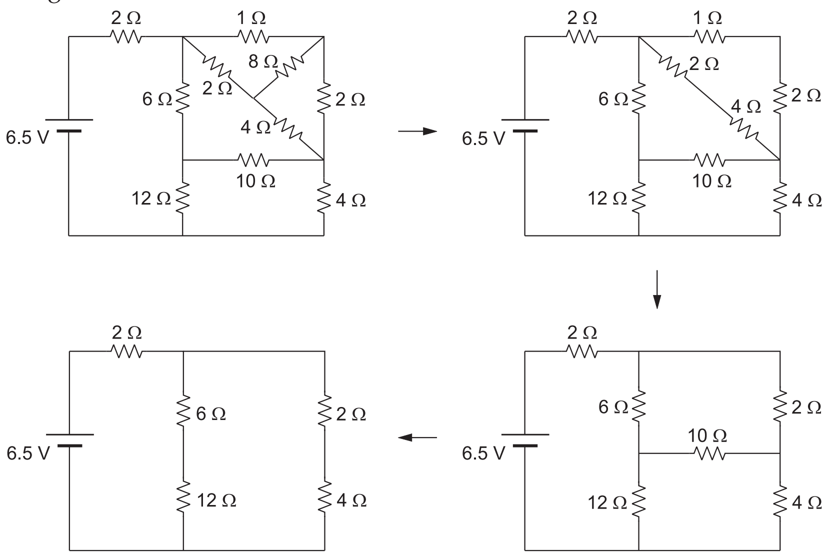
$$I_g = \frac{S}{G+S}I$$

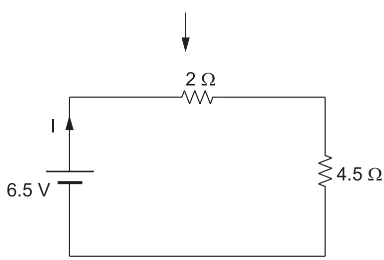


$$\Rightarrow \frac{6}{1000} = \frac{S}{10+S}\times\frac{3}{2}.$$

$$\therefore \quad S = \frac{10}{249}\,\Omega = \frac{2n}{249}\,\Omega \quad \Rightarrow n = 5.$$

4. The equivalent circuit for the given network can be redrawn in steps as given below:





$\therefore I = 1$ A.

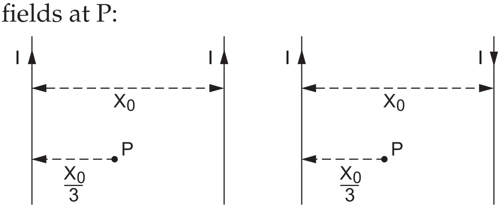
5. Time constant $\tau = RC = (10^6\ \Omega)\ (4\ \mu\text{F}) = 4$ s.

$4\text{ V} = (10\text{ V})\ [1 - e^{-t/\tau}] \Rightarrow e^{-t/4} = 0.6 = 3/5.$

$t/4 = \ln 5 - \ln 3 = 0.5 \Rightarrow t = 2$ s.

6. Net field at P $= \dfrac{\mu_0 Ja}{2} - \dfrac{\mu_0 Ja}{12} = \dfrac{5}{12}\mu_0 Ja = \dfrac{N}{12}\mu_0 Ja. \quad \therefore\ N = 5.$

7. Magnetic fields at P:



Case I   Case II

In case I. $$B_1 = \frac{\mu_0 I}{2\pi\left(\frac{X_0}{3}\right)} - \frac{\mu_0 I}{2\pi\left(\frac{2X_0}{3}\right)} = \frac{3\mu_0 I}{4\pi X_0}.$$

In case II. $$B_2 = \frac{\mu_0 I}{2\pi\left(\frac{X_0}{3}\right)} + \frac{\mu_0 I}{2\pi\left(\frac{2X_0}{3}\right)} = \frac{9\mu_0 I}{4\pi X_0}.$$

$$\therefore \quad \frac{B_2}{B_1} = 3.$$

If R_1 and R_2 be the corresponding radii,

$$\frac{R_1}{R_2} = \frac{mv/qB_1}{mv/qB_2} = \frac{B_2}{B_1} = 3.$$

8. Magnetic field in the cylindrical tube,

$$B = \mu_0 ni = \mu_0 \frac{I}{L}.$$

The emf induced in the wire loop,

$$\mathcal{E} = -\frac{d\phi}{dt} = -\frac{d}{dt}(BA) = -\frac{d}{dt}\left(\mu_0 \frac{I}{L} \cdot \pi r^2\right) = -\frac{\mu_0 \pi r^2}{L} \frac{dI}{dt}.$$

Magnetic moment $= (I_{\text{loop}})(\pi r^2) = \frac{\mathcal{E}}{R}\pi r^2.$

$$\therefore \quad -\frac{\mu_0 \pi r^2}{RL} \frac{d}{dt}(I_0 \cos 300t) \times \pi r^2 = N\mu_0 I_0 \sin 300t.$$

Solving, $N = 6$.

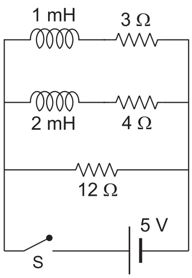
9. $$B = \frac{\mu_0 IR^2}{2(R^2+x^2)^{3/2}} = \frac{\mu_0 IR^2}{2(4R^2)^{3/2}} \qquad [\because x = \sqrt{3}R].$$

Magnetic flux, $\phi = NBA \cos 45° = \frac{\mu_0 Ia^2}{8\sqrt{2}R}.$

Now, mutual inductance,

$$M = \frac{\phi}{I} = \frac{\mu_0 a^2}{8\sqrt{2}R} = \frac{\mu_0 a^2}{2^{7/2}R} = \frac{\mu_0 a^2}{2^{p/2}R} \quad \Rightarrow \quad p = 7.$$

10. At $t = 0$, an inductor offers large resistance and at $t = \infty$, inductors behave as conductors (zero resistance).



At $t = \infty$, $I_{\max} = \frac{5\text{ V}}{R_{\text{eq}}} = \frac{5}{3/2}\text{ A} = \frac{10}{3}\text{ A}.$

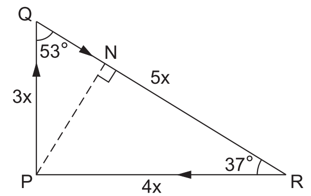
At $t = 0$, $I_{\min} = \frac{5\text{ V}}{R_{\text{eq}}} = \frac{5\text{ V}}{12\ \Omega} = \frac{5}{12}\text{ A}.$

$$\therefore \quad \frac{I_{\max}}{I_{\min}} = \frac{10/3}{5/12} = 8.$$

11. $R\sqrt{1.25} = \sqrt{R^2 + \dfrac{1}{\omega^2 C^2}}$

$\Rightarrow$ time constant of the RC circuit = 4 ms.

12. $PN = 4x \cdot \sin 37° = \dfrac{12x}{5}.$



$\therefore\ B = \dfrac{\mu_0}{4\pi} I \left[\dfrac{\cos 53° + \cos 37°}{(12x/5)}\right]$

$= 7\left(\dfrac{\mu_0 I}{48\pi x}\right)$

$= K\left(\dfrac{\mu_0 I}{48\pi x}\right). \Rightarrow K = 7.$

3.6 Ray Optics and Wave Optics

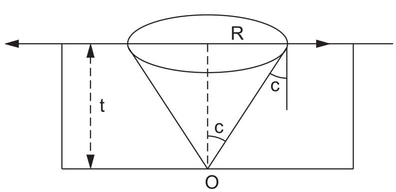
1. $f = 10$ m; for $v = \dfrac{50}{7}$ m, $u = -25$ m

for $v = \dfrac{25}{3}$ m, $u = -50$ m, $t = 30$ s

speed $= \left(\dfrac{25\text{ m}}{30\text{ s}}\right) = 3$ km h^{-1}

2. $\sin C = \dfrac{1}{n} = \dfrac{3}{5} = \dfrac{R}{(R^2 + t^2)^{\frac{1}{2}}}$

$t = 8$ cm, $R = 6$ cm.



3. From lens formula for real image, $\dfrac{1}{v} + \dfrac{1}{u} = \dfrac{1}{f}$, so magnification $m = \dfrac{f}{u - f}$

$\Rightarrow \dfrac{m_{25}}{m_{50}} = \dfrac{50 - 20}{25 - 20} = 6 \qquad [\because f = 20 \text{ cm}].$

4. For refraction through the spherical surface

$$\frac{7}{4v_1} - \frac{1}{-24} = \frac{\frac{7}{4} - 1}{6}, \text{ so } v_1 = 21 \text{ cm.}$$

Finally, $\dfrac{\frac{4}{3}}{v_2} - \dfrac{\frac{7}{4}}{21} = 0$, so $v_2 = 16$ cm.

$\therefore\ x = 18 - 16 = 2$ cm.

5. From the mirror formula, $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$, $u = 15$ cm, $f = 10$ cm.

So, $v = 30$ cm (right). Now u for the lens $= 20$ cm $= 2f_{\text{lens}}$. From the lens formula, $v_{\text{lens}} = 20$ cm (right)

$\Rightarrow$ magnification by the lens $|M_L| = 1$ (in air). When kept in liquid $\left(\mu' = \frac{7}{6}\right)$, the focal length of the lens is changed from $f = 10$ cm to f'.

$$\frac{\frac{1}{f}}{\frac{1}{f'}} = \frac{(\mu_g - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}{\left(\frac{\mu_g}{\mu'} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)} = \frac{\left(\frac{3}{2} - 1\right)}{\left(\frac{3}{2}\cdot\frac{6}{7} - 1\right)} \Rightarrow f' = \frac{35}{2} \text{ cm.}$$

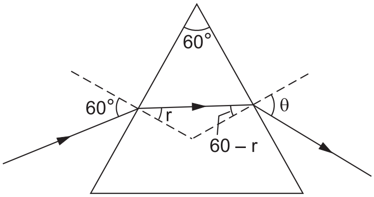
Change of medium does not affect f for mirror and hence magnification as well.

So, for lens, $u = 20$ cm and $\frac{1}{v'} - \frac{1}{-30} = \frac{2}{35} \Rightarrow v' = 140$ cm.

$\therefore M_L' = \frac{v'}{u'} = \frac{140}{20} = 7$. Now, for the system, $\frac{M'}{M} = \frac{M_L'}{M_L} = 7$.

6. $\sin 60° = n \sin r$. ...(i)

$\sin\theta = n\sin(60° - r)$. ...(ii)



Differentiating (ii) w.r.t. n,

$$\cos\theta \frac{d\theta}{dn} = -n\cos(60° - r)\frac{dr}{dn} + \sin(60° - r). \quad \text{...(iii)}$$

Differentiating (i)

$$n\cos r \cdot \frac{dr}{dn} + \sin r = 0.$$

From (iii), $\cos\theta \frac{d\theta}{dn} = -n\cos(60° - r)\left(\frac{-\tan r}{n}\right) + \sin(60° - r)$

or $\frac{d\theta}{dn} = \frac{1}{\cos\theta}[\cos(60° - r)\tan r + \sin(60° - r)]$

$\Rightarrow$ $\frac{d\theta}{dn} = \frac{1}{\cos 60°}[\cos 30° \tan 30° + \sin 30°]$

$= 2\left[\frac{1}{2} + \frac{1}{2}\right] = 2$, so $m = 2$.

7. For the internal reflection,

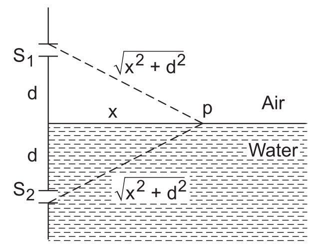
$$n \sin\theta = (n - m\Delta n) \sin 90°$$

$$\Rightarrow 1.6 \sin 30° = (1.6 - m \times 0.1)$$

$$\Rightarrow \quad 0.8 = 1.6 - \frac{m}{10} \quad \Rightarrow m = 8.$$

8. For maxima at p,

$$\Delta x = m\lambda, \text{ where } m = \text{integer}.$$



$$\mu\sqrt{x^2+d^2} - \sqrt{x^2+d^2} = m\lambda$$

$$\Rightarrow \quad \left(\frac{4}{3}-1\right)\sqrt{d^2+x^2} = m\lambda$$

$$\Rightarrow \quad \frac{d^2+x^2}{9} = m^2\lambda^2.$$

$\therefore \; x^2 = 9m^2\lambda^2 - d^2.$

Comparing with the given expression,

$$x^2 = p^2m^2\lambda^2 - d^2, \text{ we get } p = 3.$$

3.7 Modern Physics

1. KE $= \dfrac{p^2}{2m} = qV$, or momentum $p = \sqrt{2mqV}$.

de Broglie wavelength $\lambda = \dfrac{h}{p}\cdot$

$$\therefore \; \frac{\lambda_p}{\lambda_\alpha} = \frac{\sqrt{2m_\alpha q_\alpha V}}{\sqrt{2m_p q_p V}} = \sqrt{\frac{m_\alpha q_\alpha}{m_p q_p}} = \sqrt{4\times 2} = \sqrt{8} \approx 3.$$

2. Slope of line $= -\dfrac{1}{2}\cdot \quad \therefore \lambda = \dfrac{1}{2}\text{yr}^{-1}, \; t_{1/2} = \dfrac{0.693}{\lambda} = 1.386$ yr.

4.16 yr $= 3t_{1/2}$. $\quad \therefore p = 2^3 = 8.$

3. If V_s = stopping potential then

$$eV_s = \frac{hc}{\lambda} - \phi_0 = \frac{1240 \text{ eVnm}}{200 \text{ nm}} - 4.7 \text{ eV} = 1.5 \text{ eV}.$$

$$\therefore \; V_s = 1.5 \text{ V} = K\frac{Q}{r} = K\frac{ne}{r}\cdot$$

$$\therefore \; n = \frac{(1.5)(1\times 10^{-2})}{(9\times 10^9)(1.6\times 10^{-19})} = 1.05 \times 10^7.$$

4. From Einstein's photoelectric equation,

$$h\nu = \phi_0 + eV_s \quad \text{or} \quad V_s = \frac{h}{e}\nu - \frac{\phi_0}{e}\cdot$$

The plot of V_s vs ν is a straight line with a constant slope $\frac{h}{e}$ for all samples.

Thus, the ratio of slopes = 1.

5. $$\frac{p^2}{2m} = \frac{KZe^2}{r} \Rightarrow p = \sqrt{\frac{KZe^2 \cdot 2m}{r}}$$

$$\Rightarrow \lambda = \frac{h}{p} = \frac{h/e}{\sqrt{2K\frac{Zm}{r}}} = \frac{4.2 \times 10^{-15}}{\sqrt{\dfrac{2 \times 9 \times 10^9 \times 120 \times \frac{5}{3} \times 10^{-27}}{10 \times 10^{-15}}}} = 7 \text{ fm}.$$

6. Energy of absorbed photon = ionisation energy + KE of the electron

$$\Rightarrow \frac{hc}{\lambda} = \frac{13.6 \text{ eV}}{n^2} + 10.4 \text{ eV}$$

$$\Rightarrow \frac{1242 \text{ eV nm}}{90 \text{ nm}} - 10.4 \text{ eV} = \frac{13.6}{n^2} \text{ eV}$$

$$\Rightarrow n^2 = 4 \quad \Rightarrow n = 2.$$

7. $$L = \frac{nh}{2\pi} = \frac{3h}{2\pi} \Rightarrow n = 3.$$

$$\lambda = \frac{h}{p} = \frac{hr}{mvr} = \frac{hr}{L} = \frac{hr}{3h/2\pi} = \frac{2\pi r}{3}.$$

But, $r = a_0 \frac{n^2}{Z} = \frac{a_0 9}{3} = 3a_0.$

$$\Rightarrow \quad \lambda = 2\pi a_0 = p\pi a_0.$$

$$\therefore \quad p = 2.$$

8. Energy of the incident photon,

$$E = \frac{hc}{\lambda} = \frac{1.237 \times 10^{-6} \text{ eV m}}{970 \times 10^{-10} \text{ m}} = 12.75 \text{ eV}.$$

Final energy,

$$E_n = (-13.6 + 12.75) \text{ eV} = -0.85 \text{ eV} = -\frac{13.6}{4^2} \text{eV}.$$

This corresponds to the state $n = 4$ of H-atom.

The number of spectral lines present in the emission spectrum is $^4C_2 = 6$.

9. Potential energy $V \propto \frac{1}{n^2}$.

$$\Rightarrow \frac{V_i}{V_f} = \frac{n_f^2}{n_i^2} \Rightarrow 6.25 = \left(\frac{n_f}{n_i}\right)^2 \Rightarrow \left(\frac{5}{2}\right)^2 = \left(\frac{n_f}{n_i}\right)^2. \therefore \frac{n_f}{n_i} = \frac{5}{2}.$$

$\therefore$ minimum value of n_f is 5.

10. The required fraction $f = \dfrac{N_0 - N}{N_0} = 1 - e^{-\lambda t}$

$$\approx 1 - (1 - \lambda t) = \lambda t = \frac{\ln 2}{T} \times t = 0.04.$$

$\therefore$ decay $\approx 4\%$.

11. If activity at $t = 0$ and at t be A_0 and A respectively then

$$A = A_0 \left(\frac{1}{2}\right)^{\frac{t}{T}}$$

$$\Rightarrow \frac{12.5}{100} = \left(\frac{1}{2}\right)^{\frac{t}{T}} \Rightarrow \left(\frac{1}{8}\right) = \left(\frac{1}{2}\right)^{\frac{t}{T}} \Rightarrow \left(\frac{1}{2}\right)^{3} = \left(\frac{1}{2}\right)^{\frac{t}{T}}.$$

$\therefore \quad t = 3T = nT \quad \Rightarrow n = 3.$

12. For P, $\lambda_P = \dfrac{1}{\tau}$; for Q, $\lambda_Q = \dfrac{1}{2\tau}$

$$\Rightarrow \quad A_P = A_0 e^{-t/\tau};\ A_Q = A_0\, e^{-t/2\tau}$$

$$\Rightarrow \quad R_P = -\frac{dA_P}{dt} = \frac{A_0}{\tau}\, e^{-t/\tau};$$

$$R_Q = -\frac{dA_Q}{dt} = \frac{A_0}{2\tau} e^{-t/2\tau}.$$

$$\therefore \quad \frac{R_P}{R_Q} = 2\,e^{-t/2\tau} = 2e^{-1} \quad (\because t = 2\tau)$$

$$= \frac{2}{e} = \frac{n}{e} \quad \Rightarrow n = 2.$$

13. The nuclear process can be represented as

${}^{12}_{5}\mathrm{B} \longrightarrow {}^{12}_{6}\mathrm{C} + \beta^- + \gamma$, and the corresponding value of

$$Q = \left[m({}^{12}_{5}\mathrm{B}) - m({}^{12}_{6}\mathrm{C}^*)\right]c^2$$

$$= \left[m({}^{12}_{5}\mathrm{B}) - \left(m{}^{12}_{6}\mathrm{C} + \Delta m\right)\right]c^2$$

$$= \left[m({}^{12}_{5}\mathrm{B}) - m({}^{12}_{6}\mathrm{C})\right]c^2 - \Delta m{\cdot}c^2$$

$$= [12.014\ \mathrm{u} - 12u]\,c^2 - 4.041\ \mathrm{MeV}$$

$$= (0.014)\ 931.5\ \frac{\mathrm{MeV}}{c^2}{\cdot}c^2 - 4.041\ \mathrm{MeV}$$

$$= 13.041\ \mathrm{MeV} - 4.04\ \mathrm{MeV} = 9\ \mathrm{MeV}.$$

Hence β^- particle has KE of 9 MeV.

14. Final activity, $A_\text{f} = \dfrac{V_\text{body}}{V} \times A_0 \times e^{-\lambda t}$

$$\Rightarrow\ V_\text{body} = \frac{V}{A_\text{f}} \times A_0 \times e^{-\frac{\ln 2 \times t}{8 \text{ days}}}$$

$$= V \times \frac{A_0}{A_\text{f}} \times e^{-\frac{\ln 2 \times t}{192}} \simeq 4.998 \text{ litres} = 5 \text{ litres.}$$

15. Activity $\left|\dfrac{dN}{dt}\right| = \lambda N.$

$$\therefore\ N = \frac{\left(\dfrac{dN}{dt}\right)}{\lambda} = 10^{10} \times 10^{9} = 10^{19} \text{ atoms.}$$

Mass of the sample = $(10^{-25}$ kg$)(10^{19}) = 10^{-6}$ kg = 1 mg.

❑

4

Matrix-Matching-Type Questions

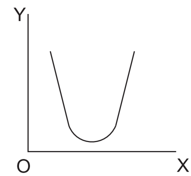
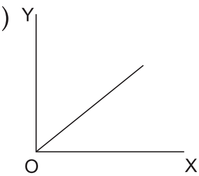
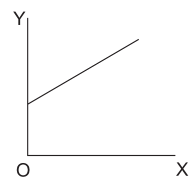
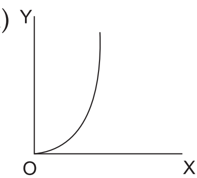
The questions given in this chapter contain two or more columns each. Match the contents of Column A with those of Column B, Column C, etc., by darkening the appropriate bubbles in the matrix given in the answer sheet.

4.1 General Physics

1. Some physical quantities are given in Column A and some SI units in which these quantities may be expressed are given in Column B.

Column A	*Column B*
(i) GM_eM_s, where G = universal gravitational constant M_e = mass of the earth M_s = mass of the sun	(a) V C m
(ii) $\dfrac{3RT}{M}$, where R = universal gas constant T = absolute temperature M = molar mass	(b) kg m^3 s^{-2}
(iii) $\dfrac{F^2}{q^2B^2}$, where F = force, q = charge B = magnetic flux density	(c) m^2 s^{-2}
(iv) $\dfrac{GM_e}{R_e}$, where G = universal gravitational constant M_e = mass of the earth R_e = radius of the earth	(d) F V^2 kg^{-1}

2. Column A gives a list of possible set of parameters measured in some experiments. The variations of the parameters in the form of graphs are shown in Column B.

Column A	*Column B*
(i) The potential energy of a simple pendulum (y-axis) as a function of its displacement (x-axis)	(a) 
(ii) Displacement (y-axis) as a function of time (x-axis) for a one-dimensional motion at zero or constant acceleration when the body is moving along the positive x-direction	(b) 
(iii) The range of a projectile (y-axis) as a function of its velocity (x-axis) when projected at a fixed angle	(c) 
(iv) The square of the time period (y-axis) of a simple pendulum as a function of its length (x-axis)	(d) 

3. Match Column A with Column B and select the correct answer from the codes given below the columns.

Column A	*Column B*
(i) Boltzmann constant	(a) $ML^2\,T^{-1}$
(ii) Coefficient of viscosity	(b) $ML^{-1}\,T^{-1}$
(iii) Planck constant	(c) $MLT^{-3}\,K^{-1}$
(iv) Thermal conductivity	(d) $ML^2T^{-2}\,K^{-1}$

Codes:

	(i)	(ii)	(iii)	(iv)
P →	(c)	(a)	(b)	(d)
Q →	(c)	(b)	(a)	(d)
R →	(d)	(b)	(a)	(c)
S →	(d)	(a)	(b)	(c)

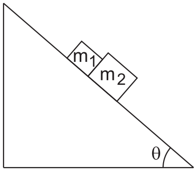
4. A person in a lift is holding a water jar which has a small hole at the lower end of its side. When the lift is at rest, the water jet coming out of the hole hits the floor of the lift at a distance d of 1.2 m from the person. In the following table, the states of the lift's motion are given in Column A and the distances where the water jet hits the floor of the lift are given in Column B. Match the statements in Column A with those in Column B and select the correct code.

Column A	*Column B*
(i) The lift is accelerating vertically up.	(a) $d = 1.2$ m
(ii) The lift is accelerating vertically down with an acceleration less than the gravitational acceleration.	(b) $d > 1.2$ m
(iii) The lift is moving vertically up with a constant speed.	(c) $d < 1.2$ m
(iv) The lift is falling freely.	(d) No water leaks out of the jar.

Codes:

P →	(i) – (b)	(ii) – (c)	(iii) – (b)	(iv) – (d)
Q →	(i) – (b)	(ii) – (c)	(iii) – (a)	(iv) – (d)
R →	(i) – (a)	(ii) – (a)	(iii) – (a)	(iv) – (d)
S →	(i) – (b)	(ii) – (c)	(iii) – (a)	(iv) – (a)

5. A block of mass $m_1 = 1$ kg and another one of mass $m_2 = 2$ kg are placed together (see figure) on an inclined plane with an angle of inclination θ. Various values of θ are given in Column A. The coefficient of friction between the block m_1 and the plane is always zero. The coefficients of static and dynamic frictions between the block m_2 and the plane are equal to $\mu = 0.3$. In Column B, the expressions for the friction on the block m_2 are given. Match the correct expressions of the friction in Column B with the angles given in Column A, and choose the correct code.



[Useful information: tan 5.5° ≈ 0.1, tan 11.5° ≈ 0.2, tan 16.5° ≈ 0.3]

Column A	*Column B*
(i) $\theta = 5°$	(a) $m_2 g \sin\theta$
(ii) $\theta = 10°$	(b) $(m_1 + m_2)g \sin\theta$
(iii) $\theta = 15°$	(c) $\mu m_2 g \cos\theta$
(iv) $\theta = 20°$	(d) $\mu(m_1 + m_2)g \cos\theta$

Codes:

P →	(i) – (a)	(ii) – (a)	(iii) – (a)	(iv) – (c)
Q →	(i) – (b)	(ii) – (b)	(iii) – (b)	(iv) – (c)
R →	(i) – (b)	(ii) – (b)	(iii) – (b)	(iv) – (d)
S →	(i) – (b)	(ii) – (b)	(iii) – (c)	(iv) – (c)

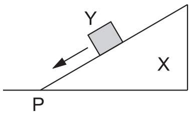
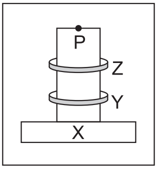
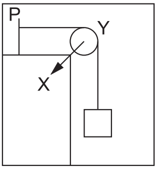
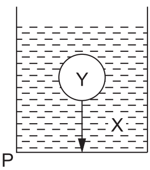
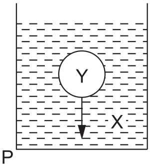
6. A particle of unit mass is moving along the x-axis under the influence of a force and its total energy is conserved. Four possible forms of the potential energy of the particle are given in Column A (a and U_0 are constants). Match the potential energies in Column A with the corresponding statement(s) in Column B.

Column A	*Column B*
(i) $U_1(x) = \frac{U_0}{2}\left[1 - \left(\frac{x}{a}\right)^2\right]^2$	(a) The force acting on the particle is zero at $x = a$.
(ii) $U_2(x) = \frac{U_0}{2}\left(\frac{x}{a}\right)^2$	(b) The force acting on the particle is zero at $x = 0$.
(iii) $U_3(x) = \frac{U_0}{2}\left(\frac{x}{a}\right)^2 \exp\left[-\left(\frac{x}{a}\right)^2\right]$	(c) The force acting on the particle is zero at $x = -a$.
(iv) $U_4(x) = \frac{U_0}{2}\left[\frac{x}{a} - \frac{1}{3}\left(\frac{x}{a}\right)^3\right]$	(d) The particle experiences an attractive force towards $x = 0$ in the region $\lvert x \rvert < a$.
	(e) The particle with total energy $\frac{U_0}{4}$ can oscillate about the point $x = -a$.

7. Column A describes some situations where a small object is in motion. Column B describes some characteristics of these motions. Match the situations in Column A with the characteristics in Column B.

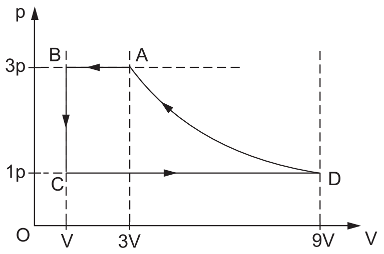
Column A	*Column B*
(i) The object moves along the x-axis under a conservative force in such a way that its speed and position satisfy $v = C_1\sqrt{C_2 - x^2}$, where C_1 and C_2 are positive constants.	(a) The object executes a simple harmonic motion.
(ii) The object moves along the x-axis in such a way that its velocity and displacement from the origin satisfy $v = -Kx$, where K is a positive constant.	(b) The object does not change its direction.
(iii) The object is attached to one end of a massless spring of a given spring constant. The other end of the spring is attached to the ceiling of an elevator. Initially everything is at rest. The elevator starts going upwards with a constant acceleration a. The motion of the object is observed from the elevator during the period it maintains the accelaration.	(c) The kinetic energy of the object keeps on decreasing.
(iv) The object is projected from the earth's surface vertically upwards with a speed $2\sqrt{GM/R_e}$, where M_e is the mass of the earth and R_e is the radius of the earth. Neglect forces from objects other than the earth.	(d) The object can change its direction only once.

8. Column B shows five systems in which two objects are labelled as X and Y. Also, in each case a point P is shown. Column A lists some statements about X and/or Y. Match these statements to the appropriate system(s) from Column B.

<table>
<tr><th>Column A</th><th>Column B</th></tr>
<tr><td>(i) The force exerted by X on Y has a magnitude Mg.</td><td>(a) Block Y of mass M left on a fixed inclined plane X, slides down it with a constant velocity.

</td></tr>
<tr><td>(ii) The gravitational potential energy of X is continuously increasing.</td><td>(b) Two ring magnets Y and Z, each of mass M are kept in a frictionless vertical plastic stand, so that they repel each other. Y rests on the base X, and Z hangs in the air in equilibrium. P is the topmost point of the stand on the common axis of the two rings. The whole system is in a lift moving up with a constant velocity.

</td></tr>
<tr><td>(iii) Mechanical energy of the system (X + Y) is continuously decreasing.</td><td>(c) A pulley Y of mass m_0 is fixed to a table through a clamp X. A block of mass M hangs from a string that goes over the pulley and is fixed at point P of the table. The whole system is kept in a lift moving upward with a constant velocity.

</td></tr>
<tr><td>(iv) The torque of the weight of Y about the point P is zero.</td><td>(d) A sphere Y of mass M is put in a nonviscous liquid X kept in a container at rest. The sphere is released and it moves down in the liquid.

</td></tr>
<tr><td></td><td>(e) A sphere Y of mass M is falling with its terminal velocity in a viscous liquid X kept in a container.

</td></tr>
</table>

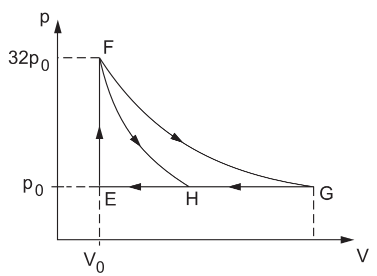
4.2 Heat and Thermodynamics

1. One mole of a monatomic gas is taken through a cycle ABCDA as shown in the p–V diagram. Column B gives the characteristics involved in the cycle. Match them with each of the processes given in Column A.



Column A	*Column B*
(i) Process A → B	(a) Internal energy decreases
(ii) Process B → C	(b) Internal energy increases
(iii) Process C → D	(c) Heat is lost
(iv) Process D → A	(d) Heat is gained
	(e) Work is done on the gas

2. One mole of a monatomic ideal gas is taken along two cyclic processes
E → F → G → E & E → F → H → E,
as shown in the p-V diagram. The processes involved are purely isochoric, isobaric, isothermal or adiabatic.



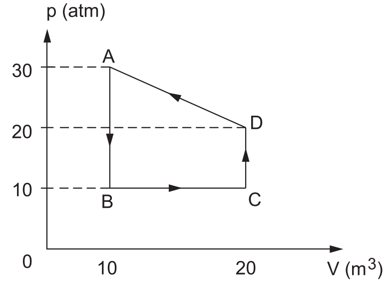
Match the paths given in Column A with the magnitudes of work done given in Column B and select the correct option (only one), using the codes given below the columns.

Column A	*Column B*
(i) G → E	(a) $160p_0V_0 \ln 2$
(ii) G → H	(b) $36p_0V_0$
(iii) F → H	(c) $24p_0V_0$
(iv) F → G	(d) $31p_0V_0$

Codes:

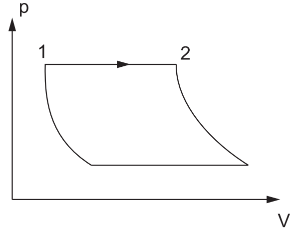
	(i)	(ii)	(iii)	(iv)
P →	(d)	(c)	(b)	(a)
Q →	(d)	(c)	(a)	(b)
R →	(c)	(a)	(b)	(d)
S →	(a)	(c)	(b)	(d)

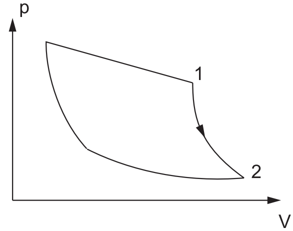
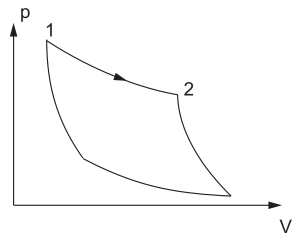
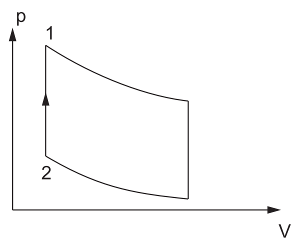
3. If the heat given to a process is considered to be positive, match the following options of Column A with their corresponding options given in Column B.



Column A	*Column B*
(i) A → B	(a) $\Delta W > 0$
(ii) B → C	(b) $\Delta Q < 0$
(iii) C → D	(c) $\Delta W < 0$
(iv) D → A	(d) $\Delta Q > 0$

4. An ideal gas is undergoing a cyclic thermodynamic process in different ways as shown in the corresponding *p*-*V* diagrams in Column C of the table. Consider only the path from the state 1 to state 2. *W* denotes the corresponding work done on the system. The equations and plots in the table have standard notations as used in the thermodynamic processes. Here γ is the ratio of the heat capacities at constant pressure and constant volume. The number of moles in the gas is n.

Column A	*Column B*	*Column C*
(i) $W_{1\to2} = \dfrac{1}{\gamma-1}(p_2V_2 - p_1V_1)$	(i) Isothermal	(a) 

(ii) $W_{1\to2} = -pv_2 + pv_1$	(ii) Isochoric	(b) 
(iii) $W_{1\to2} = 0$	(iii) Isobaric	(c) 
(iv) $W_{1\to2} = -nRT\ln\dfrac{V_2}{V_1}$	(iv) Adiabatic	(d) 

4A. Which of the following options is the only correct representation of a process, where $\Delta U = \Delta Q - p\Delta V$?

[P] $\to$ (ii) (iv) (c) [Q] $\to$ (ii) (iii) (a)

[R] $\to$ (ii) (iii) (d) [S] $\to$ (iii) (iii) (a)

4B. Which one of the following options is the correct combination?

[P] $\to$ (iii) (ii) (d) [Q] $\to$ (ii) (iv) (c)

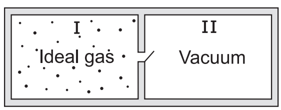
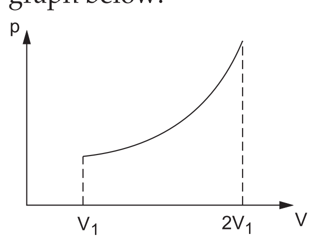
[R] $\to$ (ii) (iv) (a) [S] $\to$ (iv) (ii) (d)

4C. Which one of the following options correctly represents a thermodynamic process that is used as a correction in the determination of the speed of sound in an ideal gas?

[P] $\to$ (iii) (iv) (c) [Q] $\to$ (i) (ii) (b)

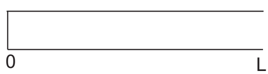
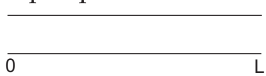
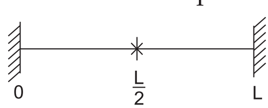
[R] $\to$ (iv) (ii) (c) [S] $\to$ (i) (iv) (b)

5. Column A contains a list of processes involving expansion of an ideal gas. Match these with the thermodynamic changes during the processes as described in Column B.

<table>
<tr><th>Column A</th><th>Column B</th></tr>
<tr><td>(i) An insulated container has two chambers separated by a valve as shown in the figure below. Chamber I contains an ideal gas and chamber II has vacuum. The valve is opened.

</td><td>(a) The temperature of the gas decreases.</td></tr>
<tr><td>(ii) An ideal monatomic gas expands to twice its original volume such that its pressure $p \propto \frac{1}{V^2}$, where V is the volume of the gas.</td><td>(b) The temperature of the gas increases or remains constant.</td></tr>
<tr><td>(iii) An ideal monatomic gas expands to twice its original volume such that its pressure $p \propto \frac{1}{V^{4/3}}$, where V is its volume.</td><td>(c) The gas loses heat.</td></tr>
<tr><td>(iv) An ideal monatomic gas expands such that its pressure p and volume V follows the behaviour as shown in the graph below.

</td><td>(d) The gas gains heat.</td></tr>
</table>

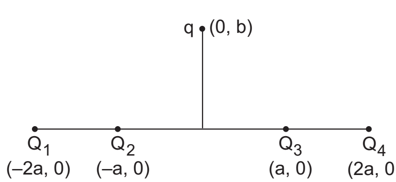
4.3 Sound Waves

1. Column A shows four systems, each with the same length L, for producing standing waves. The lowest possible natural frequency is called its fundamental frequency, whose wavelength is denoted as λ_f. Match each system of Column A with statements given in Column B describing the nature and wavelength of the standing waves.

Column A	*Column B*
(i) Pipe closed at one end 	(a) Longitudinal waves
(ii) Pipe open at both ends 	(b) Transverse waves
(iii) Stretched wire clamped at both ends 	(c) $\lambda_f = L$
(iv) Stretched wire clamped at both ends and at midpoint 	(d) $\lambda_f = 2L$
	(e) $\lambda_f = 4L$

4.4 Electrostatics

1. Four charges Q_1, Q_2, Q_3 and Q_4 of the same magnitude are fixed along the x-axis at $x = -2a$, $-a$, $+a$ and $+2a$ respectively. A positive charge q is placed



on the positive y-axis at a distance $b > 0$. Four options of the signs of these charges are given in Column A. The direction of the forces on the charge q is given in Column B. Match Column A with Column B and select the correct answer using the codes given below the columns.

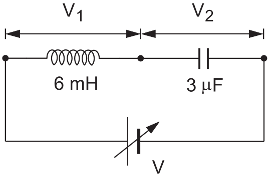
Column A	*Column B*
(i) Q_1, Q_2, Q_3, Q_4 all positive	(a) $+x$
(ii) Q_1, Q_2 positive; Q_3, Q_4 negative	(b) $-x$
(iii) Q_1, Q_4 positive; Q_2, Q_3 negative	(c) $+y$
(iv) Q_1, Q_3 positive; Q_2, Q_4 negative	(d) $-y$

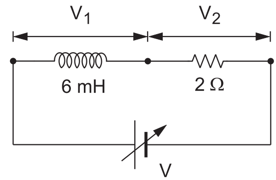
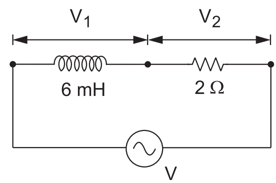
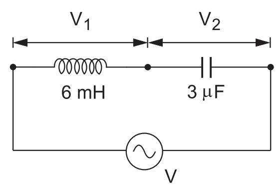
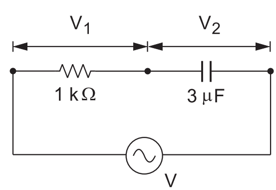
Codes:

P →	(i) – (c)	(ii) – (a)	(iii) – (d)	(iv) – (b)
Q →	(i) – (d)	(ii) – (b)	(iii) – (c)	(iv) – (a)
R →	(i) – (c)	(ii) – (a)	(iii) – (b)	(iv) – (d)
S →	(i) – (d)	(ii) – (b)	(iii) – (a)	(iv) – (c)

4.5 Current Electricity and Magnetism

1. You are given many resistors, capacitors and inductors. These are connected to a variable DC voltage source (the first two circuits) or an AC voltage source of 50 Hz frequency (the next three circuits) in different ways as shown in Column B. When a current I (steady state for DC or rms for AC) flows through the circuit, the corresponding voltage V_1 and V_2 (indicated in circuits) are related as shown in Column A. Match the two.

Column A	*Column B*
(i) $I \neq 0$, V_1 is proportional to I	(a) 

(ii) $I \neq 0, V_2 > V_1$	(b) 
(iii) $V_1 = 0, V_2 = V$	(c) 
(iv) $I \neq 0$, V_2 is proportional to I	(d) 
	(e) 

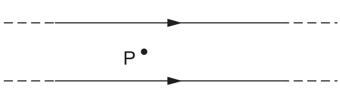
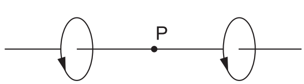
2. Match Column A with Column B.

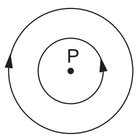
Column A	*Column B*
(i) A uniformly charged dielectric ring	(a) A time-independent electrostatic field directed out of the system
(ii) A uniformly charged dielectric ring rotating with an angular velocity ω	(b) A magnetic field
(iii) A ring carrying a constant current i_0	(c) An induced electric field
(iv) $i = i_0 \cos \omega t$	(d) A magnetic moment

3. Column A gives certain situations in which a straight metallic wire of resistance R is used and Column B gives some resulting effects. Match the statements in Column A with the statements in Column B.

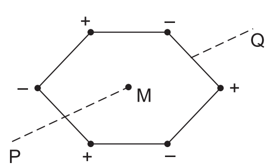
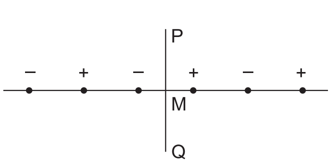
Column A	*Column B*
(i) A charged capacitor is connected to the ends of the wire.	(a) A constant current flows through the wire.
(ii) The wire is moved perpendicular to its length with a constant velocity in a uniform magnetic field perpendicular to the plane of its motion.	(b) Thermal energy is generated in the wire.
(iii) The wire is placed in a constant electric field that has its direction along the length of the wire.	(c) A constant potential difference develops between the ends of the wire.
(iv) A battery of constant emf is connected to the ends of the wire.	(d) Charges of constant magnitude appear at the ends of the wire.

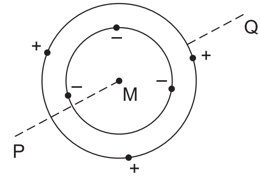
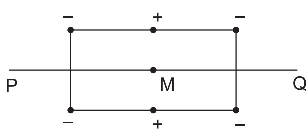
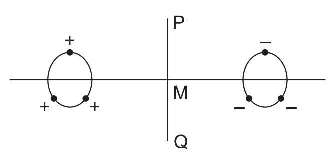
4. Two wires, each carrying a steady current I, are shown in four configurations in Column A. Some of the resulting effects are described in Column B. Match the statements in the two columns.

Column A	*Column B*
(i) P is situated between the wires. 	(a) The magnetic field (B) at P due to the currents in the wires are in the same directions.
(ii) P is situated at the midpoint of the line joining the centres of the circular wires, which have the same radii. 	(b) The magnetic fields at P due to the currents in the wires are in opposite directions.

(iii) P is situated at the midpoint of the line joining the centres of the circular wires, which have the same radii. 	(c) There is no magnetic field at P.
(iv) P is situated at the common centre of the wires. 	(d) The wires repel each other.

5. Six point charges, each of the same magnitude q, are arranged in different manners as shown in Column B. Let E be the electric field and V be the electric potential at M (potential at infinity is zero) due to the given charge distribution when it is at rest. The whole system is set into rotation with a constant angular velocity about the line PQ. Let B be the magnetic field at M and μ be the magnetic moment of the system in this condition. Assume each rotating charge to be equivalent to a steady current.

Column A	*Column B*
(i) $E = 0$	(a) The charges are at the corners of a regular hexagon. M is at the centre of the hexagon. PQ is perpendicular to the plane of the hexagon. 
(ii) $V \neq 0$	(b) The charges are on a line perpendicular to PQ, at equal intervals. M is the midpoint between the two innermost charges. 

(iii) $B = 0$	(c) The charges are placed on two coplanar insulating rings at equal intervals. M is the common centre of the rings. PQ is perpendicular to the plane of the rings. 
(iv) $\mu \neq 0$	(d) The charges are placed at the corners of a rectangle of side a and $2a$ and at the midpoints of the large sides. M is at the centre of the rectangle. PQ is parallel to the longer sides. 
	(e) The charges are placed on two coplanar identical insulating rings at equal intervals. M is the midpoint between the centres of the rings. PQ is perpendicular to the line joining the centres and coplanar with the rings. 

6. A charged particle (electron or proton) is introduced at the origin ($x = 0$, $y = 0$, $z = 0$) with a given initial velocity $\vec{v}$. A uniform electric field $\vec{E}$ and a uniform magnetic field $\vec{B}$ exist everywhere. The velocity $\vec{v}$, electric field $\vec{E}$ and magnetic field $\vec{B}$ are given in Columns A, B and C respectively. The quantities $\vec{E}_0$ and $\vec{B}_0$ are positive in magnitude.

Column A	*Column B*	*Column C*
(i) Electron with $\vec{v} = \frac{2E_0}{B_0}\hat{x}$	(i) $\vec{E} = E_0\hat{z}$	(a) $\vec{B} = -B_0\hat{x}$
(ii) Electron with $\vec{v} = \frac{E_0}{B_0}\hat{y}$	(ii) $\vec{E} = -E_0\hat{y}$	(b) $\vec{B} = B_0\hat{x}$
(iii) Proton with $\vec{v} = 0$	(iii) $\vec{E} = -E_0\hat{x}$	(c) $\vec{B} = B_0\hat{y}$
(iv) Proton with $\vec{v} = 2\frac{E_0}{B_0}\hat{x}$	(iv) $\vec{E} = E_0\hat{x}$	(d) $B = B_0\hat{z}$

6A. In which case will the particle move in a straight line with a constant velocity?

[P] $\rightarrow$ (ii) (iii) (d) [Q] $\rightarrow$ (iv) (i) (d)
[R] $\rightarrow$ (iii) (ii) (c) [S] $\rightarrow$ (iii) (iii) (a)

6B. In which case will the particle describe a helical path with axis along the positive z-direction?

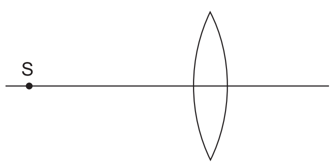
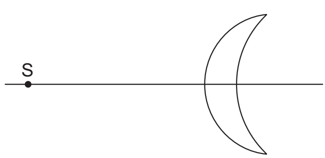
[P] $\rightarrow$ (ii) (iii) (c) [Q] $\rightarrow$ (iv) (ii) (c)
[R] $\rightarrow$ (iv) (i) (d) [S] $\rightarrow$ (iii) (iii) (a)

6C. In which case would the particle move in a straight line along the negative direction of the y-axis (i.e., move along $-\hat{y}$)?

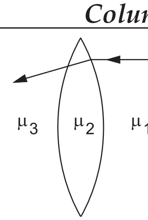
[P] $\rightarrow$ (iv) (ii) (d) [Q] $\rightarrow$ (iii) (ii) (a)
[R] $\rightarrow$ (ii) (iii) (b) [S] $\rightarrow$ (iii) (ii) (c)

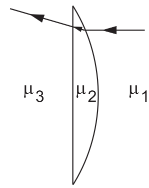
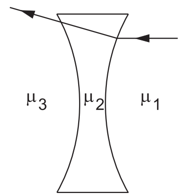
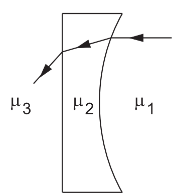
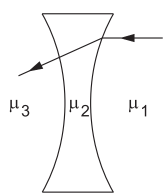
4.6 Ray Optics and Wave Optics

1. An optical component and an object S placed along its optic axis are given in Column A. The distance between the object and the component can be varied. The properties (nature) of images are given in Column B. Match all the properties of the images from Column B with the appropriate components given in Column A.

Column A	*Column B*
(i)  Concave mirror	(a) Real image
(ii)  Convex mirror	(b) Virtual image
(iii)  Convex lens	(c) Magnified image
(iv)  Concavo-convex lens	(d) Image at infinity

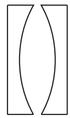
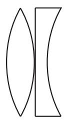
2. Two transparent media of refractive indices μ_1 and μ_3 have a solid lens-shaped transparent material of refractive index μ_2 between them as shown in the figures in Column B. A ray traversing these media is also shown in the figures. In Column A different relationships between μ_1, μ_2 and μ_3 are given. Match them to the ray diagrams shown in Column B.

Column A	*Column B*
(i) $\mu_1 < \mu_2$	(a) 

(ii) $\mu_1 > \mu_2$	(b) 
(iii) $\mu_2 = \mu_3$	(c) 
(iv) $\mu_2 > \mu_3$	(d) 
	(e) 

3. Four combinations of two thin lenses are given in Column A. The radius of curvature of each curved surface is r and the refractive index of each lens is 1.5. Match the lens combination in Column A with the focal length in Column B. Select the correct answer using the codes given below the columns.

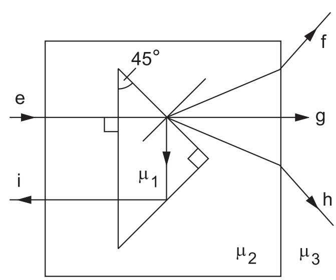
Column A	***Column B***
(i) 	(a) $2r$

(ii) 	(b) $\frac{r}{2}$
(iii) 	(c) $-r$
(iv) 	(d) r

Codes:

P →	(i) – (a)	(ii) – (b)	(iii) – (c)	(iv) – (d)
Q →	(i) – (b)	(ii) – (d)	(iii) – (c)	(iv) – (a)
R →	(i) – (d)	(ii) – (a)	(iii) – (b)	(iv) – (c)
S →	(i) – (b)	(ii) – (a)	(iii) – (c)	(iv) – (d)

4. A right-angled prism of refractive index μ_1 is placed in a rectangular block of refractive index μ_2, which is surrounded by a medium of refractive index μ_3, as shown in the figure. A ray of light *e* enters the rectangular block at normal incidence. Depending upon the relationships between μ_1, μ_2 and μ_3 it takes one of the four possible paths *ef*, *eh*, *eg* or *ei*.



Match the paths in Column A with the conditions of refractive indices in Column B and select the correct answer using the codes given below the columns.

Column A	*Column B*
(i) $e \to f$	(a) $\mu_1 > \sqrt{2}\mu_2$
(ii) $e \to g$	(b) $\mu_2 > \mu_1$ and $\mu_2 > \mu_3$
(iii) $e \to h$	(c) $\mu_1 = \mu_2$
(iv) $e \to i$	(d) $\mu_2 < \mu_1 < \sqrt{2}\mu_2$ and $\mu_2 > \mu_3$

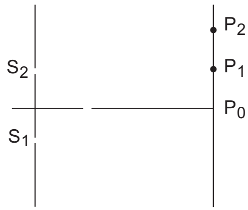
Codes:

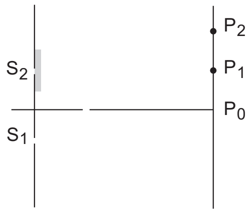
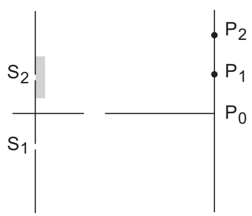
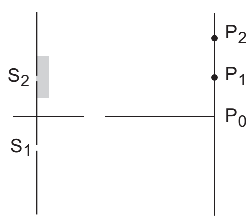
	(i)	(ii)	(iii)	(iv)
P →	(b)	(c)	(a)	(d)
Q →	(a)	(b)	(d)	(c)
R →	(d)	(a)	(b)	(c)
S →	(b)	(c)	(d)	(a)

5. A simple telescope used to view distant objects has eyepiece and objective lens of focal lengths f_e and f_o respectively. Match the columns A and B.

Column A	*Column B*
(i) Intensity of light received by the lens	(a) Radius of aperture (R)
(ii) Angular magnification	(b) Dispersion of lens
(iii) Length of telescope	(c) Focal length f_o, f_e
(iv) Sharpness of image	(d) Spherical aberration

6. The figures in Column A relate to Young's double-slit arrangement with the screen placed far away from the slits S_1 and S_2. In each of these cases, $S_1P_0 = S_2P_0$, $S_1P_1 - S_2P_1 = \frac{\lambda}{4}$ and $S_1P_2 - S_2P_2 = \frac{\lambda}{3}$, where λ is the wavelength of the light used. In (ii), (iii) and (iv), a transparent sheet of refractive index n and thickness t is pasted on the slit S_2. The thickness of the sheets are different in different cases. The phase difference between the light waves reaching a point P on the screen from the two slits is denoted by $\delta(P)$ and the intensity by $I(P)$. Match the columns A and B.

Column A	*Column B*
(i) 	(a) $\delta(P_0) = 0$

(ii) $(n-1)t = \lambda/4$ 	(b) $\delta(P_1) = 0$
(iii) $(n-1)t = \lambda/2$ 	(c) $I(P_1) = 0$
(iv) $(n-1)t = 3\frac{\lambda}{4}$ 	(d) $I(P_0) > I(P_1)$
	(e) $I(P_2) > I(P_1)$

4.7 Modern Physics

1. Match Column A showing the nuclear processes with Column B containing the parent nucleus and one of the end products of each process, and then select the correct answer using the codes given below the columns.

Column A	*Column B*
(i) Alpha decay	(a) ${}^{15}_{8}\mathrm{O} \rightarrow {}^{15}_{7}\mathrm{N} + \ldots$
(ii) β^+ decay	(b) ${}^{238}_{92}\mathrm{U} \rightarrow {}^{234}_{90}\mathrm{Th} + \ldots$

(iii) Fission	(c) $^{185}_{83}Bi \rightarrow ^{184}_{82}Pb +$
(iv) Proton emission	(d) $^{239}_{94}Pu \rightarrow ^{140}_{57}La + ...$

Codes:

	(i)	(ii)	(iii)	(iv)
P →	(d)	(b)	(a)	(c)
Q →	(a)	(c)	(b)	(d)
R →	(b)	(a)	(d)	(c)
S →	(d)	(c)	(b)	(a)

2. Match the nuclear processes given in Column A with the appropriate option(s) in Column B.

Column A	*Column B*
(i) Nuclear fusion	(a) Absorption of thermal neutrons by $^{235}_{92}U$
(ii) Fission in a nuclear reactor	(b) $^{60}_{27}Co$ nucleus
(iii) β-Decay	(c) Energy production starts via the conversion of hydrogen into helium
(iv) γ-Ray emission	(d) Heavy water
	(e) Neutrino emission

3. Match the following.

Column A	*Column B*
(i) The transition of a hydrogen atom from one energy level to another	(a) Characteristic X-rays
(ii) Electron emission from a material	(b) Photoelectric effect
(iii) Moseley's law	(c) Hydrogen spectrum
(iv) Change of photon energy into the kinetic energy of electrons	(d) β-Decay

4. Column B gives certain systems, each undergoing a process. Column A suggests changes in some of the parameters related to the systems. Match the statements in Column A with the appropriate process(es) in Column B.

Column A	*Column B*
(i) The energy of the system is increased.	(a) System: A capacitor initially uncharged Process: Connected to a battery
(ii) Mechanical energy provided to the system is converted into energy of random motion of its parts.	(b) System: A gas in an adiabatic container fitted with an adiabatic piston Process: The gas is compressed by pushing the piston
(iii) Internal energy of the system is converted into mechanical energy.	(c) System: A gas in a rigid container Process: The gas getting cooled due to colder atmosphere surrounding it
(iv) Mass of the system is decreased	(d) System: A heavy nucleus, initially at rest Process: Nuclear fission—two fragments of nearly equal masses and some neutrons emitted
	(e) System: A resistance-wire loop Process: The loop placed in a time-varying magnetic field perpendicular to its plane

Answers

4.1 General Physics

1.

	a	b	c	d
(i)	●	●	○	○
(ii)	○	○	●	●
(iii)	○	○	●	●
(iv)	○	○	●	●

2.

	a	b	c	d
(i)	●	○	○	●
(ii)	○	●	○	●
(iii)	○	○	○	●
(iv)	○	●	○	○

3.

	a	b	c	d
(i)	○	○	○	●
(ii)	○	●	○	○
(iii)	●	○	○	○
(iv)	○	○	●	○

Code: R

4.

	a	b	c	d
(i)	●	○	○	○
(ii)	●	○	○	○
(iii)	●	○	○	○
(iv)	○	○	○	●

Code: R

5.

	a	b	c	d
(i)	○	●	○	○
(ii)	○	●	○	○
(iii)	○	○	●	○
(iv)	○	○	●	○

Code: S

6.

	a	b	c	d	e
(i)	●	●	●	○	●
(ii)	○	●	○	●	○
(iii)	●	●	●	●	○
(iv)	●	○	●	○	●

7.

	a	b	c	d
(i)	●	○	○	○
(ii)	○	●	●	○
(iii)	○	●	○	○
(iv)	○	●	●	○

8.

	a	b	c	d	e
(i)	●	○	○	○	●
(ii)	○	●	○	●	●
(iii)	●	○	●	○	●
(iv)	○	●	○	○	○

4.2 Heat and Thermodynamics

1.

	a	b	c	d	e
(i)	●	○	●	○	●
(ii)	●	○	●	○	○
(iii)	○	●	○	●	○
(iv)	○	○	●	○	●

2.

	a	b	c	d
(i)	○	○	○	●
(ii)	○	○	●	○
(iii)	○	●	○	○
(iv)	●	○	○	○

Code: P

3.

	a	b	c	d
(i)	○	●	○	○
(ii)	●	○	○	●
(iii)	○	○	○	●
(iv)	○	●	●	○

4A. Q **4B.** P **4C.** S

5.

	a	b	c	d
(i)	○	●	○	○
(ii)	●	○	●	○
(iii)	●	○	○	●
(iv)	○	●	○	●

4.3 Sound Waves

1.

	a	b	c	d	e
(i)	●	○	○	○	●
(ii)	●	○	○	●	○
(iii)	○	●	○	●	○
(iv)	○	●	●	○	○

4.4 Electrostatics

1.

	a	b	c	d
(i)	○	○	●	○
(ii)	●	○	○	○
(iii)	○	○	○	●
(iv)	○	●	○	○

Code: P

4.5 Current Electricity and Magnetism

1.

	a	b	c	d	e
(i)	○	○	●	●	●
(ii)	○	●	●	●	●
(iii)	●	●	○	○	○
(iv)	○	●	●	●	●

2.

	a	b	c	d
(i)	●	○	○	○
(ii)	○	●	○	●
(iii)	○	●	○	●
(iv)	○	●	●	●

3.

	a	b	c	d
(i)	○	●	○	○
(ii)	○	○	●	●
(iii)	○	○	●	●
(iv)	●	●	●	○

4.

	a	b	c	d
(i)	○	●	●	○
(ii)	●	○	○	○
(iii)	○	●	●	○
(iv)	○	●	○	○

5.

	a	b	c	d	e
(i)	●	○	●	●	○
(ii)	○	○	●	●	○
(iii)	●	●	○	○	●
(iv)	○	○	●	●	○

6A. R **6B.** R **6C.** S

4.6 Ray Optics and Wave Optics

1.

	a	b	c	d
(i)	●	●	●	●
(ii)	○	●	○	○
(iii)	●	●	●	●
(iv)	●	●	●	●

2.

	a	b	c	d	e
(i)	●	○	●	○	○
(ii)	○	●	○	●	●
(iii)	●	○	●	○	●
(iv)	○	●	○	●	○

3.

	a	b	c	d
(i)	○	●	○	○
(ii)	○	○	○	●
(iii)	○	○	●	○
(iv)	●	○	○	○

Code: Q

4.

	a	b	c	d
(i)	○	●	○	○
(ii)	○	○	●	○
(iii)	○	○	○	●
(iv)	●	○	○	○

Code: S

5.

	a	b	c	d
(i)	●	○	○	○
(ii)	○	○	●	○
(iii)	○	○	●	○
(iv)	●	●	○	●

6.

	a	b	c	d	e
(i)	●	○	○	●	○
(ii)	○	●	○	○	○
(iii)	○	○	○	○	●
(iv)	○	○	●	●	●

4.7 Modern Physics

1.

	a	b	c	d
(i)	○	●	○	○
(ii)	●	○	○	○
(iii)	○	○	○	●
(iv)	○	○	●	○

Code: R

2.

	a	b	c	d	e
(i)	○	○	●	○	●
(ii)	●	○	○	●	○
(iii)	○	●	○	○	●
(iv)	○	○	●	○	○

3.

	a	b	c	d
(i)	●	○	●	○
(ii)	○	●	○	●
(iii)	●	○	○	○
(iv)	○	●	○	○

4.

	a	b	c	d	e
(i)	●	●	○	○	●
(ii)	○	●	○	○	○
(iii)	○	○	○	●	○
(iv)	○	○	○	●	○

Hints and Solutions

4.1 General Physics

1. (i) → (a), (b) : $GM_eM_s = Fr^2 = \text{kg m}^3\text{ s}^{-2}$.

$$\text{V C m} = \text{J m} = \text{N m m} = \text{kg m}^3\text{ s}^{-2}.$$

(ii) → (c), (d) : $\dfrac{3RT}{M} = C_{rms}^2 \to (\text{m s}^{-1})^2 = \text{m}^2\text{ s}^{-2}$.

$$\text{F V}^2\text{ kg}^{-1} = \text{C V kg}^{-1} = \text{J kg}^{-1}$$
$$= \text{kgm}^2\text{ s}^{-2}\text{ kg}^{-1}$$
$$= \text{m}^2\text{ s}^{-2}.$$

(iii) → (c), (d) : $\dfrac{F^2}{q^2B^2} = \dfrac{(IlB)^2}{(qB)^2} = (\text{s}^{-1}\text{ m})^2 = \text{m}^2\text{ s}^{-2} = \text{FV}^2\text{ kg}^{-1}$.

(iv) → (c), (d) : $\dfrac{GM_e}{R_e} = \dfrac{\text{force} \times \text{distance}}{\text{mass}}$

$$= \frac{\text{N m}}{\text{kg}} = \frac{\text{kg m s}^{-2}\cdot\text{m}}{\text{kg}} = \text{m}^2\text{ s}^{-2}.$$

2. (i) → (a), (d) : $U = \frac{1}{2}kx^2$ represents a parabola.

(ii) → (b), (d) : With zero acceleration, motion is uniform and slope of (x–t) is constant. Hence (b). With constant acceleration $\left(x = \frac{1}{2}at^2\right)$, the slope increases with time, hence (d).

(iii) → (d) : Range $R = u^2\dfrac{\sin 2\theta}{g}$, $R \propto u^2$.

This represents a parabola passing through the origin.

(iv) → (b) : $T^2 = \left(\dfrac{4\pi^2}{g}\right)l = kl$.

3. (i) : Energy $= \frac{3}{2}kT \equiv \text{ML}^2\text{T}^{-2}$

$$\Rightarrow [k] = \frac{\text{ML}^2\text{ T}^{-2}}{\text{Temperature}} = \text{ML}^2\text{T}^{-2}\text{K}^{-1}.$$

(ii) : $F = 6\pi\eta rv \Rightarrow \eta = \text{ML}^{-1}\text{T}^{-1}$.

(iii) : Energy $= h\nu \Rightarrow h = \text{ML}^2\text{ T}^{-1}$.

(iv) : $Q = K\dfrac{\Delta\theta}{\Delta x}At \;\Rightarrow [K] = MLT^{-3}K^{-1}$.

Thus, (i) → (d), (ii) → (b), (iii) → (a), (iv) → (c) [Code R].

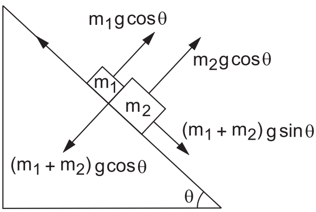
4. In column A, in situations (i), (ii) and (iii), no horizontal velocity is imparted to the falling water, so d remains the same.

Hence, (i) → (a), (ii) → (a), (iii) → (a).

For free fall in situation (iv), effective value of g is zero, so no water leaks out of the jar. So, (iv) → (d).

∴ code (R) is correct.

5. Condition for not sliding,



$$f_{max} > (m_1 + m_2)g \sin\theta$$

$$\Rightarrow \quad \mu N > (m_1 + m_2)g \sin\theta$$

$$\Rightarrow \quad 0.3\, m_2 g \cos\theta \geq 30 \sin\theta$$

$$\Rightarrow \quad 6 \geq 30 \tan\theta$$

$$\Rightarrow \quad \tan\theta \leq 0.2, \theta = 11°.$$

∴ for (i) and (ii), $f = (m_1 + m_2)g \sin\theta$.

For (iii) and (iv), $F = f_{max} = \mu m_2 g \cos\theta$.

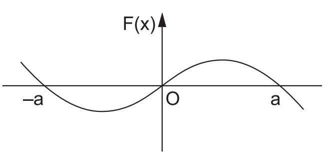
Thus, (i) → (b), (ii) → (b), (iii) → (c), (iv) → (c).

Combining all, code (S) is correct.

6. (i) $$F_x = -\frac{dU}{dx} = -\frac{d}{dx}\left[\frac{U_0}{2}\left(1 - \frac{x^2}{a^2}\right)^2\right]$$

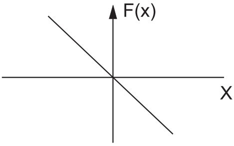
$$= -\frac{U_0}{2}2\left(1 - \frac{x^2}{a^2}\right)\left(-\frac{2}{a^2}x\right)$$

$$= \frac{2U_0}{a^4}(x - a)x(x + a).$$



The variation of $F(x)$ with (x) is shown in the graph. The correct options are (a), (b), (c) and (e).

(ii) $$F_x = -\frac{dU}{dx} = -\frac{d}{dx}\left[\frac{U_0}{2}\frac{x^2}{a^2}\right] = -\frac{U_0}{a^2}x.$$



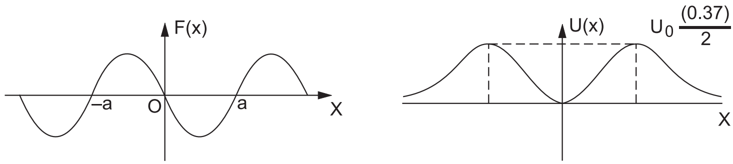
The plot $F(x)$–x is shown here.

The correct options are (b) and (d).

(iii) $$Fx = -\frac{dU}{dx} = -\frac{d}{dx}\left[\frac{U_0}{2}\left(\frac{x}{a}\right)^2 \exp\left(-\frac{x^2}{a^2}\right)\right]$$

$$= U_0\, e^{-x^2/a^2} \cdot \frac{1}{a^4}(x + a)x(x - a).$$

This variation is given by the following graphs.

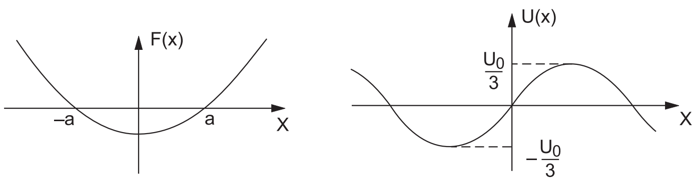


The correct options are (a), (b), (c) and (d).

(iv) $F_x = -\frac{dU}{dx} = -\frac{d}{dx}\left[\frac{U_0}{2}\left\{\frac{x}{a} - \frac{1}{3}\left(\frac{x}{a}\right)^3\right\}\right]$

$= -\frac{U_0}{2a^3}[(x-a)(x+a)].$

The variation of F_x and $U(x)$ are shown below.



Correct options are (a), (c) and (e).

7. (i) → (a) : In SHM, $v = \omega\sqrt{A^2 - x^2} \equiv C_1\sqrt{C_2 - x^2}$.
The object executes SHM.

(ii) → (b), (c) : $v = -kx = \frac{dx}{dt} \Rightarrow x = e^{-kt}$.
$\therefore\ v = -ke^{-kt}$.
The object keeps on moving on the x-axis [option (b)] and its kinetic energy decreases exponentially [(option (c)].

(iii) → (b) : $T = 2\pi\sqrt{\frac{m}{k}}$, the oscillation is independent of the frame and depends only on spring constant k.

(iv) → (b), (c) : Since the speed of projection is greater than the escape speed ($v_e = \sqrt{GM_e/R_e}$), the kinetic energy keeps on decreasing [option (c)], and the object does not change its direction [option (b)].

8. (i) → (a), (e) : For uniform motion,
$Mg \sin\theta = f = \mu Mg \cos\theta \Rightarrow \mu = \tan\theta.$

Net force on the block Y by the incline X is

$$\sqrt{f^2+N^2} = \sqrt{M^2g^2\sin^2\theta + M^2g^2\cos^2\theta}$$
$$= Mg \text{ [option (a)].}$$

Weight $(Mg) = F_{\text{viscous}} + F_{\text{buoyancy}}$ [option (e)].

(ii) $\to$ (b), (d), (e) : Lift moves up, gravitational potential energy of X goes on increasing [option (b)]. Similarly for options (d) and (e).

(iii) $\to$ (a), (c), (e) : Block Y moves down, gravitational potential energy of the system $(X + Y)$ goes on decreasing [option (a)]. Same is true for options (c) and (e).

(iv) $\to$ (b) : Weight of Y passes through P, its torque about P is zero [option (b)].

4.2 Heat and Thermodynamics

1. (i) Process $A \to B$: Isobaric compression [options (a), (c), (e)].

(ii) Process $B \to C$: Isochoric decrease in pressure [options (a), (c)].

(iii) Process $C \to D$: Isobaric expansion [options (b), (d)].

(iv) Process $D \to A$: Polytropic compression with $T_A = T_D$ [options (c), (e)].

2. Let the temperature at E be T_0.

Process E $\to$ F is isochoric, so the temperature at F is $32T_0$. Process F $\to$ G is isothermal, so the temperature at G is also $32T_0$. Process F $\to$ H is adiabatic, so $(32\,p_0)V_0^{5/3} = p_0\,(V_H)^{5/3}$, since $\gamma = \frac{5}{3}$ for a monoatomic gas. Hence $V_H = 8V_0$. Since the work done is given by the area under the p–V graph, the magnitudes of work done during the processes are

(i) $G \to E$: $p_0(V_G - V_E) = p_0(32V_0 - V_0) = 31p_0V_0$.

(ii) $G \to H$: $p_0(V_G - V_H) = p_0\,(32V_0 - 8V_0) = 24p_0V_0$.

(iii) $F \to H$: $\frac{1}{\gamma - 1}(p_FV_F - p_HV_H) = \frac{3}{2}(32p_0V_0 - 8p_0V_0) = 36p_0V_0$.

(iv) $F \to G$: $RT\ln(V_G/V_F) = p_FV_F\ln\frac{V_G}{V_F} = p_FV_F\ln\frac{p_F}{p_G}$

$$= (32p_0)V_0\ln\left(\frac{32\,p_0}{p_0}\right) = 32\,p_0V_0(5\ln 2)$$
$$= 160p_0V_0\ln 2.$$

Hence, the code (P) is correct.

3. (i) $\to$ (b) : AB is an isochoric process, $\Delta W = 0$ and heat is expelled.

(ii) $\to$ (a), (d) : BC is an isobaric expansion, work is done by the gas ($\Delta W > 0$). Heat is absorbed ($\Delta Q > 0$).

(iii) $\to$ (d) : CD is an isochoric process with increase in pressure ($p \propto T$); temperature increases, heat is absorbed, so $\Delta Q > 0$.

(iv) $\to$ (b), (c) : DA represents compression, so $\Delta W < 0$.

$$T_D = \frac{p_D V_D}{nR} = \frac{(20)(20)}{nR} = \frac{400}{nR}.$$

$$T_A = \frac{p_A V_A}{nR} = \frac{(30)(10)}{nR} = \frac{300}{nR}.$$

Since $T_A < T_D$, heat is expelled, $\Delta Q < 0$.

4A. In the isobaric process (iii), $W_{1\to 2} = pV_1 - pV_2$. ...(ii)
Pressure remains constant, (a). Hence code Q.

4B. $W_{1\to 2} = 0$, (iii), is applicable for isochoric process [(ii) and (d)]: hence code P.

4C. Expression for the work done during adiabatic expansion:

$W_{1\to 2} = \dfrac{1}{\gamma - 1}(p_2V_2 - p_1V_1)$, corresponds to options (i) and (iv) respectively and also represented by (b). Thus code S.

5. (i) $\to$ (b) : Opening of the valve causes free expansion under adiabatic condition for which $\Delta Q = 0$, $\Delta W = 0$, $\Delta U = 0$, so the temperature of the gas remains constant.

(ii) $\to$ (a), (c) : Given $p = \dfrac{k}{V^2}$. And $pV = nRT$

$$\Rightarrow V = \frac{k}{nRT} \Rightarrow VT = \text{constant}.$$

when $V \to 2V$, $T \to \dfrac{T}{2}$.

Further, change in internal energy,

$$\Delta U = nC_V\Delta T = n\left(\frac{3}{2}R\right)\left(-\frac{T}{2}\right) = n\left(\frac{3}{2}R\right)\left(-\frac{k}{2nRV}\right) = -\frac{3}{4}\left(\frac{k}{V}\right).$$

Work done by the gas,

$$\Delta W = \int p\,dV = \int_V^{2V} \frac{k}{V^2}\,dV = \frac{k}{2V}.$$

From the first law, $\Delta Q = \Delta U + \Delta W = \dfrac{-k}{4}$.

Heat is expelled.

(iii) → (a), (d) : Given $p = \frac{k}{V^{4/3}}$. And $p = \frac{nRT}{V}$

$$\Rightarrow \frac{k}{V^{1/3}} = nRT.$$

During expansion, $V \to 2V$, $T \to \frac{T}{2^{1/3}} < T$.

Hence T decreases [option (a)].

Further, change in internal energy,

$$\Delta U = nC_V\Delta T = n\left(\frac{3}{2}R\right)T\left(\frac{1}{2^{1/3}} - 1\right) = \frac{3k}{2V^{1/3}}\left(\frac{1}{2^{1/3}} - 1\right).$$

Work done during expansion,

$$\Delta W = \int pdV = k\int_V^{2V} \frac{dV}{V^{4/3}} = \frac{3K}{V^{1/3}}\left(1 - \frac{1}{2^{1/3}}\right).$$

$$\Rightarrow \Delta Q = \Delta U + \Delta W = \frac{3k}{2V^{1/3}}\left(\frac{1}{2^{1/3}} - 1\right) + \frac{3k}{V^{1/3}}\left(1 - \frac{1}{2^{1/3}}\right)$$

$$= \frac{3k}{2V^{1/3}}\left(1 - \frac{1}{2^{2/3}}\right).$$

$\therefore \quad \Delta Q > 0$ [option (d)].

(iv) → (b), (d) : $p_1V_1 = nRT_1$; $p_2(2V_1) = nRT_2$.

$$\therefore \frac{T_2}{T_1} = \frac{2p_2}{p_1} \Rightarrow T_2 > T_1.$$

Now, $\Delta U = nC_V\Delta T = nC_V(T_2 - T_1) > 0$;

ΔW = area under p–V diagram is positive during expansion, so $\Delta W > 0$.

Heat $\Delta Q > 0$ (absorption).

4.3 Sound Waves

1. (i) → (a), (e) : Closed organ pipe: longitudinal wave; $\lambda_f = 4L$

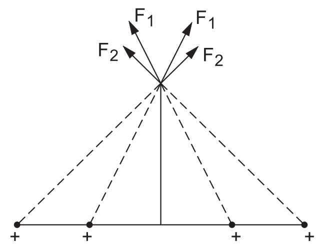
   (ii) → (a), (d) : Open organ pipe: longitudinal wave; $\lambda_f = 2L$

   (iii) → (b), (d) : Stretched string clamped at both ends: transverse wave; $\lambda_f = 2L$

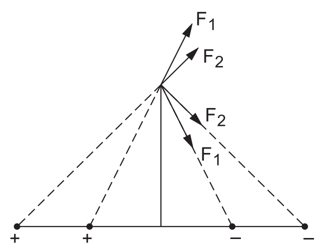
   (iv) → (b), (c) : String clamped at both ends and at midpoint; transverse wave; $\lambda_f = L$

4.4 Electrostatics

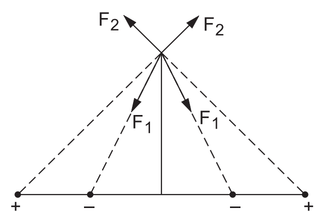
1. (i) → (c) : The x-components of the forces will cancel out and the net force will act along the $+y$-axis.



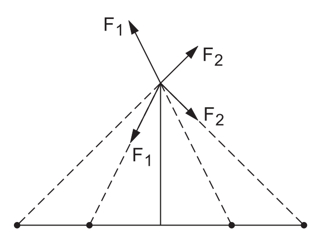
(ii) → (a) : The y-components of the forces will cancel out and the net force will act along the $+x$-axis.



(iii) → (d) : The x-components of the forces cancel out and the net force will act along the $-y$-axis.



(iv) → (b) : The y-components of the forces will cancel out and the net force will be along the $-x$-axis.



Combining all, code P is correct.

4.5 Current Electricity and Magnetism

1. (i) → (c), (d), (e) : Current I is zero in case (a) in steady state but $I \neq 0$ for cases (b) to (e).

In case (b), across the inductor V_1 is not proportional to I, so (c), (d) and (e) are correct options.

(ii) $\rightarrow$ (b), (c), (d), (e) : $V_1 = 0$, $V_2 = V$. $\therefore V_2 > V_1$ ... case (b).

$X_L = \omega L \approx 1.88\,\Omega < R = 2\,\Omega$

$\Rightarrow V_2 > V_1$ ... case (c).

$$X_C = \frac{1}{\omega C} = \frac{1}{100\pi(3\times 10^{-6}\text{ F})} = 1061\,\Omega > X_L$$

$\Rightarrow V_L > V_1$ ... case (d).

(iii) $\rightarrow$ (a), (b) : In DC, L acts as a short circuit in steady state ($V_1 = 0$) and $V_2 = V$.

(iv) $\rightarrow$ (b), (c), (d), (e) : As explained above, $I \neq 0$ and the voltage drop across the circuit elements is proportional to I.

2. (i) $\rightarrow$ (a) : $\vec{E} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Qx}{(R^2+x^2)^{3/2}}$.

Time-independent electrostatic field at an axial point.

(ii) $\rightarrow$ (b), (d) : A charged ring rotating uniformly is equivalent to a current $I = \dfrac{Q}{T} = \dfrac{Q\omega}{2\pi}$, which produced a magnetic field $B = \dfrac{\mu_0 IR^2}{2(R^2+x^2)^{3/2}}$.

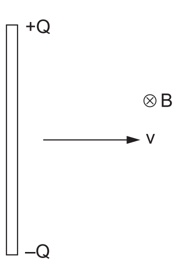
Its magnetic moment $\vec{m} = I\vec{A}$.

(iii) $\rightarrow$ (b), (d) : Constant current I_0 in ring is the same case as in (ii), hence (b) and (d) are correct options.

(iv) $\rightarrow$ (b), (c), (d) : Time-varying magnetic field is produced by time-varying current [option (b)]. The time-varying magnetic flux creates induced emf, hence induced electric field [option (c)]; finally, $\vec{m} = I\vec{A}$ [option (d)].

3. (i) $\rightarrow$ (b) : Energy of the charged capacitor $\left(U = \dfrac{Q^2}{2C}\right)$ appear as heat in the connecting wire.

(ii) $\rightarrow$ (c), (d) : Motional emf ($\mathcal{E} = Blv$) is induced in the wire. As shown in the figure, charges of constant magnitude, $\pm Q$, appear at the ends of the wire.



(iii) $\rightarrow$ (c), (d) : Free electrons in the conducting wire experience electric force $F = eE$, thus charges with opposite sign accumulate at the ends. This in turn develops potential difference across the ends.

(iv) $\rightarrow$ (a), (b), (c) : Connecting a wire of resistance R across a battery causes a constant current I (= $\mathcal{E}/R$) through the wire. Heat (I^2RT) is developed in the wire. The terminal voltage V (= $\mathcal{E} - Ir$) is developed between the ends of the wire.

4. (i) $\rightarrow$ (b), (c) : Magnetic field is at P due to two straight currents which are in opposite directions [option (b)]; if P is at the midpoint, $B = 0$ [option (c)].

(ii) $\rightarrow$ (a) : Magnetic moment ($\vec{m} = I\vec{A}$) due to both the coils is towards the right, so magnetic field at P is in the same direction.

(iii) $\rightarrow$ (b), (c) : Magnetic fields at P are due to two coils which are in opposite direction [option (b)]. For symmetrical position of P, $B = 0$ [option (c)].

(iv) $\rightarrow$ (b) : Magnetic fields at P are due to two concentric circular coils which are in opposite directions and unequal in magnitude.

5. (i) $\rightarrow$ (a), (c), (d) : Field $E = 0$ for charge distribution.

(ii) $\rightarrow$ (c), (d) : Potential is nonzero.

(iii) $\rightarrow$ (a), (b), (e) : Magnetic field is produced by a current loop. Net current during rotation in each of (a), (b) and (e) is zero, so $B = 0$.

(iv) $\rightarrow$ (c), (d) : Magnetic moment $\vec{m} = I\vec{A}$ will be nonzero if $\vec{m} \neq 0$, so $I \neq 0$, which is true for (c) and (d) only.

6A. For a constant velocity, $\vec{F} = \vec{F_E} + \vec{F_B} = 0$ (so net force $\vec{F}$ must be zero).

$q\vec{E} + q(\vec{v} \times \vec{B}) = 0, \vec{E} = -(\vec{v} \times \vec{B})$. Hence code R.

6B. $\vec{v} = 2\dfrac{E_0}{B_0}\hat{x}, \vec{E} = E_0\hat{z}, \vec{B} = B_0\hat{z}.$

Since velocity is along the x-direction, magnetic field will rotate it about the z-axis and electric field will provide the helical path. Hence code R.

6C. Magnetic force is always zero and electric field will move it along the $-y$ direction. Hence code S.

4.6 Ray Optics and Wave Optics

1. (i) $\rightarrow$ (a), (b), (c), (d) : For a concave mirror, with $f \leq u \leq \infty$, the image is real; magnified for $f \leq u \leq 2f$; virtual for $u < f$; at infinity for $u = f$.

(ii) $\rightarrow$ (b) : Image formed by a convex mirror is always virtual and diminished.

(iii) $\rightarrow$ (a), (b), (c), (d) : Image formed by a convex lens is real for $u > f$; virtual for $u < f$; magnified for $f < u < 2f$ and formed at infinity for $u = f$.

(iv) $\rightarrow$ (a), (b), (c), (d) : Image formed by a concavo-convex lens may be real, virtual, magnified and at infinity.

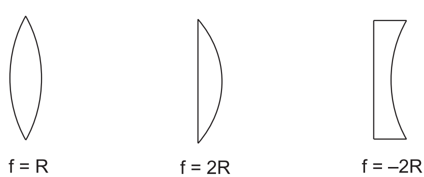
2. (i) $\rightarrow$ (a), (c) : The ray bends towards the normal while passing from optically denser to rarer ($\mu_1 < \mu_2$) medium.

(ii) $\rightarrow$ (b), (d), (e) : The ray bends away from the normal while passing from optically denser to rarer ($\mu_1 > \mu_2$) medium.

(iii) $\rightarrow$ (a), (c), (e) : Path of the ray remains unchanged when refractive indices of the two media are same ($\mu_2 = \mu_3$).

(iv) $\rightarrow$ (b), (d) : The ray bends away from the normal while passing from denser to rarer ($\mu_2 > \mu_3$) medium.

3. From $\dfrac{1}{f} = (\mu - 1)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$, the focal lengths of the three component lenses are



For lens combination $\dfrac{1}{f} = \dfrac{1}{f_1} + \dfrac{1}{f_2}$, so

for (i) : $\dfrac{1}{f_{eq}} = \dfrac{1}{R} + \dfrac{1}{R} = \dfrac{2}{R}$, $f_{eq} = \dfrac{R}{2} \rightarrow$ (b)

for (ii) : $\dfrac{1}{f_{eq}} = \dfrac{1}{2R} + \dfrac{1}{2R} = \dfrac{1}{R}$, $f_{eq} = R \rightarrow$ (d)

for (iii) : $\dfrac{1}{f_{eq}} = \dfrac{1}{-2R} + \dfrac{1}{-2R} = -\dfrac{1}{R}$, $f_{eq} = -R \rightarrow$ (c)

for (iv) : $\dfrac{1}{f_{eq}} = \dfrac{1}{R} + \dfrac{1}{-2R} = \dfrac{1}{2R}$, $f_{eq} = 2R \rightarrow$ (a)

4. (i) $\rightarrow$ (b) : $\mu_2 > \mu_1$ ... (towards the normal);

$\mu_2 > \mu_3$ ... (away from the normal).

(ii) $\rightarrow$ (c) : $\mu_1 = \mu_2$ ... (no change in path);

(iii) $\rightarrow$ (d) : $\mu_1 > \mu_2$ ... (away from the normal);

$\mu_2 > \mu_3$ ... (away from the normal);

$$\mu_1 \times \frac{1}{\sqrt{2}} = \mu_2 \sin r \Rightarrow \sin r = \frac{\mu_1}{\sqrt{2}\,\mu_2}.$$

Since $\sin r < 1$, so $\mu_1 < \sqrt{2}\mu_2$.

(iv) $\rightarrow$ (a) : For total reflection, $45° > \theta_c$.

So, $\sin 45° > \sin \theta_c$,

$$\Rightarrow \frac{1}{\sqrt{2}} > \frac{\mu_2}{\mu_1} \quad \Rightarrow \mu_1 > \sqrt{2}\,\mu_2.$$

Hence, code S is the correct answer.

5. (i) $\rightarrow$ (a) : Intensity $\propto$ aperture (R)

(ii) $\rightarrow$ (c) : Angular magnification $= f_o/f_e$

(iii) $\rightarrow$ (c) : Length of telescope $= f_o \pm f_e$

(iv) $\rightarrow$ (a), (b), (d) : Sharpness of image depends on aperture [option (a)], dispersion [option (b)] and spherical aberration [option (d)].

6. (i) $\rightarrow$ (a), (d) : Path difference, $\Delta = S_1P_0 - S_2P_0 = 0$, so phase difference

$$\delta(P_0) = \frac{2\pi}{\lambda} \cdot \Delta = 0.$$

$I(P_0) = 4I_0$, where I_0 = intensity due to each source.

$$I(P_1) = 2I_0\,(1 + \cos\phi) = 4I_0 \cos^2\frac{\phi}{2}$$

$\Rightarrow I(P_0) > I(P_1)$.

(ii) $\rightarrow$ (b) : Path difference at P_1 is $\Delta = S_1P_1 - [S_2P_1 + (n-1)t]$

$$= (S_1P_1 - S_2P_1) - \frac{\lambda}{4} = \frac{\lambda}{4} - \frac{\lambda}{4} = 0.$$

$\therefore \delta(P_1) = 0$.

(iii) $\rightarrow$ (e) : Path difference at P_1 is $S_1P_1 - [S_2P_1 + (n-1)t]$

$$= \frac{\lambda}{4} - \frac{\lambda}{2} = -\frac{\lambda}{4}, \quad \text{so } \phi = -\frac{\pi}{2}.$$

$\therefore$ intensity $I(P_1) = 2I_0$.

Similarly, for point P_2, $\Delta = S_1P_2 - [S_2P_2 + (n-1)t]$

$$= \frac{\lambda}{3} - \frac{\lambda}{2} = -\frac{\lambda}{6}.$$

The corresponding phase difference,

$$\phi = \frac{2\pi}{\lambda}\left(-\frac{\lambda}{6}\right) = -\frac{\pi}{3}.$$

$\therefore\ I(P_2) = I_0 + I_0 + 2I_0 \cos\left(-\frac{\pi}{3}\right) = 3I_0 < 2I_0.$

$\Rightarrow\ I(P_2) < I(P_1).$

(iv) $\rightarrow$ (c), (d), (e) : Path difference at P_1,

$$S_1P_1 - [S_2P_1 + (n-1)t] = \frac{\lambda}{4} - \frac{3\lambda}{4} = -\frac{\lambda}{2}, \quad \text{so } \phi = -\pi.$$

$\therefore\ I(P_1) = 2I_0\,(1 + \cos\phi) = 2I_0 \times 0 = 0.$

For P_0, path difference

$$\Delta = S_2P_0 + (n-1)t - S_1P_0$$

$$= \frac{3\lambda}{4}, \text{ so phase difference } \phi = \frac{3}{2}\pi.$$

$$\therefore\ I(P_0) = 2I_0\left(1 + \cos\frac{3\pi}{2}\right) = 2I_0$$

$\Rightarrow\ I(P_0) > I(P_1).$

Finally at P_2,

$$\Delta = S_1P_2 - [S_2P_2 + (n-1)t] = \frac{\lambda}{3} - \frac{3\lambda}{4} = -\frac{5\lambda}{12}.$$

$$\therefore \text{ phase difference } \phi = -\frac{5\pi}{6};\ \cos\phi = \frac{-\sqrt{3}}{2}.$$

$$\therefore\ I(P_2) = 2I_0(1 + \cos\phi) = 2I_0\left(1 - \frac{\sqrt{3}}{2}\right)$$

$\Rightarrow\ I(P_2) > I(P_1).$

4.7 Modern Physics

1. (i) $\rightarrow$ (b) : In α-decay, ${}^{A}_{Z}\mathrm{X} \rightarrow {}^{A-4}_{Z-2}\mathrm{Y} + \ldots$

(ii) $\rightarrow$ (a) : In β^+-decay, charge number reduces by 1 and mass number remains unchanged.

(iii) $\rightarrow$ (d) : In a fission reaction, the heavy nucleus splits into two lighter nuclei as in ${}^{239}_{94}\mathrm{U} \rightarrow {}^{140}_{57}\mathrm{La} + \ldots$

(iv) $\rightarrow$ (c) : In proton (${}^{1}_{1}\mathrm{H}$) emission, charge number and mass number both reduce by 1 as in ${}^{185}_{83}\mathrm{Bi} \rightarrow {}^{184}_{82}\mathrm{Po} + \ldots$

2. (i) $\rightarrow$ (c), (e) : In a fusion reaction, the lighter nuclei fuse to form heavier nuclei with neutrino emission.

(ii) $\rightarrow$ (a), (d) : Absorption of thermal neutrons $({}_0^1n)$ by ${}_{92}^{235}U$ produces fission. Heavy water is used to slow down the fast-moving neutrons.

(iii) $\rightarrow$ (b), (e) : Unstable ${}_{27}^{60}Co$ nucleus disintegrates with emission of β-particle and neutrino.

(iv) $\rightarrow$ (c) : During a fusion reaction, γ-rays are emitted.

3. (i) $\rightarrow$ (a), (c) : Transition between two energy levels occurs in hydrogen spectrum and when characteristic X-rays are produced.

(ii) $\rightarrow$ (b), (d) : β-decay is electron emission; photoelectric effect also occurs with electron emission.

(iii) $\rightarrow$ (a) : Moseley's law relates wavelength of characteristic X-rays.

(iv) $\rightarrow$ (b) : From Einstein's equation $h\nu = \phi_0 + \frac{1}{2}mv_{\max}^2$.

4. (i) $\rightarrow$ (a), (b), (e) : Energy of a charged capacitor $= Q^2/2C$.

Adiabatic compression increases the internal energy.

Time-varying magnetic field induces electric field and the induced current dissipitates heat.

(ii) $\rightarrow$ (b) : Same as in case (i).

(iii) $\rightarrow$ (d) : During fission, energy is released due to loss in mass.

(iv) $\rightarrow$ (d) : Same as in case (iii).

❑

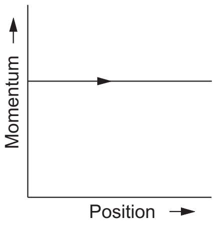
5

Linked-Comprehension-Type Questions

*Each passage in this chapter is followed by multiple-choice questions, which have to be answered on the basis of the passage. Each question has four choices (a, b, c and d), out of which **only one** is true.*

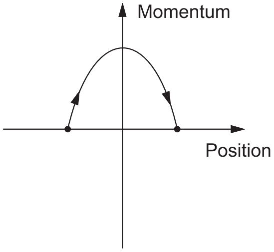
5.1 General Physics

- Phase-space diagrams are useful tools in analyzing all kinds of dynamical problems. They are specially useful in studying the changes in motion as initial position and momentum are changed. Here we consider some simple dynamical systems in one dimension. For such systems, phase space is a plane in which position is plotted along the horizontal axis and momentum is plotted along the vertical axis. The phase-space diagram is $x(t)$ vs $p(t)$ curve in this plane. The arrow on the curve indicates the time flow. For example, the phase-space diagram for a particle moving with a constant velocity is a straight line as shown in the figure. We use the sign convention in which the position or the momentum upwards (or to the right) is positive and downward (or to the left) is negative.

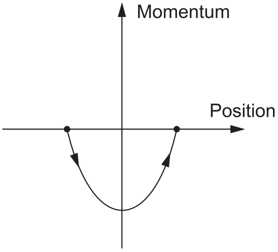


1. The phase-space diagram for a ball thrown vertically up from the ground is

(a)



(b)



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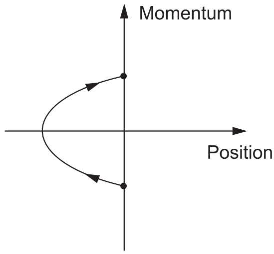
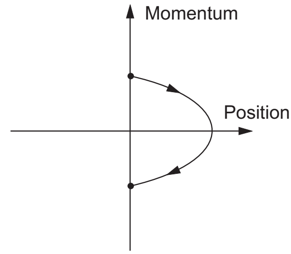
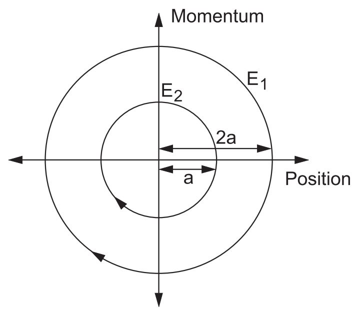


(d)

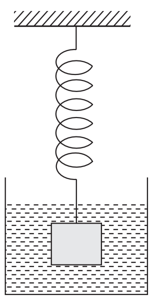


2. The phase-space diagram for simple harmonic motion is a circle centred at the origin. In the figure, the two circles represent the same oscillator but with different initial conditions and E_1 and E_2 are the total mechanical energies respectively. Then

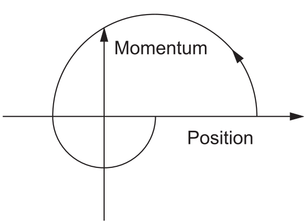


(a) $E_1 = \sqrt{2}E_2$
(b) $E_1 = 2E_2$
(c) $E_1 = 4E_2$
(d) $E_1 = 16E_2$

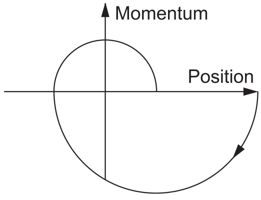
3. Consider the spring–mass system, with the mass submerged in water, as shown in the figure. The phase-space diagram for one cycle of this system is



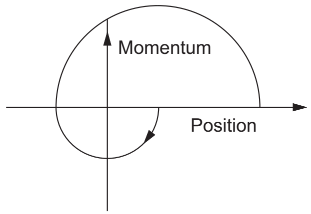
(a)



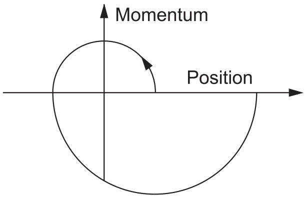
(b)



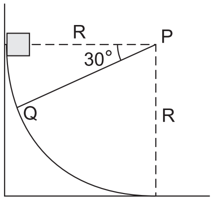
(c)



(d)



- A small block of mass 1 kg is released from rest at the top of a rough track. The track is a circular arc of radius 40 m. The block slides along the track without toppling and a frictional force acts on it in the direction opposite to the instantaneous velocity. The work done in overcoming the friction up to the point Q as shown in the figure is 150 J. [Take $g = 10$ m s^{-2}.]



4. The speed of the block when it reaches the point Q is

(a) 5 m s^{-1} (b) 10 m s^{-1}

(c) $10\sqrt{3}$ m s^{-1} (d) 20 m s^{-1}

5. The magnitude of normal reaction that acts on the block at the point Q is

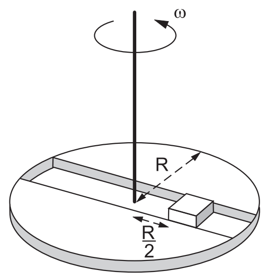
(a) 7.5 N (b) 8.6 N

(c) 11.5 N (d) 22.5 N

- A frame of reference that is accelerated with respect to an inertial frame of reference is called a noninertial frame of reference. A coordinate system fixed on a circular disc rotating about a fixed axis with a constant angular velocity ω is an example of a noninertial frame of reference. The relationship between the force $\vec{F}_{rot}$ experienced by a particle of mass m moving on a rotating disc and the force F_{in} experienced by the particle in an inertial frame of reference is

$$\vec{F}_{rot} = \vec{F}_{in} + 2m(\vec{v}_{rot} \times \vec{\omega}) + m(\vec{\omega} \times \vec{r}) \times \vec{\omega},$$

where $\vec{v}_{rot}$ is the velocity of the particle in the rotating frame of reference and $\vec{r}$ is the position vector of the particle with respect to the centre of the disc.



Now consider a smooth slot along a diameter of a disc of radius R rotating counterclockwise with a constant angular speed ω about its vertical axis through its centre. We assign a coordinate system with the origin at the centre of the disc, the x-axis along the slot, the y-axis perpendicular to the slot and the z-axis along the rotation axis ($\vec{\omega} = \omega\hat{k}$). A small block of mass m is gently placed in the slot at $\vec{r} = (R/2)\hat{i}$ at $t = 0$ and is constrained to move only along the slot.

6. The distance r of the block at time t is

(a) $\frac{R}{4}(e^{2\omega t} + e^{-2\omega t})$ (b) $\frac{R}{2}\cos 2\omega t$

(c) $\frac{R}{2}\cos \omega t$ (d) $\frac{R}{4}(e^{\omega t} + e^{-\omega t})$

7. The net reaction of the disc on the block is

(a) $-m\omega^2 R\cos\omega t\hat{j} - mg\hat{k}$ (b) $m\omega^2 R\sin\omega t\hat{j} - mg\hat{k}$

(c) $\frac{1}{2}m\omega^2 R(e^{\omega t} - e^{-\omega t})\hat{j} + mg\hat{k}$ (d) $\frac{1}{2}m\omega^2 R(e^{2\omega t} - e^{-2\omega t})\hat{j} + mg\hat{k}$

- Two discs A and B are mounted coaxially on a vertical axle. The discs have moments of inertia I and $2I$ respectively about the common axis. Disc A is imparted an initial angular velocity 2ω using the entire potential energy of a spring compressed by a distance x_1. Disc B is imparted an angular velocity ω by a spring having the same spring constant and compressed by a distance x_2. Both the discs rotate in the clockwise direction.

8. The ratio x_1/x_2 equals

(a) 2 (b) $\frac{1}{2}$

(c) $\sqrt{2}$ (d) $\frac{1}{\sqrt{2}}$

9. When the disc B is brought in contact with the disc A, they acquire a common angular velocity in time t. The average frictional torque on one disc by the other during this period is

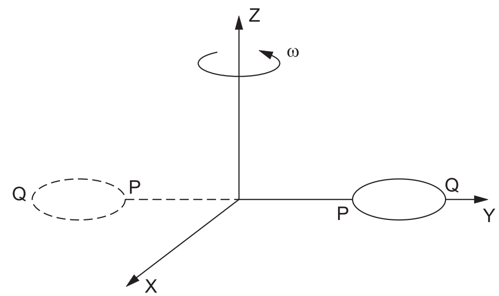
(a) $\dfrac{2I\omega}{3t}$ (b) $\dfrac{9I\omega}{2t}$

(c) $\dfrac{9I\omega}{4t}$ (d) $\dfrac{3I\omega}{2t}$

10. The loss of kinetic energy during the above process is

(a) $\dfrac{I\omega^2}{2}$ (b) $\dfrac{I\omega^2}{3}$

(c) $\dfrac{I\omega^2}{4}$ (d) $\dfrac{I\omega^2}{2t}$



- The general motion of a rigid body can be considered to be a combination of (i) a motion of its centre of mass about an axis, and (ii) its motion about an instantaneous axis passing through the centre of its mass. These axes need not be stationary. Consider, for example, a thin uniform disc welded (rigidly fixed) horizontally at its rim to a massless stick as shown in the figure. When the disc–stick system is rotated about the origin on a horizontal frictionless plane with angular speed ω, the motion at any instant can be taken as a combination of (i) a rotation of the centre of mass of the disc about the z-axis, and (ii) a rotation of the disc through an instantaneous vertical axis passing through its centre of mass (as is seen from the changed orientation of points P and Q). Both these motions have the same angular speed ω in this case.

Now consider two similar systems as shown in the figures ahead. In Figure I, the disc is with its face vertical and parallel to the xz-plane. In Figure II, the disc is with its face making an angle of 45° with the xy-plane and its horizontal diameter is parallel to the x-axis. In both the cases, the disc is welded at point P and the systems are rotated with a constant angular speed ω about the z-axis.

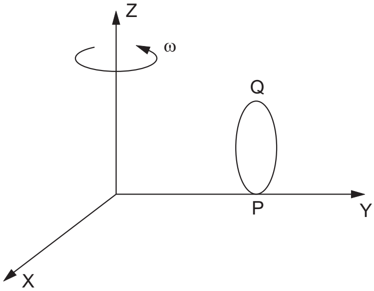


Fig. I

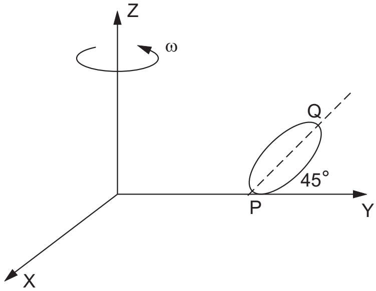


Fig. II

11. Which of the following statements about the instantaneous axis (passing through the centre of mass) is correct?

(a) It is vertical in both the figures.

(b) It is vertical in Figure I, and it is at 45° to the *xz*-plane and lies in the plane of the disc in Figure II.

(c) It is horizontal in Figure I, and it is at 45° to the *xz*-plane and normal to the plane of the disc in Figure II.

(d) It is vertical in Figure I, and it is at 45° to the *xz*-plane and normal to the plane of the disc in Figure II.

12. Which of the following statements regarding the angular speed about the instantaneous axis (passing through the centre of mass) is correct?

(a) It is $\sqrt{2}\,\omega$ in both the figures.

(b) It is ω in Figure I and $\dfrac{\omega}{\sqrt{2}}$ in Figure II.

(c) It is ω in Figure I and $\sqrt{2}\,\omega$ in Figure II.

(d) It is ω in both the figures.

- One twirls a circular ring (of mass M and radius R) near the tip of one's finger as shown in Figure 1. In the process, the finger never loses contact with the inner rim of the ring. The finger traces out the surface of a cone, shown by the dotted line. The radius of the path traced out by the point where the ring and the finger is in contact is r. The finger rotates with an angular velocity ω_0. The rotating ring rolls without slipping on the outside of a smaller circle described by the point where the ring and the finger is in contact (Figure 2). The coefficient of friction between the ring and the finger is μ and the acceleration due to gravity is g.

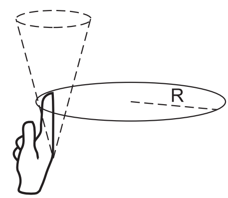


Fig. 1

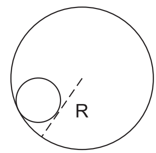


Fig. 2

13. The minimum value of ω_0 below which the ring will drop down is

(a) $\sqrt{\dfrac{2g}{\mu(R-r)}}$ (b) $\sqrt{\dfrac{g}{\mu(R-r)}}$

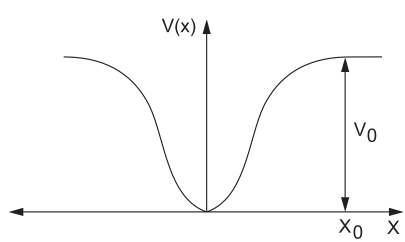
(c) $\sqrt{\dfrac{3g}{2\mu(R-r)}}$ (d) $\sqrt{\dfrac{g}{2\mu(R-r)}}$

14. The total kinetic energy of the ring is

(a) $\dfrac{3}{2}M\omega_0^2(R-r)^2$ (b) $\dfrac{1}{2}M\omega_0^2(R-r)^2$

(c) $M\omega_0^2(R-r)^2$ (d) $M\omega_0^2 R^2$

- When a particle of mass m moves on the x-axis in a potential of the form $V(x) = kx^2$, it performs simple harmonic motion. The corresponding time period is proportional to $\sqrt{\dfrac{m}{k}}$, as can be seen easily by using dimensional analysis. However, the motion of a particle can be periodic even when its potential energy increases on both sides of $x = 0$ in a way different from kx^2 and its total energy is such that the particle does not escape to infinity. Consider a particle of mass m moving on the x-axis. Its potential energy is $V(x) = \alpha x^4 (\alpha > 0)$ for $|x|$ near the origin and becomes a constant equal to V_0 for $|x| \geq X_0$ (see figure).



15. If the total energy of the particle is E, it will perform periodic motion only if

(a) $E < 0$ (b) $E > 0$

(c) $V_0 > E > 0$ (d) $E > V_0$

16. For periodic motion of small amplitude A, the time period T of this particle is proportional to

(a) $A\sqrt{\frac{m}{\alpha}}$ (b) $\frac{1}{A}\sqrt{\frac{m}{\alpha}}$

(c) $A\sqrt{\frac{\alpha}{m}}$ (d) $\frac{1}{A}\sqrt{\frac{\alpha}{m}}$

17. The acceleration of this particle for $|x| > X_0$ is

(a) proportional to V_0 (b) proportional to $\frac{V_0}{mX_0}$

(c) proportional to $\sqrt{\frac{V_0}{mX_0}}$ (d) zero

- When a liquid medicine of density ρ is to be put in the eye, it is done with the help of a dropper. As the bulb on the top of the dropper is pressed, a drop forms at the opening of the dropper. We wish to estimate the size of the drop. We first assume that the drop formed at the opening is spherical because that requires a minimum increase in its surface energy. To determine the size, we calculate the net vertical force due to the surface tension T when the radius of the drop is R. When this force becomes smaller than the weight of the drop, the drop gets detached from the dropper.

18. If the radius of the opening of the dropper is r, the vertical force due to the surface tension on the drop of radius R (assuming $r << R$) is

(a) $2\pi rt$ (b) $2\pi RT$

(c) $\frac{2\pi r^2 T}{R}$ (d) $\frac{2\pi R^2 T}{r}$

19. If $r = 5 \times 10^{-4}$ m, $\rho = 10^3$ kg m^{-3}, $g = 10$ m s^{-2}, $T = 0.11$ N m^{-1}, the radius of the drop when it detaches from the dropper is approximately

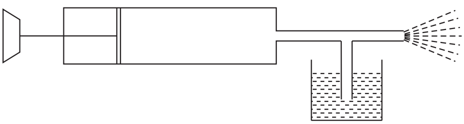
(a) 1.4×10^{-3} m (b) 3.3×10^{-3} m

(c) 2.0×10^{-3} m (d) 4.1×10^{-3} m

20. After the drop detaches, its surface energy is

(a) 1.4×10^{-6} J (b) 2.7×10^{-6} J

(c) 5.4×10^{-6} J (d) 8.1×10^{-6} J



- A spray gun is shown in the figure, where a piston pushes air out of a nozzle. A thin tube of uniform cross section is connected to the nozzle. One end of the tube is in a small liquid container. As the piston pushes air through the nozzle, the liquid from the container rises into the nozzle and is sprayed out. For the spray gun shown, the radii of the piston and nozzle are 20 mm and 1 mm respectively. The upper end of the container is open to the atmosphere.

21. If the piston is pushed at a speed of 5 mm s^{-1}, the air comes out of the nozzle with a speed of

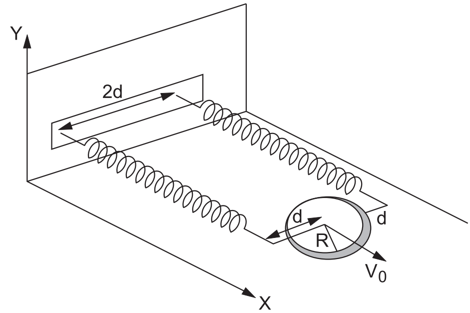
(a) 0.1 m s^{-1} (b) 1 m s^{-1}

(c) 2 m s^{-1} (d) 8 m s^{-1}

22. If the density of air is ρ_a and that of the liquid is ρ_l then for a given piston the speed (i.e., volume per unit time) at which the liquid is sprayed will be proportional to

(a) $\sqrt{\frac{\rho_a}{\rho_l}}$ (b) $\sqrt{\rho_a \rho_l}$

(c) $\sqrt{\frac{\rho_l}{\rho_a}}$ (d) ρ_l



- A uniform thin cylindrical disc of mass M and radius R is attached to two identical massless springs of spring constant k which are fixed to the wall as shown in the figure. The springs are attached to the axle of the disc symmetrically on either side at a distance d from the centre. The axle is massless, and both the springs and the axle are in a horizontal plane. The unstretched length of each spring is L. The disc is initially at its equilibrium position with its centre of mass (CM) at a distance L from the wall. The disc rolls without slipping with velocity $\vec{v_0} = v_0\hat{i}$. The coefficient of friction is μ.

23. The net external force acting on the disc when its centre of mass is at a distance x with respect to its equilibrium position is

(a) $-kx$ (b) $-2kx$

(c) $-\frac{2kx}{3}$ (d) $-\frac{4kx}{3}$

24. The centre of mass of the disc undergoes simple harmonic motion with angular frequency ω equal to

(a) $\sqrt{\frac{k}{M}}$ (b) $\sqrt{\frac{2k}{M}}$

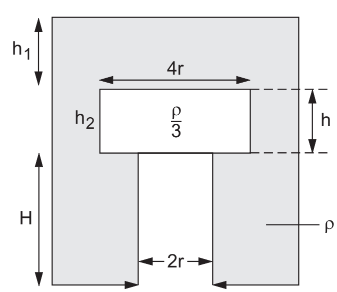
(c) $\sqrt{\frac{2k}{3M}}$ (d) $\sqrt{\frac{4k}{3M}}$

25. The maximum value of v_0 for which the disc will roll without slipping is

(a) $\mu g\sqrt{\frac{M}{k}}$ (b) $\mu g\sqrt{\frac{M}{2k}}$

(c) $\mu g\sqrt{\frac{3M}{k}}$ (d) $\mu g\sqrt{\frac{5M}{2k}}$

- A wooden cylinder of diameter $4r$, height h and density $\rho/3$ is kept on a hole of diameter $2r$ of a tank filled with water of density ρ, as shown in the figure. The height of the base of the cylinder from the base of the tank is H.



26. If the water level starts decreasing slowly then when it (the water) is at a height h_1 above the cylinder, the block just starts moving up. Then the value of h_1 is

(a) $\frac{2h}{3}$ (b) $\frac{5h}{4}$

(c) $\frac{5h}{3}$ (d) $\frac{5h}{2}$

27. The cylinder is prevented from moving up by applying a force and the water level is further decreased. Then the height of the water level (h_2 in the figure) for which the cylinder remains in original position without application of force is

(a) $\frac{h}{3}$ (b) $\frac{4h}{9}$

(c) $\frac{2h}{3}$ (d) h

28. If the height h_2 of the water level is further decreased then

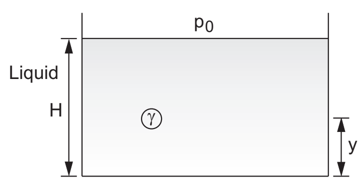
(a) the cylinder remains at its original position and will not move up

(b) for $h_2 = \frac{h}{3}$, the cylinder again starts to move up

(c) for $h_2 = \frac{h}{5}$, the cylinder again starts to move up

(d) for $h_2 = \frac{h}{4}$, the cylinder again starts to move up

5.2 Heat and Thermodynamics



- A small spherical monatomic ideal gas bubble $\left(\gamma = \frac{5}{3}\right)$ is trapped inside a liquid of density ρ_l (see the adjacent figure). Assume that the bubble does not exchange any heat with the liquid. The bubble contains n moles of the gas. The temperature of the gas when the bubble is at the bottom is T_0. The height of the liquid is H and the atmospheric pressure is p_0 (neglect the surface tension).

1. As the bubble moves upwards, besides the buoyant force, which of the following forces is/are acting on it?

(a) Only the force of gravity

(b) The force of gravity and the force due to the pressure of the liquid

(c) The force of gravity, the force due to the pressure of the liquid, and the force due to the viscosity of the liquid

(d) The force of gravity and the force due to the viscosity of the liquid

2. When the gas bubble is at a height y from the bottom, its temperature is

(a) $T_0\left(\dfrac{p_0+\rho_1 gH}{p_0+\rho_1 gy}\right)^{2/5}$ (b) $T_0\left(\dfrac{p_0+\rho_1 g(H-y)}{p_0+\rho_1 gH}\right)^{2/5}$

(c) $T_0\left(\dfrac{p_0+\rho_1 gH}{p_0+\rho_1 gy}\right)^{3/5}$ (d) $T_0\left(\dfrac{p_0+\rho_1 g(H-y)}{p_0+\rho_1 gH}\right)^{3/5}$

3. The buoyancy force acting on the gas bubble is (assume R to be the molar gas constant)

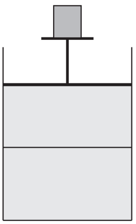
(a) $\rho_1 nRgT_0\dfrac{(p_0+\rho_1 gH)^{2/5}}{(p_0+\rho_1 gy)^{7/5}}$

(b) $\dfrac{\rho_1 nRgT_0}{(P_0+\rho_1 gH)^{2/5}[p_0+\rho_1 g(H-y)]^{3/5}}$

(c) $\rho_1 nRgT_0\dfrac{(p_0+\rho_1 gH)^{3/5}}{(p_0+\rho_1 gy)^{8/5}}$

(d) $\dfrac{\rho_1 nRgT_0}{(p_0+\rho_1 gH)^{3/5}[p_0+\rho_1 g(H-y)]^{2/5}}$

- In the figure, a container is shown to have a movable (without friction) piston on the top. The container and the piston are all made of perfectly insulating material allowing no heat transfer between outside and inside the container. The container is divided into two compartments by a rigid partition made of a thermally conducting



material that allows transfer of heat. The lower compartment of the container is filled with 2 moles of an ideal monatomic gas at 700 K and the upper compartment is filled with 2 moles of an ideal diatomic gas at 400 K. The heat capacities per mole of an ideal monatomic gas are $C_V = \frac{3}{2}R$ and $C_p = \frac{5}{2}R$, and those for an ideal diatomic gas are $C_V = \frac{5}{2}R$ and $C_p = \frac{7}{2}R$.

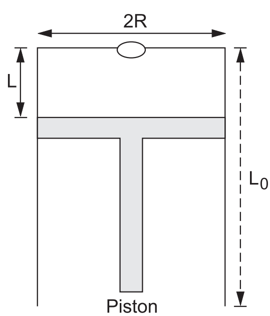
4. Consider the partition to be rigidly fixed so that it does not move. When equilibrium is achieved, the final temperature of the gases will be

(a) 550 K (b) 525 K
(c) 513 K (d) 490 K

5. Now consider the partition to be free to move without friction so that the pressure of gas in both the compartments is the same. The total work done by the gases till the time they achieve equilibrium will be

(a) $250R$ (b) $200R$
(c) $100R$ (d) $-100R$

• A fixed thermally conducting cylinder has its radius R and height L_0. The cylinder is open at its bottom and has a small hole at its top. A piston of mass M is held at a distance L from the top surface as shown in the figure. The atmospheric pressure is p_0.



6. The piston is pulled out slowly and held at a distance $2L$ from the top. The pressure in the cylinder between its top and the piston will then be

(a) p_0 (b) $\frac{p_0}{2}$

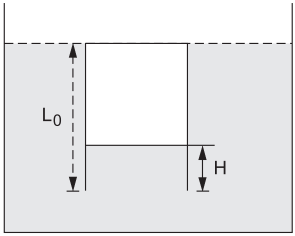
(c) $\frac{p_0}{2} + \frac{Mg}{\pi R^2}$ (d) $\frac{p_0}{2} - \frac{Mg}{\pi R^2}$

7. While the piston is at a distance $2L$ from the top, the hole at the top is sealed. The piston is then released to a position where it can stay in equilibrium. In this condition, the distance of the piston from the top is

(a) $\left(\dfrac{2p_0\pi R^2}{\pi R^2 p_0 + Mg}\right)(2L)$ (b) $\left(\dfrac{p_0\pi R^2 - Mg}{\pi R^2 p_0}\right)(2L)$

(c) $\left(\dfrac{p_0\pi R^2 + Mg}{\pi R^2 p_0}\right)(2L)$ (d) $\left(\dfrac{p_0\pi R^2}{\pi R^2 p_0 - Mg}\right)(2L)$

8. The piston is taken completely out of the cylinder. The hole at the top is sealed. A water tank is brought below the cylinder and put in a position so that the water surface in the tank is at the same level as the top of the cylinder, as shown in the figure. The density of water is ρ. In equilibrium, the height H of the water column in the cylinder satisfies



(a) $\rho g(L_0 - H)^2 + p_0(L_0 - H) + L_0p_0 = 0$

(b) $\rho g(L_0 - H)^2 - p_0(L_0 - H) - L_0p_0 = 0$

(c) $\rho g(L_0 - H)^2 + p_0(L_0 - H) - L_0p_0 = 0$

(d) $\rho g(L_0 - H)^2 - p_0(L_0 - H) + L_0p_0 = 0$

5.3 Sound Waves

- Two waves $y_1 = A\cos(0.5\pi x - 100\pi t)$ and $y_2 = A\cos(0.46\pi x - 92\pi t)$ are travelling in a pipe placed along the x-axis.

1. Find the number of times the intensity is maximum in the time interval of 1 s.

(a) 4 (b) 6

(c) 8 (d) 10

2. Find the wave velocity of the louder sound.

(a) 100 m s^{-1} (b) 192 m s^{-1}

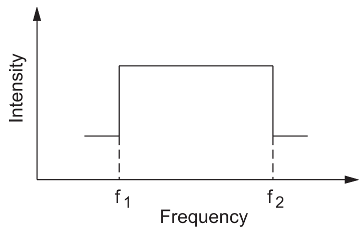
(c) 200 m s^{-1} (d) 96 m s^{-1}

3. Find the number of times $y_1 + y_2 = 0$ at $x = 0$ in 1 s.

(a) 100 (b) 46

(c) 191 (d) 96

- Two trains A and B are moving with speeds 20 m s^{-1} and 30 m s^{-1} respectively in the same direction on the same straight track with B ahead of A. The engines are at the front ends. The engine of A blows a long whistle. Assume that the sound of the whistle is composed of components varying in frequency from $f_1 = 800$ Hz to $f_2 = 1120$ Hz as shown in the figure. The spread in the frequency (highest frequency–lowest frequency) is thus 320 Hz. The speed of sound in still air is 340 m s^{-1}.



4. The speed of sound of the whistle is

(a) 340 m s^{-1} for the passengers in A and 310 m s^{-1} for the passengers in B

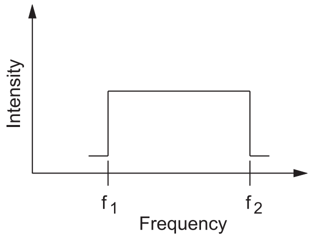
(b) 360 m s^{-1} for the passengers in A and 310 m s^{-1} for the passengers in B

(c) 310 m s^{-1} for the passengers in A and 360 m s^{-1} for the passengers in B

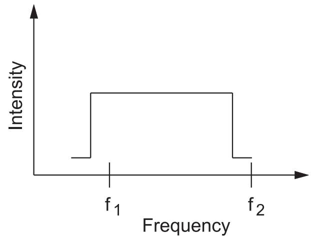
(d) 340 m s^{-1} for the passengers in both the trains

5. The distribution of the intensity of sound of the whistle as heard by the passengers in train A is best represented by

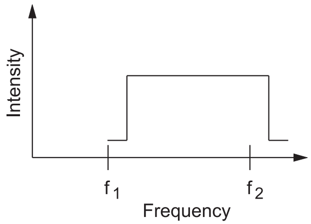
(a)



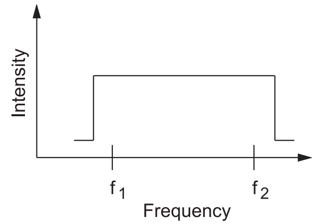
(b)



(c)



(d)

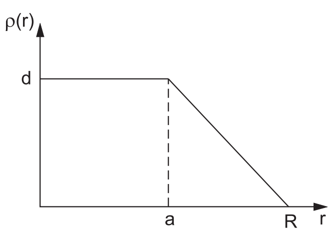


6. The spread of frequency as heard by the passengers in train B is

(a) 310 Hz (b) 330 Hz
(c) 350 Hz (d) 390 Hz

5.4 Electricity and Magnetism

- The nuclear charge (Ze) is nonuniformly distributed within a nucleus of radius R. The charge density, i.e., charge per unit volume, $\rho(r)$ is dependent only on the radial distance r from the centre of the nucleus, as shown in the figure. The electric field is only along the radial direction.



1. The electric field at $r = R$ is

(a) independent of a (b) directly proportional to a
(c) inversely proportional to a (d) none of these

2. For $a = 0$, the value of d (maximum value of ρ as shown in the figure) is

(a) $\dfrac{3Ze}{4\pi R^3}$ (b) $\dfrac{3Ze}{\pi R^3}$

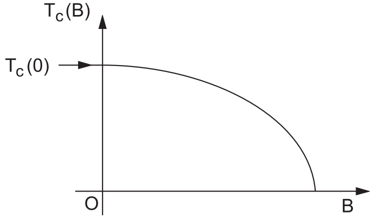
(c) $\dfrac{4Ze}{3\pi R^3}$ (d) $\dfrac{Ze}{3\pi R^3}$

3. The electric field within the nucleus is generally observed to be linearly dependent on r. This implies that

(a) $a = 0$ (b) $a = \frac{R}{2}$

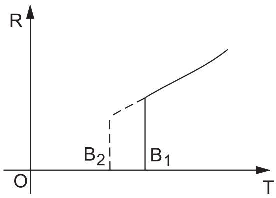
(c) $a = R$ (d) $a = \frac{2R}{3}$

- The electrical resistance of certain materials known as superconductors, changes abruptly from a nonzero value to zero as their temperature is lowered below a critical temperature $T_C(0)$. An interesting property of superconductors is that their critical temperature becomes smaller than $T_C(0)$ if they are placed in a magnetic field, i.e., the critical temperature $T_C(B)$ is a function of the magnetic field strength B. The dependence of $T_C(B)$ on B is shown in the figure.

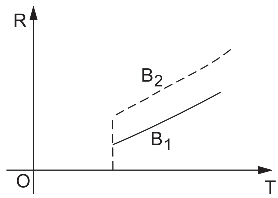


4. In the graphs below, the resistance R of a superconductor is shown as a function of its temperature T for two different magnetic fields B_1 (solid line) and B_2 (dashed line). If B_2 is larger than B_1, which of the following graphs shows the correct variation of R with T in these fields?

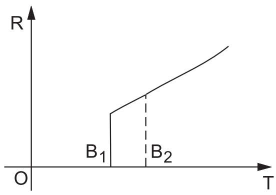
(a)



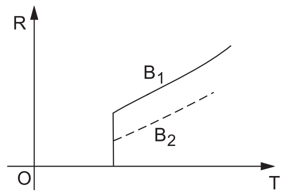
(b)



(c)



(d)



5. A superconductor has $T_C(0) = 100$ K. When a magnetic field of 7.5 tesla is applied, its T_C decreases to 75 K. For this material, one can definitely say that when

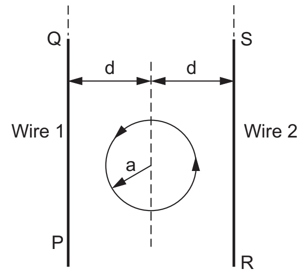
(a) $B = 5$ tesla, $T_C(B) = 80$ K

(b) $B = 5$ tesla, $75\text{ K} < T_C(B) < 100$ K

(c) $B = 10$ tesla, $75\text{ K} < T_C(B) < 100$ K

(d) $B = 10$ tesla, $T_C(B) = 70$ K

- The figure shows a circular loop of radius a with two long parallel wires (Wire 1 and Wire 2), all in the plane of the paper. The distance of each wire from the centre of the loop is d. The loop and the wire are carrying the same current I. The current in the loop is in the counterclockwise direction if seen from above.



6. When $d \approx a$, but the wires are not touching the loop, it is found that the net magnetic field at the axis of the loop is zero at a height h above the loop.

(a) The currents in Wire 1 and Wire 2 flow in the directions PQ and RS respectively and $h \approx a$.

(b) The currents in Wire 1 and Wire 2 flow in the directions PQ and SR respectively and $h \approx a$.

(c) The currents in Wire 1 and Wire 2 flow in the directions PQ and SR respectively and $h \approx 1.2a$.

(d) The currents in Wire 1 and Wire 2 flow in the directions PQ and RS respectively and $h \approx 1.2a$.

7. Consider $d >> a$ and the loop is rotated about its diameter parallel to the wires by 30° from the position shown in the figure. If the currents in the wires are flowing in the opposite directions, the torque on the loop at its new position will be (assume that the net field due to the wires is constant over the loop)

(a) $\dfrac{\mu_0 I^2 a^2}{d}$ (b) $\dfrac{\mu_0 I^2 a^2}{2d}$

(c) $\dfrac{\sqrt{3}\,\mu_0 I^2 a^2}{d}$ (d) $\dfrac{\sqrt{3}\,\mu_0 I^2 a^2}{2d}$

- A dense collection of equal number of electrons and positive ions is called neutral plasma. Certain solids containing fixed positive ions surrounded by free electrons can be treated as neutral plasma. Let N be the number density of free electrons, each of mass m. When electrons are subjected to an electric field, they are displaced relatively away from the heavy positive ions. If the electric field becomes zero, the electrons begin to oscillate about the positive ions with a natural frequency ω_P which is called the plasma frequency. To sustain the oscillations, a time-varying electric field needs to be applied that has an angular frequency ω, where a part of the energy is absorbed and a part of it is reflected. As ω approaches ω_P, all the free electrons are set to resonate together and all the energy is reflected. This is the explanation of high reflectivity of metals.

8. Taking the electronic charge e and the permittivity ε_0, use dimensional analysis to determine the correct expression for ω_0.

(a) $\sqrt{\dfrac{Ne}{m\varepsilon_0}}$ (b) $\sqrt{\dfrac{m\varepsilon_0}{Ne}}$

(c) $\sqrt{\dfrac{Ne^2}{m\varepsilon_0}}$ (d) $\sqrt{\dfrac{m\varepsilon_0}{Ne^2}}$

9. Estimate the wavelength at which plasma reflection will occur for a metal having the density of electrons $N = 4 \times 10^{27}\ \text{m}^{-3}$. Take $\varepsilon_0 = 10^{-11}$ and mass $m = 10^{-30}$, where these quantities are in proper SI units.

(a) 800 nm (b) 600 nm

(c) 300 nm (d) 200 nm

- A thermal power plant produces electric power of 600 kW at 4000 V, which is to be transmitted to a place 20 km away. The power can be transmitted either directly through a cable of large current-carrying capacity, or by using a combination of step-up and step-down transformers at the two ends. The drawback of the direct transmission is large energy dissipation. In the method using transformers, the dissipation is much smaller. By this method, a step-up transformer is used at the plant end so that the current is reduced to a smaller value. At the consumers' end, a step-down transformer is used to supply power to the consumers at the specified lower voltage. It is reasonable to assume that the power cable is purely resistive and the transformers are ideal with a power factor of unity. All the currents and voltages mentioned are rms values.

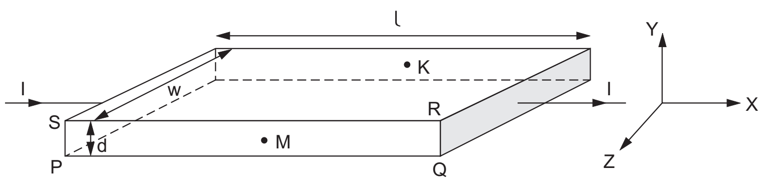
10. If the direct transmission method is used with a cable of resistance $0.4\ \Omega\ \text{km}^{-1}$, the power dissipation (in %) during transmission is

(a) 20 (b) 30
(c) 40 (d) 50

11. In the method using the transformers, assume that the ratio of the number of turns in the primary to that in the secondary in the step-up transformer is 1 : 10. If the power to the consumers' end has to be supplied at 200 V, the ratio of the number of turns in the primary to that in the secondary in the step-down transformer is

(a) 200 : 1 (b) 150 : 1
(c) 100 : 1 (d) 50 : 1

- In a thin rectangular metallic strip a constant current I flows along the positive x-direction, as shown in the figure. The length, width and thickness of the strip are l, w and d respectively. A uniform magnetic field $\vec{B}$ is applied on the strip along the positive y-direction. Due to this, the charge carriers experience a net deflection along the z-direction. This results in accumulation of charge carriers on the surface PQRS and appearance of equal and opposite charges on the face opposite PQRS. A potential difference along the z-direction is thus developed. Charge accumulation continues until the magnetic force is balanced by the electric force. The current is assumed to be uniformly distributed on the cross section of the strip and carried by electrons.



12. Consider two different metallic strips (1 and 2) of the same material. Their lengths are the same, widths are w_1 and w_2 and thicknesses are d_1 and d_2 respectively. Two points K and M are symmetrically located on the opposite faces parallel to the xy-plane (see figure) of each strip. V_1 and V_2 are the potential differences between K and M in strips 1 and 2 respectively. Then for a given current I flowing through each of them in a given magnetic field strength B, the correct statement(s) is/are:

(a) If $w_1 = w_2$ and $d_1 = 2d_2$ then $V_2 = 2V_1$
(b) If $w_1 = w_2$ and $d_1 = 2d_2$ then $V_1 = V_2$
(c) If $w_1 = 2w_2$ and $d_1 = d_2$ then $V_2 = 2V_1$
(d) If $w_1 = 2w_2$ and $d_1 = d_2$ then $V_2 = V_1$

13. Consider two different metallic strips (1 and 2) of same dimensions (length l, width w and thickness d) with carrier densities n_1 and n_2 respectively. Strip 1 is placed in a magnetic field B_1 and strip 2 is placed in a magnetic field B_2, both along the positive y-direction. Then V_1 and V_2 are the potential differences developed between K and M in strips 1 and 2 respectively. Assuming that the current I is the same for both the strips, the correct option(s) is/are:

(a) If $B_1 = B_2$ and $n_1 = 2n_2$, then $V_2 = 2V_1$
(b) If $B_1 = B_2$ and $n_1 = 2n_2$ then $V_2 = V_1$
(c) If $B_1 = 2B_2$ and $n_1 = n_2$ then $V_2 = 0.5V_1$
(d) If $B_1 = 2B_2$ and $n_1 = n_2$ then $V_2 = V_1$

- A point charge Q is moving in a circular orbit of radius R in the xy-plane with an angular speed ω. This can be considered as being equivalent to a loop carrying a steady current $\frac{Q\omega}{2\pi}$. A uniform magnetic field along the positive z-axis is now switched on, which increases at a constant rate from O to B in one second. Assume that the radius of the orbit remains constant. The application of the magnetic field induces an emf in the orbit. The induced emf is defined as the work done by an induced electric field in moving a unit positive charge around a closed loop. It is known that for an orbiting charge, the magnetic dipole moment is proportional to the angular momentum with a proportionality constant γ.

14. The magnitude of the induced electric field in the orbit at any instant of time during the time interval of the magnetic field change is

(a) $\dfrac{BR}{4}$ (b) $\dfrac{BR}{2}$

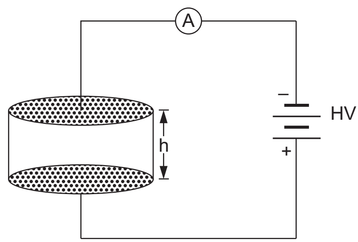
(c) BR (d) $2BR$

15. The change in the magnetic dipole moment associated with the orbit at the end of the time interval during which the magnetic field changes is

(a) $-\gamma BQR^2$ (b) $-\dfrac{\gamma}{2}BQR^2$

(c) $\dfrac{\gamma}{2}BQR^2$ (d) γBQR^2

- Consider an evacuated cylindrical chamber of height h having rigid conducting plates at the ends and an insulating curved surface as shown in the figure. A number of spherical balls made of a lightweight and soft material and coated with a conducting material are placed on the bottom plate. Each ball has a radius $r << h$. Now a high-voltage source (HV) is connected across the conducting plates such that the bottom plate is at $+V_0$ and the top plate is at $-V_0$. Due to their conducting surfaces, the balls will get charged, become equipotential with the plate and be repelled by it. The balls will eventually collide with the top plate where the coefficient of restitution can be taken to be zero due to the soft nature of the material of the balls. The electric field in the chamber can be considered to be that of a parallel-plate capacitor. Assume that there are no collisions between the balls, and the interaction between them is negligible. (Ignore gravity.)



16. Which one of the following statements is correct?

(a) The balls will bounce back to the bottom plate carrying the opposite charge they went up with.

(b) The balls will execute SHM between the two plates.

(c) The balls will bounce back to the bottom plate carrying the same charge they went up with.

(d) The balls will stick to the top plate and remain there.

17. The average current in the steady state registered by the ammeter in the circuit will be

(a) proportional to $V_0^{1/2}$ (b) proportional to V_0^2
(c) proportional to V_0 (d) zero

- Consider a simple RC circuit as shown in Figure 1.

Process 1: In the circuit the switch S is closed at $t = 0$ and the capacitor is fully charged to voltage V_0 (i.e., charging continues for time $T >> RC$). In the process, some dissipation (E_D) occurs across the resistance R. The amount of energy finally stored in the fully charged capacitor is E_C.

Process 2: In a different process the voltage is first set to $\frac{V_0}{3}$ and maintained for a charging time $T >> RC$. Then the voltage is raised to $\frac{2V_0}{3}$ without discharging the capacitor and again maintained for a time $T >> RC$. The process is repeated one more time by raising the voltage to V_0 and the capacitor is charged to the same final voltage V_0 as in Process 1. These two processes are depicted in Figure 2.

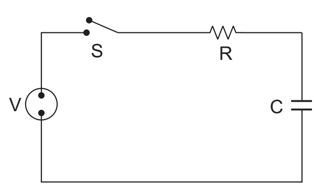


Fig. 1

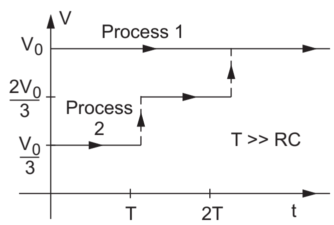


Fig. 2

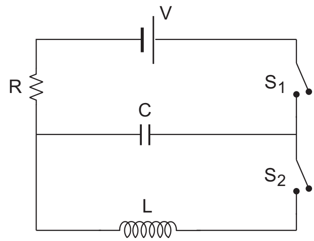
18. In Process 1, the energy stored in the capacitor, E_C, and heat dissipated across the resistance, E_D, are related by

(a) $E_C = \frac{1}{2}E_D$ (b) $E_C = 2E_D$
(c) $E_C = E_D$ (d) $E_C = E_0 \ln 2$

19. In Process 2, total energy dissipated across the resistance, E_D, is

(a) $E_D = 3\left(\frac{1}{2}CV_0^2\right)$ (b) $E_D = \frac{1}{2}CV_0^2$
(c) $E_D = 3CV_0^2$ (d) $E_D = \frac{1}{3}\left(\frac{1}{2}CV_0^2\right)$

• A capacitor of capacitance C can be charged (with the help of a resistance R) by a voltage source V, by closing the switch S_1 while keeping the switch S_2 open. The capacitor can be connected in series with an inductor L by closing switch S_2 and opening S_1.



20. Initially the capacitor was uncharged. Then the switch S_1 is closed and S_2 is kept open. If the time constant of this circuit is τ then

(a) after the time interval τ, charge on the capacitor is $CV/2$
(b) work done by the voltage source will be half of the heat dissipated when the capacitor is fully charged
(c) after time interval 2τ, charge on the capacitor is $CV(1 - e^{-2})$
(d) after time interval 2τ, charge on the capacitor is $CV(1 - e^{-1})$

21. After the capacitor gets fully charged, S_1 is opened and S_2 is closed so that the inductor is connected in series with the capacitor. Then

(a) at $t = 0$, the energy stored in the circuit is purely in the form of magnetic energy
(b) at any time $t > 0$, the current in the circuit is in the same direction
(c) at $t > 0$, there is no exchange of energy between the inductor and the capacitor
(d) at any time $t > 0$, the instantaneous current in the circuit may be $V\sqrt{\dfrac{C}{L}}$

22. If the total charge stored in the LC circuit is Q_0 then for $t \geq 0$,

(a) the charge on the capacitor is $Q = Q_0 \cos\left(\dfrac{\pi}{2} + \dfrac{1}{\sqrt{LC}}\right)$

(b) the charge on the capacitor is $Q = Q_0 \cos\left(\dfrac{\pi}{2} - \dfrac{1}{LC}\right)$

(c) the charge on the capacitor is $Q = -LC\dfrac{d^2Q}{dt^2}$

(d) the charge on the capacitor is $Q = -\dfrac{1}{\sqrt{LC}}\dfrac{d^2Q}{dt^2}$

- Modern trains are based on the Maglev technology in which a train is magnetically levitated and runs on its EDS Maglev system. There are coils on both sides of each wheel. Due to the motion of the train, a current is induced in the coil of the track, which levitates it. This is in accordance with Lenz's law. If the train comes down then according to Lenz's law, a repulsive force increase due to which the train gets uplifted and if it goes much higher then there is a net downward force due to gravity. The advantage of a Maglev train is that there is no friction between the train and the track, thereby reducing the power consumption and enabling the train to attain a very high speed. The major disadvantage of the Maglev train is that as it slows down, the electromagnetic forces decrease and it becomes difficult to keep the train levitated and as it moves forward, according to Lenz's law, there is an electromagnetic drag force.

23. What is the advantage of this system?

(a) There is no friction hence no power loss.
(b) No electric power is used.
(c) Gravitation force is zero.
(d) By Lenz's law, the train experiences a drag.

24. What is the major disadvantage of this system?

(a) The train experiences an upward force according to Lenz's law.
(b) The friction force creates a drag on the train.
(c) Retardation is caused.
(d) By Lenz's law, the train experiences a drag.

25. Which force causes the train to elevate upward?

(a) Electrostatic force (b) Time-varying electric field
(c) Magnetic force (d) Induced electric field

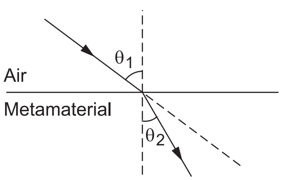
5.5 Ray Optics and Wave Optics

- Most materials have the refractive index $n > 1$. So, when a light ray from air enters a naturally occurring material then by Snell's law $\left(\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1}\right)$, it is understood that the refracted ray bends towards the normal. But it never emerges on the same side of the normal as the incident ray. According to electromagnetism, the refractive index of a medium is given by the relation $n = \frac{c}{v} = \pm\sqrt{\varepsilon_r \mu_r}$, where c is the speed of the electromagnetic wave in vacuum, v is its speed in the medium, ε_r and μ_r are the relative permittivity and permeability of the medium respectively.

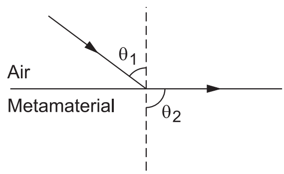
In normal materials, both ε_r and μ_r are positive, implying positive n for the medium. When both ε_r and μ_r are negative, one must choose the negative root of n. Materials with a negative refractive index can now be artificially prepared and are called metamaterials. They exhibit significantly different optical behaviour without violating any physical law. Since n is negative, it results in a change in the direction of propagation of the refracted light. However, similar to the normal materials, the frequency of light remains unchanged upon refraction even in metamaterials.

1. For the light incident from air on a metamaterial, the appropriate ray diagram is

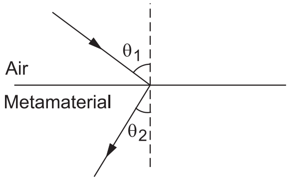
(a)



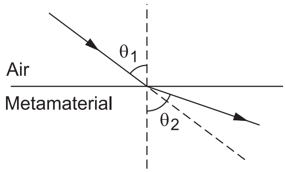
(b)



(c)



(d)



2. Choose the correct statement.

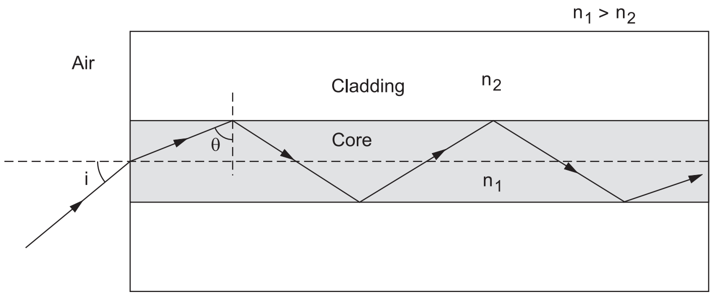
(a) The speed of light in the metamaterial is $v = c|n|$.

(b) The speed of light in the metamaterial is $v = \frac{c}{|n|}$.

(c) The speed of light in the metamaterial is $v = c$.

(d) The wavelength of light in the metamaterial (λ_m) is given by $\lambda_m = \lambda_{air}|n|$, when λ_{air} is the wavelength of light in air.

- Light guidance in an optical fibre can be understood by considering a structure comprised of a thin solid glass cylinder of refractive index n_1, surrounded by a medium of lower refractive index n_2. The light guidance in the structure takes place due to successive internal reflections at the interface of the media n_1 and n_2 as shown in the figure. All rays with the angle of incidence i less than a particular value i_m are confined in the medium of refractive index n_1. The numerical aperture (NA) of the structure is defined as $\sin i_m$.



3. For the two structures, namely S_1 with $n_1 = \frac{\sqrt{45}}{4}$ and $n_2 = \frac{3}{2}$, and S_2 with $n_1 = \frac{8}{5}$ and $n_2 = \frac{7}{5}$ and taking the refractive index of water to be $\frac{4}{3}$ and that of air to be 1, choose the correct statement(s).

(a) The NA of S_1 immersed in water is the same as that of S_2 immersed in a liquid of refractive index $\frac{16}{3\sqrt{15}}$.

(b) The NA of S_1 immersed in a liquid of refractive index $\frac{16}{\sqrt{15}}$ is the same as that of S_2 immersed in water.

(c) The NA of S_1 placed in air is the same as that of S_2 immersed in a liquid of refractive index $\frac{4}{\sqrt{15}}$.

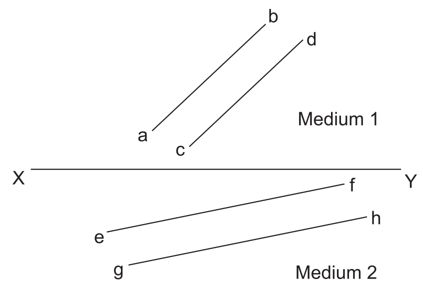
(d) The NA of S_1 placed in air is the same as that of S_2 placed in water.

4. If two structures of the same cross-sectional area, but different numerical apertures NA_1 and NA_2 ($NA_2 < NA_1$) are joined longitudinally, the numerical aperture of the combined structure is

(a) $\dfrac{NA_1 NA_2}{NA_1 + NA_2}$ (b) $NA_1 + NA_2$

(c) NA_1 (d) NA_2

- The figure shows a surface XY separating two transparent media, Medium 1 and Medium 2. The lines *ab* and *cd* represent wavefronts of light travelling in Medium 1 and incident on XY. The lines *ef* and *gh* represent wavefronts of the light in Medium 2 after refraction.



5. Light travels as a

(a) parallel beam in each medium

(b) convergent beam in each medium

(c) divergent beam in each medium

(d) divergent beam in one medium and convergent beam in the other medium

6. The phases of the light wave at *c, d, e* and *f* are ϕ_c, ϕ_d, ϕ_e and ϕ_f respectively. It is given that $\phi_c \neq \phi_f$.

(a) ϕ_c cannot be equal to ϕ_d

(b) ϕ_d can be equal to ϕ_e

(c) $(\phi_d - \phi_f)$ is equal to $(\phi_c - \phi_e)$

(d) $(\phi_d - \phi_c)$ is not equal to $(\phi_f - \phi_e)$

7. The speed of light is

(a) the same in the two media

(b) greater in Medium 1 than in Medium 2

(c) greater in Medium 2 than in Medium 1

(d) different at *b* and *d*

5.6 Modern Physics

- The key feature of Bohr theory of spectrum of hydrogen atom is the quantization of angular momentum when an electron is revolving around a proton. We will extend this to a general rotational motion to find quantized rotational energy of a diatomic molecule assuming it to be rigid. The rule to be applied is Bohr quantization condition.

1. A diatomic molecule has moment of inertia I. By Bohr quantization condition, its rotational energy in the nth level ($n = 0$ is not allowed) is

(a) $\frac{1}{n^2}\left(\frac{h^2}{8\pi^2 I}\right)$ (b) $\frac{1}{n}\left(\frac{h^2}{8\pi^2 I}\right)$

(c) $n\left(\frac{h^2}{8\pi^2 I}\right)$ (d) $n^2\left(\frac{h^2}{8\pi^2 I}\right)$

2. It is found that the excitation frequency from ground to the first excited state of rotation for the CO molecule is close to $\frac{4}{\pi} \times 10^{11}$ Hz. Then, the moment of inertia of CO molecule about its centre of mass is close to (take $h = 2\pi \times 10^{-34}$ J s)

(a) 2.76×10^{-46} kg m^2 (b) 1.87×10^{-46} kg m^2

(c) 4.67×10^{-47} kg m^2 (d) 1.17×10^{-47} kg m^2

3. In a CO molecule, the distance between C (mass = 12 amu) and O (mass = 16 amu), where 1 amu $= \frac{5}{3} \times 10^{-27}$ kg, is close to

(a) 2.4×10^{-10} m (b) 1.9×10^{-10} m

(c) 1.3×10^{-10} m (d) 4.4×10^{-11} m

- The β-decay process, discovered around 1900, is basically the decay of a neutron (n). In the laboratory, a proton (p) and an electron (e^-) are observed as the decay products of the neutron. Considering the decay of a neutron as a two-body decay process, it was predicted that the kinetic energy of the electrons should be a constant. But it was observed experimentally that the kinetic energy of the electron has a continuous spectrum. Considering a three-body decay process, i.e., $n \to p + e^- + \bar{\nu}_e$, around 1930, Pauli explained the observed energy spectrum of the electron. If one assumes the antinutrino ($\bar{\nu}_e$) to be massless and possessing negligible energy and the neutron to be at rest, and applies the principle of conservation of momentum and energy, the maximum kinetic energy of the electron is found to be 0.8×10^6 eV. The kinetic energy of the proton is only the recoil energy.

4. If the antinutrino had a mass 3 eV/c^2 (where c is the speed of light) instead of zero mass, what should be the range of the kinetic energy, K, of the electron?

(a) $0 \le K \le 0.8 \times 10^6$ eV (b) $3.0 \text{ eV} \le K \le 0.8 \times 10^6$ eV
(c) $2.0 \text{ eV} \le K \le 0.8 \times 10^6$ eV (d) $0 \le K \le 0.8 \times 10^6$ eV

5. What is the maximum energy of the antineutrino?

(a) Zero (b) Much less than 0.8×10^6 eV
(c) Nearly 0.8×10^6 eV (d) Much larger than 0.8×10^6 eV

- The mass of any nucleus ${}^{A}_{Z}X$ is less than the sum of the masses of $(A - Z)$ neutrons and Z protons in the nucleus. The energy corresponding to this mass difference is known as the binding energy of the nucleus. A heavy nucleus of mass M can break into two lighter nuclei of masses m_1 and m_2 only if $(m_1 + m_2) < M$. Also, two light nuclei of masses m_3 and m_4 can undergo complete fusion and form a heavy nucleus of mass M' only if $(m_3 + m_4) > M'$. The masses of some neutral atoms are given in the table below.

${}^{1}_{1}H$	1.007825 u	${}^{2}_{1}H$	2.014102 u	${}^{3}_{1}H$	3.016050 u	${}^{4}_{2}He$	4.002603 u
${}^{6}_{3}Li$	6.015123 u	${}^{7}_{3}Li$	7.016004 u	${}^{70}_{30}Zn$	69.925325 u	${}^{82}_{34}Se$	81.916709 u
${}^{152}_{64}Gd$	151.919803 u	${}^{206}_{82}Pb$	205.974455 u	${}^{209}_{83}Bi$	208.980388 u	${}^{210}_{84}Po$	209.982876 u

6. Which of the following statements is correct?

(a) The nucleus ${}^{6}_{3}Li$ can emit an alpha particle.
(b) The nucleus ${}^{210}_{84}Po$ can emit a proton.
(c) Deuterons and alpha particles can undergo complete fusion.
(d) The nuclei ${}^{70}_{30}Zn$ and ${}^{82}_{34}Se$ can undergo complete fusion.

7. The kinetic energy (in keV) of the alpha particle released when the nucleus ${}^{210}_{84}Po$ at rest undergoes alpha decay is

(a) 5319 (b) 5422 (c) 5707 (d) 5818

- When a particle is restricted to move along x-axis between $x = 0$ and $x = a$, where a is of nanometre dimension, its energy can take only certain specific values. The allowed energies of the particle moving in such a restricted region correspond to the formation of standing waves with nodes at its ends $x = 0$ and $x = a$. The wavelength of this standing wave is related to the linear momentum p of the particle according to the de Broglie relation. The energy of the particle of mass m is related to the linear momentum as $E = \frac{p^2}{2m}\cdot$ Thus, the energy of the particle can be described by a quantum number n, taking values $n = 1, 2, 3, \ldots$, corresponding to the number of loops in the standing wave.

  Use the model described above to answer the following three questions for a particle moving in the line $x = 0$ to $x = a$. Take $h = 6.6 \times 10^{-34}$ J s and $e = 1.6 \times 10^{-19}$ C.

8. The allowed energy of the particle for a particular value of n is proportional to

(a) a^{-2} (b) $a^{-3/2}$ (c) a^{-1} (d) a^2

9. If the mass of the particle is $m = 1.0 \times 10^{-30}$ kg and $a = 6.6$ nm, the energy of the particle in its ground state is closest to

(a) 0.8 meV (b) 8 meV
(c) 80 meV (d) 800 meV

10. The speed of the particle that can take discrete values, is proportional to

(a) $n^{-3/2}$ (b) n^{-1} (c) $n^{1/2}$ (d) n

- Scientists are working hard to develop a nuclear fusion reactor. Nuclei of heavy hydrogen, ${}_1^2\text{H}$, known as deuteron and denoted by D can be thought of as a candidate for such a reactor. The D–D reaction is ${}_1^2\text{H} + {}_1^2\text{H} \rightarrow {}_2^3\text{He} + \text{n} + \text{energy}$. In the core of the fusion reactor, a gas of heavy hydrogen is fully ionized into deuteron nuclei and electrons. This collection of ${}_1^2\text{H}$ nuclei and electrons is known as plasma. The nuclei move randomly in the reactor core and occasionally come close enough for nuclear fusion to take place. Usually, the temperatures in the reactor core are too high and no material wall can be used to confine the plasma. Special techniques are used which confine the plasma for a time t_0 before the particles fly away from the core. If n is the number density (number/volume) of deuterons, the product nt_0 is called Lawson number. In one of the criteria, a reaction is termed successful if Lawson number is greater than 5×10^{14} s cm^{-3}.

It may be helpful to use the following: Boltzmann constant $k = 8.6 \times 10^{-5}$ eV K^{-1}; $\dfrac{e^2}{4\pi\varepsilon_0} = 1.44 \times 10^{-9}$ eV m.

11. In the core of a nuclear fusion reactor, the gas becomes plasma because of the

(a) strong nuclear force acting between the deuterons
(b) Coulomb force acting between the deuterons
(c) Coulomb force acting between the deuteron–electron pair
(d) high temperature

12. Assume that two deuteron nuclei in the core of a fusion reactor at temperature T are moving towards each other, each with a kinetic energy 1.5 kT, when the separation between them is large enough to neglect Coulomb potential energy. Also, neglect any interaction with other particles in the core. The minimum temperature T required for them to reach a separation of 4×10^{-15} m is in the range

(a) 1.0×10^9 K $< T < 2.0 \times 10^9$ K (b) 2.0×10^9 K $< T < 3.0 \times 10^9$ K
(c) 3.0×10^9 K $< T < 4.0 \times 10^9$ K (d) 4.0×10^9 K $< T < 5.0 \times 10^9$ K

13. The results of calculations for four different designs of a fusion reactor using D–D reaction are given below. Which of these is the most promising based on Lawson criterion?

(a) Deuteron density = 2.0×10^{12} cm^{-3},
confinement time = 5.0×10^{-3} s
(b) Deuteron density = 8.0×10^{14} cm^{-3},
confinement time = 9.0×10^{-1} s
(c) Deuteron density = 4.0×10^{23} cm^{-3},
confinement time = 1.0×10^{-11} s
(d) Deuteron density = 1.0×10^{24} cm^{-3},
confinement time = 4.0×10^{-12} s

Answers

5.1 General Physics

1. d	**2.** c	**3.** b	**4.** b	**5.** a	**6.** d
7. c	**8.** c	**9.** a	**10.** b	**11.** a	**12.** d
13. b	**14.** c	**15.** c	**16.** b	**17.** d	**18.** c
19. a	**20.** b	**21.** c	**22.** a	**23.** d	**24.** d
25. c	**26.** c	**27.** b	**28.** a		

5.2 Heat and Thermodynamics

1. d	**2.** b	**3.** b	**4.** d	**5.** d	**6.** a
7. d	**8.** c				

5.3 Sound Waves

1. a	**2.** c	**3.** d	**4.** b	**5.** a	**6.** a

5.4 Electricity and Magnetism

1. a	**2.** b	**3.** c	**4.** a	**5.** b	**6.** c
7. b	**8.** c	**9.** b	**10.** b	**11.** a	**12.** a, d
13. a, c	**14.** b	**15.** b	**16.** a	**17.** b	**18.** c
19. d	**20.** c	**21.** d	**22.** c	**23.** a	**24.** d
25. d					

5.5 Ray Optics and Wave Optics

1. c	**2.** b	**3.** a, c	**4.** d	**5.** a	**6.** c
7. b					

5.6 Modern Physics

1. d	**2.** b	**3.** c	**4.** d	**5.** c	**6.** c
7. a	**8.** a	**9.** b	**10.** d	**11.** d	**12.** a
13. b					

<u>*Hints and Solutions*</u>

5.1 General Physics

1. During upward motion the momentum and displacement both are positive and as position increases, momentum decreases.
2. Energy $E \propto A^2$, where A is the amplitude.

   $\therefore\ \frac{E_1}{E_2} = \left(\frac{2a}{a}\right)^2 \ \Rightarrow\ E_1 = 4E_2.$

3. Total energy $E = \frac{p^2}{2m} + \frac{1}{2}Kx^2$.

   However, $E = \frac{1}{2}KA(t)^2$, where $A(t)$ is the amplitude

   $$\therefore\quad \frac{1}{2}KA(t)^2 = \frac{p^2}{2m} + \frac{1}{2}Kx^2$$
   or $\ p^2 + mK^2x^2 = mKA(t)^2.$

   Hence the equation of motion is a circle in phase space whose radius is decreasing due to the friction of the liquid.

4. From work–energy principle: $W_g + W_f = \Delta KE$.

   So, $\quad mgR\sin 30° + W_f = \frac{1}{2}mv^2$

   $$\Rightarrow\quad (1\text{ kg})(10\text{ m s}^{-2})(40\text{ m})\left(\frac{1}{2}\right) - 150\text{ J} = \frac{1}{2}(1\text{ kg})v^2$$

   $$\Rightarrow\quad v^2 = (10\text{ m s}^{-1})^2 \ \Rightarrow\ v = 10\text{ m s}^{-1}.$$

5. The centripetal force $\left(\frac{mv^2}{R}\right)$ is provided by $(\mathcal{N} - mg\cos 60°)$, so the normal reaction, $\mathcal{N} = mg\cos 60° + \frac{mv^2}{R} = 7.5$ N.

6. Centripetal acceleration, $a = \omega^2 r$.

   $\therefore\ v\frac{dv}{dr} = r\omega^2$

   $$\Rightarrow\ \int_0^v v\,dv = \int_{R/2}^{r} \omega^2\, r\,dr \ \Rightarrow\ v = \omega\sqrt{r^2 - \frac{R^2}{4}}.$$

$$\therefore \quad \int_{R/2}^{r} \frac{dr}{\sqrt{r^2 - R^2/4}} = \omega \int_0^t dt.$$

Let $r = \frac{R}{2} \sec\theta$, $dr = \frac{R}{2} \sec\theta \tan\theta \, d\theta$

$$\Rightarrow \quad \int_{R/2}^{r} \frac{\frac{R}{2}\sec\theta\tan\theta}{\frac{R}{2}\tan\theta} d\theta = \omega \int dt$$

$$\Rightarrow \quad \omega t = \ln\left(2r + \frac{\sqrt{4r^2 - R^2}}{R}\right)$$

$$\Rightarrow \quad r = \frac{R}{4}(e^{\omega t} + e^{-\omega t}).$$

7. Term $2m(\vec{v}_{\text{rot}} \times \vec{\omega})$ gives the normal reaction from the edge ($\mathcal{N}_e$)

$$v_{\text{rot}} = \frac{dr}{dt} = \frac{R}{4}(e^{\omega t} - e^{-\omega t})\,\omega.$$

$$\therefore \quad \mathcal{N}_e = 2m\frac{R\omega}{4}(e^{\omega t} - e^{-\omega t})\cdot\omega.$$

$$\Rightarrow \quad \vec{\mathcal{N}}_e = \frac{mR\omega^2}{2}(e^{\omega t} - e^{-\omega t})\hat{j}.$$

Normal reaction from the bottom of the slot = $mg\hat{k}$.

$\therefore$ net reaction from the slot is

$$\vec{R} = \frac{mR\omega^2}{2}(e^{\omega t} - e^{-\omega t})\hat{j} + mg\hat{k}.$$

8. $\frac{1}{2}I(2\omega)^2 = \frac{1}{2}kx_1^2;\ \frac{1}{2}(2I)\omega^2 = \frac{1}{2}kx_2^2$

$$\Rightarrow \quad \frac{x_1}{x_2} = \sqrt{2}.$$

9. Conserving angular momentum

$$I(2\omega) + 2I(\omega) = (I + 2I)\,\omega'.$$

$\therefore$ common angular velocity $\omega' = \frac{4}{3}\omega$.

Now, angular impulse = (average frictional torque) × time

$\Rightarrow \quad \tau \cdot t$ = change in angular momentum

$$= 2I\left(\frac{4}{3}\omega - \omega\right) = \frac{2I\omega}{3}$$

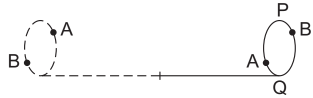
$$\Rightarrow \quad \tau = \frac{2I\omega}{3t}.$$

10. Initial kinetic energy $= \frac{1}{2}I(2\omega)^2 + \frac{1}{2}(2I)\omega^2 = 3I\omega^2$.

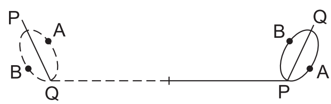
Final kinetic energy $= \frac{1}{2}(I+2I)\left(\frac{4}{3}\omega\right)^2$.

$\therefore$ loss in kinetic energy $= 3I\omega^2 - \frac{8}{3}I\omega^2 = \frac{I\omega^2}{3}$.

11. As shown in the figure, when the system as a whole turns by 180°, the disc also turns by 180° about its vertical axis. Hence the instantaneous axis that passes through the centre of mass is vertical in both the cases (a) and (b).



12. As shown in the adjoining figure, when the system as a whole is turned by 180°, the disc also turns by 180° about the vertical axis. Hence for both the cases (a) and (b), the angular speed about the instantaneous axis that passes through the centre of mass is ω.



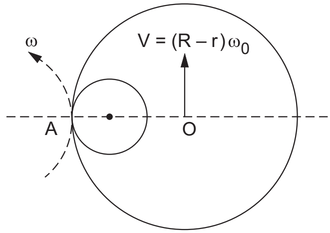
13.

$$N = M\omega^2(R-r).$$

$$F_{max} = \mu M\omega^2(R-r) \le Mg$$

$$\Rightarrow \quad \omega^2 \le \frac{g}{\mu(R-r)}.$$

$$\therefore \omega_{min} = \sqrt{\frac{g}{\mu(R-r)}}.$$



14. $\omega R = \omega_0(R-r) \Rightarrow \omega = \omega_0\left(\frac{R-r}{R}\right)$.

Rotational kinetic energy $= \frac{1}{2}I\omega^2$

$$= \frac{1}{2}(2MR^2)\omega_0^2\left(\frac{R-r}{R}\right)^2 = M\omega_0^2(R-r)^2.$$

15. For an oscillating system, the kinetic energy must be zero periodically for a finite value of x. Also, $E < V_0$.

16. $V(x) = \alpha x^4$. $\therefore$ $[\alpha] = ML^{-2}T^{-2}$.

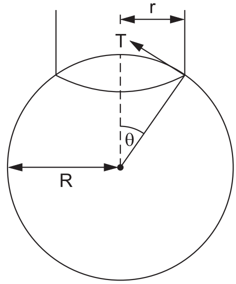
The only expression which has the dimension M^0L^0T is (b).

17. For $x > X_0$, V is constant.

Hence, force = 0 and therefore acceleration = 0.

18. Vertical upward force due to surface tension = $(2\pi r)(T \sin\theta)$

$= 2\pi r T\left(\frac{r}{R}\right) = 2\pi r^2 T/R.$



19. $2\pi r^2 T/R = mg = \left(\frac{4}{3}\pi R^3\right)\rho g.$

20. Surface energy = $4\pi R^2 T$.

21. From continuity equation,

$$A_1 V_1 = A_2 V_2$$

$$\Rightarrow \quad \pi(20)^2 \times 5 = \pi(1)^2 V_2$$

$$\Rightarrow \quad V_2 = 2000 \text{ mm s}^{-1} = 2 \text{ m s}^{-1}.$$

22. $\frac{1}{2}\rho_a V_a^2 = \frac{1}{2}\rho_l V_l^2$. For a given value of V_a, $V_l \propto \sqrt{\frac{\rho_a}{\rho_l}}$.

23. For translation: $2kx - f = Ma_{CM}$. ...(i)

For rotation: torque due to friction (f)

$$fR = I_{CM}\,\alpha = \frac{MR^2}{2}\frac{a_{CM}}{R}$$

$$\Rightarrow \quad f = Ma_{CM/2}. \qquad \text{...(ii)}$$

Adding equations (i) and (ii),

$$2kx = \frac{3}{2}Ma_{CM} \Rightarrow a_{CM} = \frac{4}{3}\frac{kx}{M}.$$

$\therefore$ net external force on the disc is

$Ma_{CM} = \frac{4}{3}kx$ (–ve sign indicates retardation).

24. The equation for angular motion is

torque = $I\alpha = -2kx \cdot R$

$$\Rightarrow \quad \alpha = -\frac{2kR}{I}x = -\frac{2kR}{\frac{3MR^2}{2}}(R\theta)$$

$$\Rightarrow \quad \alpha = -\frac{4k}{3M}\theta = -\omega^2\theta$$

$$\Rightarrow \quad \omega = \sqrt{\frac{4k}{3M}}.$$

25. From equation (ii), $f = \frac{1}{2}Ma_{CM} = \mu N = \mu Mg$

$$\Rightarrow \quad Ma_{CM} = 2\mu Mg = \frac{4}{3}kx$$

$$\Rightarrow \quad x = \frac{3}{2}\left(\frac{\mu Mg}{k}\right).$$

At this value of x, friction (static) is limiting and slipping just starts. At this position the velocity v_0 can be obtained from energy-conservation:

$$\frac{1}{2}Mv_0^2 + \frac{1}{2}I\omega^2 = 2\left(\frac{1}{2}kx^2\right). \qquad \text{...(iii)}$$

Take $I = \frac{1}{2}MR^2$, $\omega = \frac{v_0}{R}$ and $x = \frac{3}{2}\frac{\mu Mg}{k}$ to solve (iii), from which we get

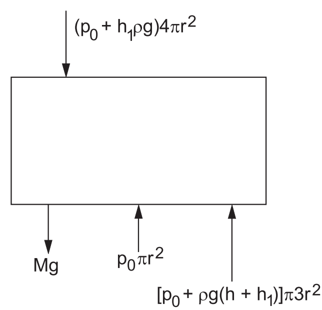
$$v_0 = \mu g\sqrt{\frac{3M}{k}}.$$

26. $M = \pi(4r^2)h\frac{\rho}{3}g$.

For equilibrium,

$$(p_0 = \rho g h_1)\pi 4r^2 + Mg$$
$$= (p_0 + \rho g(h + h_1)) \cdot \pi \cdot 3r^2 + p_0\pi r^2.$$

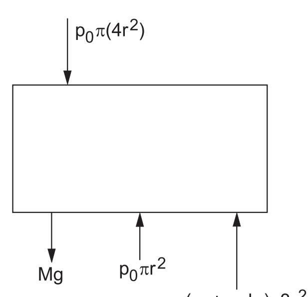
Simplifying, $h_1 = \frac{5}{3}h$.



27. $p_0\pi 4r^2 + Mg = p_0\pi r^2 + (\rho g h_1 + p_0)\pi 3r^2$.

Substituting M and simplifying,

$$h_1 = 4h/9.$$



28. The cylinder will not move up if $h_2 < \frac{4}{9}h$.

5.2 Heat and Thermodynamics

1. In addition to the buoyant force, the weight (mg) downward and the viscous force against the motion (downward) act together.

2. $p_1^{1-\gamma} T_1^{\gamma} = p_2^{1-\gamma} T_2^{\gamma}$.

$T_1 = T_0$, $p_1 = p_0 + \rho_l gH$, and

$p_2 = p_0 + \rho_l g(H - y)$.

3. Buoyant force $= \rho_l Vg = \rho_l g \dfrac{nRT_2}{p_2}$

and $\rho_2 = \rho_0 + \rho_l g(H - y)$

$$\Rightarrow \quad T_2 = T_0 \left[\frac{p_0 + \rho_l g(H-y)}{p_0 + \rho_l gH}\right]^{2/5}.$$

4. Let the final temperature of the gases $= T$.

Heat expelled by the lower compartment (nC_v, ΔT)

$$= 2\left(\frac{3}{2}R\right)(700 - T).$$

Heat absorbed by the upper compartment

$$(nC_p \Delta T) = 2\left(\frac{7}{2}R\right)(T - 400).$$

For equilibrium,

$7\text{T} - 2800 = 2100 - 3T$

$\Rightarrow \qquad T = 490$ K.

5. $\Delta W_1 + \Delta U_1 = \Delta Q_1$ and $\Delta W_2 + \Delta U_2 = \Delta Q_2$.

For the total system, $\Delta Q_1 + \Delta Q_2 = 0$

$\Rightarrow \quad \frac{7}{2}R(T - 400) = \frac{5}{2}R(700 - T) \Rightarrow T = 525$ K.

Work done by gas 1, $\Delta W_1 = nR\Delta T = 2R(525 - 400) = 250R$.

Work done by gas 2, $\Delta W_2 = nR\Delta T = 2R(525 - 700) = -350R$.

$\Rightarrow$ net work $= \Delta W_1 + \Delta W_2 = -100R$.

6. With the hole open, the space between the top and the piston is exposed to the atmosphere so the pressure $= p_0$.

7. For equilibrium,

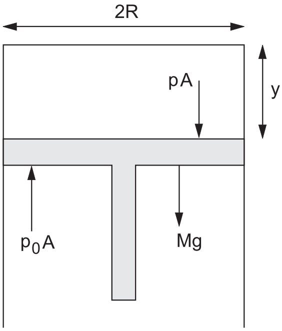
$$Mg + p\pi R^2 = p_0\pi R^2$$

$\Rightarrow\ p\pi R^2 = p_0\pi R^2 - Mg.$

For isothermal expansion,

$$p_0(\pi R^2 \cdot 2L) = p(\pi R^2 \cdot y)$$

$$\Rightarrow\ y = \left(\frac{p_0\pi R^2}{p_0\pi R^2 - Mg}\right)2L.$$



8. $\pi R^2 p_0 L_0 = p(L_0 - H)\pi R^2$

$\Rightarrow\ p = p_0 L_0/(L_0 - H).$

For equilibrium,

$$p = p_0 + \rho g(L_0 - H).$$

Equating, we get

$$\rho g(L_0 - H)^2 + p_0(L_0 - H) - L_0 p_0 = 0.$$

5.3 Sound Waves

1. Beat frequency = difference in frequency.

Here, $f_1 = \dfrac{100\pi}{2\pi} = 50\ \text{s}^{-1}$, $f_2 = \dfrac{92\pi}{2\pi} = 46\ \text{s}^{-1}$.

$\Rightarrow$ required number $= f_1 - f_2 = 4.$

2. Wave velocity, $v = \dfrac{\omega}{k}$.

$$\therefore\ v_1 = \frac{100\pi}{0.5\pi} = 200\ \text{m s}^{-1},$$

$$v_2 = \frac{92\pi}{0.46\pi} = 200\ \text{m s}^{-1}.$$

$\Rightarrow\ v_1 = v_2 = 200\ \text{m s}^{-1}.$

3. At $x = 0$

$$y_1 + y_2 = A\cos 100\pi t + A\cos 92\pi t = 0,$$

$\therefore\ \cos(100\pi t) = -\cos(92\pi t) = \cos[(2n+1)\pi - (92\pi t)]$

$\Rightarrow\ 100\pi t = (2n+1)\pi - 92\pi t$

$\Rightarrow\ t = \dfrac{2n+1}{192}.$

$\therefore\ \Delta t = t_{n+1} - t_n = \dfrac{2}{192}.$

Hence, the required number $= \dfrac{1}{\Delta t} = \dfrac{192}{2} = 96.$

4. The speed of sound depends on the frame of reference. Thus the speed of sound relative to train A is (340 + 20) m s^{-1} = 360 m s^{-1}. Similarly, relative to train B, speed of sound = (340 – 30) m s^{-1} = 310 m s^{-1}.

5. As there is no relative motion between the trains and the passengers in both the trains, the distribution of intensity of sound of the whistle will be uniform. Hence, the best graph for this situation must be (a).

6. According to Doppler effect, apparent frequency

$$f' = \frac{v \pm v_o}{v + v_s} f.$$

Here v = 340 m s^{-1}, v_o(train B) = 30 m s^{-1},

v_s(source ≡ train A) 20 m s^{-1} and f = true frequency.

∴ lowermost frequency,

$$f_1' = \left(\frac{340-30}{340-20}\right)(800 \text{ Hz}) = 775 \text{ Hz}.$$

Similarly, the uppermost frequency,

$$f_2' = \left(\frac{340-30}{340-20}\right)(1120 \text{ Hz}) = 1085 \text{ Hz}.$$

⇒ frequency spread = 310 Hz.

5.4 Electricity and Magnetism

1. From Gauss's theorem

$$E \cdot 4\pi R^2 = \frac{Q}{\varepsilon_0} = \frac{Ze}{\varepsilon_0}$$

$$\Rightarrow \qquad E = \frac{Ze}{4\pi\varepsilon_0 R^2}, \text{ hence independent of } a.$$

2. Charge density as a function of r is

$$\rho(r) = d\left(1 - \frac{r}{R}\right).$$

Total charge enclosed

$$Q = Ze = \int_0^R \rho(r) \cdot 4\pi r^2 dr = \frac{\pi d R^3}{3}.$$

$$\therefore \quad d = \frac{3Ze}{\pi R^3}.$$

3. Linear dependence of the electric field within the nucleus with radial distance is possible only if the charge density is uniform throughout, so $a = R$.

From Gauss's theorem,

$$E \cdot 4\pi r^2 = \frac{Ze}{\varepsilon_0} = \frac{\frac{4}{3}\pi r^3 d}{\varepsilon_0}$$

$$\Rightarrow \qquad E = \left(\frac{1}{3\varepsilon_0} d\right) r.$$

$$\therefore \qquad E \propto r.$$

4. Critical temperature decreases with the increase in magnetic field. As shown in option (a), magnetic field B_2 is larger than B_1 (given) and the critical temperature is lower.

5. For $B = 0$, $\quad T_C = 100$ K;

$B = 7.5$ T, $\quad T_C = 75$ K;

$B = 5$ T, $\quad 75 \text{ K} < T_C < 100$ K.

6. The net magnetic field at the given points will be zero, if

$$|\vec{B}_{\text{wires}}| = |\vec{B}_{\text{ring}}|.$$

Due to the ring,

$$B_{\text{ring}} = \frac{\mu_0 I a^2}{2(a^2+h^2)^{3/2}}, \text{ vertically up the plane.}$$

Due to the wires,

$$B_{\text{wires}} = 2\frac{\mu_0 I}{2\pi\sqrt{a^2+h^2}} \times \frac{a}{\sqrt{a^2+h^2}} = \frac{\mu_0 I a}{\pi(a^2+h^2)}$$

$$\Rightarrow \qquad \frac{\mu_0 I a}{\pi(a^2+h^2)} = \frac{\mu_0 I a^2}{2(a^2+h^2)^{3/2}}$$

$$\Rightarrow \qquad h = a\sqrt{\frac{\pi^2}{4} - 1} \approx 1.2a.$$

For the net field to be zero, the magnetic field due to straight currents will be directed vertically down the plane, so the current flows in the directions PQ and SR.

7. Magnetic field at the midpoint of the two straight wires, $B = \dfrac{\mu_0 I}{\pi d}$.

Magnetic moment of the loop, $m = I\pi a^2$.

Magnitude of torque, $|\vec{\tau}| = |\vec{m} \times \vec{B}|$

$$= mB \sin 150° = \frac{1}{2} mB$$

$$= \frac{\mu_0 I^2 a^2}{2d}.$$

8. Check dimensionally: $\sqrt{\dfrac{Ne^2}{m\varepsilon_0}} = T^{-1} = [\omega_0]$.

9. $\omega = 2\pi\nu = 2\pi\dfrac{c}{\lambda}\cdot$

10. Input power = 600 kW, voltage = 4000 V.

Total line resistance = $(0.4\ \Omega\ \text{km}^{-1})(20\ \text{km}) = 8\ \Omega$.

$\because$ $P = VI \Rightarrow (600\ \text{kV})(10^3\ \text{A}) = (4000\ \text{V})I \Rightarrow I = 150$ A.

$\therefore$ energy loss due to heat dissipation,

$P = I^2R = (150\ \text{A})^2(8\ \Omega) = 180000\ \text{W} = 180\ \text{kW}$.

$\therefore$ % loss $= \left(\dfrac{180\ \text{kW}}{600\ \text{kW}}\right) \times 100\% = 30\%$.

11. Given: $\dfrac{N_p}{N_s} = \dfrac{1}{10}$ in the step-up transformer.

Now, $\dfrac{V_s}{V_p} = \dfrac{N_s}{N_p} \Rightarrow \dfrac{V_s}{4000\ \text{V}} = \dfrac{10}{1} \Rightarrow V_s = 40000$ V.

Hence, the required ratio in the step-down transformer

$$\frac{N_p}{N_s} = \frac{V_p}{V_s} = \frac{40{,}000}{200} = 200:1.$$

12. $qvB = qE = q\dfrac{V_M - V_K}{W}$.

$\therefore \quad V_M - V_K = WvB.$

But current $I = neAv = ne(Wd)v$

$\Rightarrow \quad Wv = \dfrac{I}{ned}\cdot$

$\therefore \quad V_M - V_K = \left(\dfrac{I}{ned}\right)B.$

For $\quad w_1 = w_2, d_1 = 2d_2; \qquad V_2 = 2V_1.$

For $\quad w_1 = 2w_2, d_1 = d_2; \qquad V_1 = V_2.$

13. $V_M - V_K = \left(\dfrac{I}{ned}\right)B.$

For $n_1 = 2n_2, B_1 = B_2 \quad \Rightarrow V_2 = 2V_1.$

For $B_1 = 2B_2, n_1 = n_2 \quad \Rightarrow V_1 = 2V_2.$

or $V_2 = 0.5V_1.$

14. Let E = magnitude of the induced electric field, so

$$E(2\pi R) = \frac{d\phi_B}{dt} = \pi R^2\frac{dB}{dt} = \pi R^2 B$$

$$\Rightarrow \quad E = \frac{RB}{2}\cdot$$

15. Change in angular momentum,

$$\Delta L = \int \tau \, dt = \int EQRdt = \frac{RB}{2} QR \times 1 = \frac{QBR^2}{2}.$$

Now, change in the magnitude of the dipole moment,

$$\Delta\mu = \gamma \Delta L$$

$$= -\frac{\gamma}{2} BQR^2 \text{ (considering direction).}$$

16. The balls get negatively charged after hitting the top plate and get attracted towards the positively charged bottom plate. The motion executed will be periodic but not SHM.

17. As the balls keep on carrying charge from one plate to the other, current will continue to flow even in steady state.

While at the bottom plate, $V_0 = \dfrac{Kq}{r}$

$$\Rightarrow \qquad q = \frac{V_0 r}{K}.$$

In the cavity of the cylinder, electric field,

$$E = [V_0 - (-V_0)]/h = \frac{2V_0}{h}.$$

$$\left(\therefore \text{ time to reach the other plate, } t = \sqrt{\frac{2h}{a}} = \frac{1}{V_0}\sqrt{\frac{Kmh}{r}}. \right)$$

Acceleration, $a = \dfrac{F}{m} = \dfrac{qE}{m} = \dfrac{2rV_0^2}{mK}.$

If there are n balls,

average current $= \dfrac{nq}{t} = n\left(\dfrac{V_0 r}{K}\right) V_0 \sqrt{\dfrac{r}{Kmh}} = \dfrac{nr}{K}\sqrt{\dfrac{r}{Kmh}}\, V_0^2.$

$$\Rightarrow \qquad I_{av} \propto V_0^2.$$

18. $E_C = \dfrac{1}{2} C V_0^2.$

Energy delivered by the cell $= Q_0 V_0 = (CV_0)(V_0) = CV_0^2.$

$$E_D = CV_0^2 - E_C = \frac{1}{2} CV_0^2$$

$$\Rightarrow \qquad E_C = E_D.$$

19. If the capacitor is charged from V_i to V_f ($V_f > V_i$), heat dissipated is

$$\Delta E = W_{battery} - \Delta U$$

$$= C(V_f - V_i) \cdot V_f - \frac{1}{2} C(V_f^2 - V_i^2)$$

$$= \frac{1}{2} C(V_f - V_i)^2.$$

Total heat dissipated in the two processes taken together,

$$E_D = (\Delta E)_1 + (\Delta E)_2 + (\Delta E)_3$$
$$= \frac{1}{2}C\left(\frac{V_0}{3} - 0\right)^2 + \frac{1}{2}C\left(\frac{2V_0}{3} - \frac{V_0}{3}\right)^2 + \frac{1}{2}C\left(V_0 - \frac{2V_0}{3}\right)^2$$
$$= \frac{1}{6}CV_0^2.$$

20. Instantaneous charge on the capacitor during charging, $Q = Q_0\left(1 - e^{-t/\tau}\right)$, where $Q_0 = CV$ = steady state charge.
At $t = 2\tau$, $Q = CV(1 - e^{-2})$.

21. With S_1 open and S_2 closed, LC oscillation in the circuit gets set in, for which $Q = Q_0 \cos \omega t$, where $\omega^2 = \frac{1}{LC}$, $Q_0 = CV$.

Now, $I = -\frac{dQ}{dt} = CV\,\omega \sin \omega t = CV\frac{1}{\sqrt{LC}}\sin \omega t.$

$\therefore \quad I_{max} = V\sqrt{\frac{C}{L}}.$

22. Instantaneous charge, $Q = Q_0 \cos \omega t$.

Differentiating, $\frac{d^2Q}{dt^2} = -\omega^2 Q_0 \cos \omega t = -\omega^2 Q$

$\Rightarrow \; Q = -\frac{1}{\omega^2}\frac{d^2Q}{dt^2} = -LC\frac{d^2Q}{dt^2}.$

23. Absence of friction, hence no power loss.

24. According to Lenz's law, drag is experienced by the train.

25. The upward elevation is caused by the induced electric field.

5.5 Ray Optics and Wave Optics

1. A metamaterial has negative refractive index.

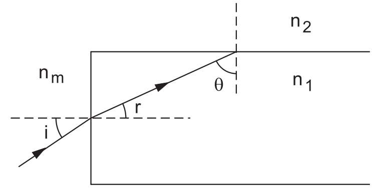
Now, $\sin\theta_2 = \frac{n_1}{n_2}\sin\theta_1.$

$\because \; n_2$ is negative, so θ_2 is negative.

2. $n = \frac{c}{v}$, for metamaterial. $\therefore \; v = \frac{c}{|n|}.$

3. For total internal reflection, $\theta \geq C$.

$\Rightarrow \quad 90° - r \geq C$



$\Rightarrow \quad \sin(90° - r) \geq \sin C$

$\Rightarrow \quad \cos r \geq \sin C.$

From Snell's law,

$$n_m \sin i_m \equiv n_1 \sin r$$

and $\sin C = \dfrac{n_2}{n_1}$

$$\Rightarrow \quad \sin^2 i_m = \frac{n_1^2}{n_m^2}(1 - \cos^2 r) = \frac{n_1^2}{n_m^2}(1 - \sin^2 C)$$

$$\Rightarrow \quad \text{NA} = \sin i_m = \sqrt{\frac{n_1^2}{n_m^2}\left(1 - \frac{n_2^2}{n_1^2}\right)} = \frac{\sqrt{n_1^2 - n_2^2}}{n_m}.$$

Putting the proper values, the correct options are found to be (a) and (c).

4. For total internal reflection to take place in both the structures, the numerical aperture should be the least one for the combined structure. Hence, the correct option is (d).

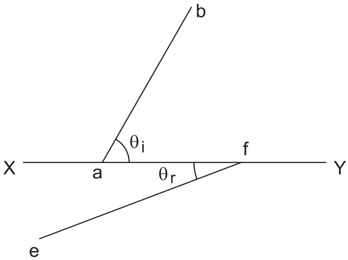
5. The normal to the wavefront represents the direction of the rays. In both the media the wavefronts are parallel, so light travels as parallel beam in each case.

6. We know that a wavefront is represented as a surface with identical phase during wave propagation. Hence,

$$\phi_c = \phi_d \text{ and } \phi_e = \phi_f$$

$\Rightarrow \quad \phi_d - \phi_f = \phi_c - \phi_e.$

7. The angle between the incident wavefront (*ab*) and the refracting surface (XY) is the angle of incidence (θ_i). Similarly, θ_r is the angle of refraction between XY and the refracted wave front *ef*.



From Snell's law,

$$n_1 \sin \theta_1 = n_2 \sin \theta_2 \Rightarrow \frac{c_0}{c_1}\sin\theta_1 = \frac{c_0}{c_2}\sin\theta_2,$$

where c's represent the speed of light.

$$\Rightarrow \quad \frac{1}{c_1}\sin\theta_1 = \frac{1}{c_2}\sin\theta_2 \Rightarrow \frac{\sin\theta_1}{\sin\theta_2} = \frac{c_1}{c_2}.$$

$\because \quad \theta_1 > \theta_2$, hence $c_1 > c_2$.

Hence, the speed of light is greater in Medium 1.

5.6 Modern Physics

1. According to Bohr quantum condition, angular momentum $L = \frac{nh}{2\pi}$.

Hence, rotational kinetic energy $= \frac{L^2}{2I} = \left(\frac{nh}{2\pi}\right)^2 \frac{1}{2I} = n^2 \frac{h^2}{8\pi^2 I}$.

2. Rotational KE for the CO molecule in the nth state is $E_n = n^2 \frac{h^2}{8\pi^2 I}$.

Hence for excitation from ground state ($n = 1$) to the first excited state ($n = 2$), we have

$$\frac{4h}{\pi}\times 10^{11} = \frac{(2^2 - 1^2)h^2}{8\pi^2 I}.$$

Simplifying, the moment of inertia of CO molecule about its centre of mass,

$$I = \left(\frac{3h}{8\pi}\right)\left(\frac{10^{-11}}{4}\right)\text{kg m}^2 = 1.87 \times 10^{-46}\ \text{kg m}^2.$$

3. Moment of inertia $I = \frac{m_1 m_2}{m_1 + m_2} r^2$.

$$\therefore \qquad r = \sqrt{\frac{I(m_1 + m_2)}{m_1 m_2}}.$$

Substituting the values,

$$r = \sqrt{\frac{(1.87 \times 10^{-46}\ \text{kg m}^2)(12\ \text{amu} + 16\ \text{amu})}{(12\ \text{amu})(16\ \text{amu})}}$$

$$= \sqrt{\frac{(1.87 \times 10^{-46})(28)}{(12 \times 16) \times \frac{5}{3} \times 10^{-27}}}\ \text{m} = 1.3 \times 10^{-10}\ \text{m}.$$

4. $0 \le (\text{KE})_{\beta^-} \le Q - (\text{KE})_p - (\text{KE})_{\bar{\nu}}$

$\Rightarrow\ 0 \le (\text{KE})_{\beta^-} < Q.$

5. $Q = (0.8)(10^6\ \text{eV})$.

$(\text{KE})_p + (\text{KE})_{\beta^-} + (\text{KE})_{\bar{\nu}} = Q.$

$(\text{KE})_p$ is nearly zero, when $(\text{KE})_{\beta^-}$ is zero.

So, $(\text{KE})_{\bar{\nu}} = Q - (\text{KE})_p \simeq Q (\simeq 0.8 \times 10^6\ \text{eV})$.

6. For the nuclear reaction ${}^{6}_{3}\text{Li} \rightarrow {}^{4}_{2}\text{He} + {}^{2}_{1}\text{H}$,

$$\frac{Q}{c^2} = 6.01523 - 4.002603 - 2.014102 = -0.001582.$$

This being negative, α-decay is not possible.

For ${}^{210}_{84}\text{Po} \rightarrow {}^{1}_{1}\text{H} + {}^{209}_{83}\text{Bi}$,

$$\frac{Q}{c^2} = 209.9828766 - 1.007825 - 208.980388 = -0.005337 < 0.$$

This reaction is also not possible.

For ${}^{2}_{1}\text{H} + {}^{4}_{2}\text{He} \rightarrow {}^{6}_{3}\text{Li}$,

$$\frac{Q}{c^2} = 2.014102 + 4.002603 - 6.015123 = 0.001627 > 0.$$

This reaction is possible [option (c)].

For ${}^{70}_{30}\text{Ze} + {}^{82}_{34}\text{Se} \rightarrow {}^{152}_{64}\text{Ge}$,

$$\frac{Q}{c^2} = 69.925325 + 81.916709 - 151.919803 = -0.077769 < 0.$$

This reaction is not possible.

7. For ${}^{210}_{84}\text{Po} \rightarrow {}^{4}_{2}\text{He} + {}^{206}_{82}\text{Pb}$,

$$Q = (209.982876 - 4.002603 - 205.97455)\, c^2 = 5.422 \text{ MeV}.$$

Conserving linear momentum,

$$\sqrt{2K_1(4)} = \sqrt{2K_2(206)}$$

$$\Rightarrow \quad 4K_1 = 206K_2.$$

$$\therefore \quad K_1 = \frac{103}{2}K_2.$$

But, $\quad K_1 + K_2 = 5.422 \text{ MeV}$

$$\Rightarrow \quad K_1 + \frac{2}{103}K_1 = 5.422 \text{ MeV}$$

$$\Rightarrow \quad \frac{105}{103}K_1 = 5.422 \text{ MeV}$$

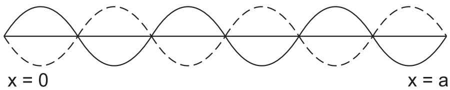
$$\Rightarrow \quad K_1 = 5.319 \text{ MeV} = 5319 \text{ keV}.$$

8. If n segments are contained in length a then $a = n\left(\frac{\lambda}{2}\right)$

$$\Rightarrow \quad \lambda = \frac{2a}{n} = \frac{h}{p}, \text{ so the linear momentum } p = n\frac{h}{2a}.$$

$\Rightarrow$ Kinetic energy, $E = \dfrac{p^2}{2m} = \dfrac{n^2h^2}{8ma^2}$.

$\therefore$ $E \propto a^{-2}$.



9. For the ground state, $n = 1$,

$$E_1 = \frac{h^2}{8\,ma^2} = \frac{(6.6 \times 10^{-34}\text{ J s})^2}{8(1 \times 10^{-30}\text{ kg})\,(6.6 \times 10^{-9}\text{ m})^2} = 0.125 \times 10^{-20}\text{ J}$$

$$= \frac{0.125 \times 10^{-20}}{e}\text{ eV} \approx 8\text{ meV}.$$

10. $\because$ $p = mv = n\dfrac{h}{2a}$,

$\therefore$ velocity, $v = n\dfrac{h}{2ma}$.

$\therefore$ $v \propto n$.

11. Fusion is a nuclear reaction (as in the sun) which requires a very high temperature.

12. Mean translational kinetic energy per molecule $= \dfrac{3}{2}$ kT, where $k\left(=\dfrac{R}{N}\right)$ is the Boltzmann constant.

Now, $2\left(\dfrac{3}{2}kT\right) = \dfrac{1}{4\pi\varepsilon_0}\left(\dfrac{e^2}{d}\right)$.

Solving, we get $T = 1.4 \times 10^9$ K, which exists in the temperature range of $1.0 \times 10^9\text{ K} < T < 2.0 \times 10^9$ K.

13. As given, the Lawson number $nt_0 > 5 \times 10^{14}\text{ s cm}^{-3}$.

The deuteron density $n = 8 \times 10^{14}\text{ cm}^{-3}$ and the confinement time $t_0 = 9 \times 10^{-1}$ s satisfies the requirement.

❑

6

Assertion–Reason-Type Questions

*Each of the following questions contains an assertion (**Statement 1**) and a reason (**Statement 2**). Each question has four choices (a, b, c and d), out of which only one is correct.*

(a) Statement 1 is true, Statement 2 is true; Statement 2 is a correct explanation of Statement 1.

(b) Statement 1 is true, Statement 2 is true; Statement 2 is not a correct explanation of Statement 1.

(c) Statement 1 is true, Statement 2 is false.

(d) Statement 1 is false, Statement 2 is true.

1. **Statement 1:** The formula connecting u, v and f for a spherical mirror is valid only for those mirrors whose sizes are very small compared to their radii of curvature.

   Statement 2: The laws of reflection are strictly valid for plane surfaces but not for large spherical surfaces.

2. **Statement 1:** If the accelerating potential in an X-ray tube is increased, the wavelengths of the characteristic X-rays do not change.

   Statement 2: When an electron beam strikes the target in an X-ray tube, part of the kinetic energy is converted into X-ray energy.

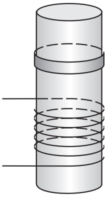
3. **Statement 1:** A block of mass m starts moving on a rough horizontal surface with a velocity v. It stops due to friction between the block and the surface after moving through a certain distance. The surface is now tilted to an angle of 30° with the horizontal, and the same block is made to go up on the surface with the same initial velocity v. The decrease in mechanical energy in the second situation is smaller than that in the first situation.

   Statement 2: The coefficient of friction between the block and the surface decreases with the increase in the angle of inclination.

4. **Statement 1:** For an elastic collision between two bodies, the relative speed of the bodies after the collision is equal to the relative speed before the collision.

   Statement 2: In an elastic collision, the linear momentum of the system is conserved.

5. **Statement 1:** A vertical iron rod has a coil of wire wound over it at the bottom end. An alternating current flows in the coil. The rod goes through a conducting ring as shown in the figure. The ring can float at a certain height above the coil.

   Statement 2: In the above situation, a current is induced in the ring, which interacts with the horizontal component of the magnetic field to produce an average force in the upward direction.

6. **Statement 1:** If there is no external torque on a body about its centre of mass, the velocity of the centre of mass remains constant.

   Statement 2: The linear momentum of an isolated system remains.

7. **Statement 1:** The total translational kinetic energy of the molecules of a given mass of an ideal gas is 1.5 times the product of its pressure and its volume.

   Statement 2: The molecules of a gas collide with each other and the velocities of the molecules change due to the collisions.

8. **Statement 1:** A piece of cloth covers a table. Some dishes are kept on it. The cloth can be pulled out without dislodging the dishes from the table.

   Statement 2: For every action there is an equal and opposite reaction.

9. **Statement 1:** The stream of water flowing at a high speed from a garden hosepipe tends to spread like a fountain when held vertically up, but tends to narrow down when held vertically down.

   Statement 2: In any steady flow of an incompressible fluid, the volume flow rate of the fluid remains constant.

10. **Statement 1:** Two cylinders, one hollow (metallic) and the other solid (wooden) with the same mass and identical dimensions are simultaneously allowed to roll without stopping, down an inclined

plane from the same height. The hollow cylinder will reach the bottom of the inclined plane first.

Statement 2: By the principle of conservation of energy, the total kinetic energy of both the cylinders are identical when they reach the bottom of the incline.

11. Statement 1: An astronaut in an orbiting space station above the earth experiences weightlessness.

Statement 2: An object moving around the earth under the influence of the earth's gravitational force is in a state of free fall.

12. Statement 1: In a metre-bridge experiment, the null point for an unknown resistance is measured. Now, the unknown resistance is put inside an enclosure maintained at a higher temperature. The null point can be obtained at the same point as before by decreasing the value of the standard resistance.

Statement 2: Resistance of a metal increases with temperature.

13. Statement 1: It is easier to pull a heavy object than to push it on a level ground.

Statement 2: The magnitude of frictional force depends on the nature of the two surfaces in contact.

14. Statement 1: For practical purposes, the earth is used as a reference at zero potential in electrical circuits.

Statement 2: The electrical potential of a sphere of radius R with charge Q uniformly distributed on the surface is given by $\dfrac{Q}{4\pi\varepsilon_0 R}$.

15. Statement 1: The sensitivity of a moving-coil galvanometer is increased by placing a suitable magnetic material as a core.

Statement 2: Soft iron has a high magnetic permeability and cannot be easily magnetized or demagnetized.

16. Statement 1: For an observer looking out through the window of a fast-moving train, the nearby objects appear to move in the opposite direction, while the distant objects appear to be stationary.

Statement 2: If the observer and the object are moving with the velocities v_1 and v_2 respectively with reference to a laboratory frame, the velocity of the object vis-a-vis the observer is $v_1 - v_2$.

Answers

1. c	**2.** b	**3.** c	**4.** b	**5.** a	**6.** d
7. b	**8.** b	**9.** a	**10.** d	**11.** a	**12.** d
13. b	**14.** b	**15.** c	**16.** b		

Hints and Solutions

1. All the formulae relating u, v and f are valid for paraxial rays.

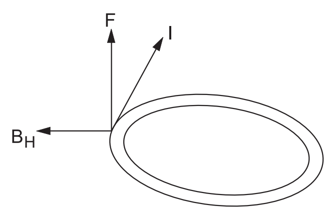
2. The increase in accelerating potential (V) decreases the magnitude of cut-off wavelength $\left(\lambda_{\min} = \frac{hc}{eV}\right)$ but does not affect the characteristic spectrum.

3. In the second case, work is done against friction as well as gravity while moving up the incline. The coefficient of friction is the property of the nature of the surfaces in contact, and not of the inclination.

4. The coefficient of restitution (e) for an elastic collision is unity, where velocity of separation = e × (velocity of approach).

Linear momentum is always conserved.

5. The interaction between the induced current (I) and the horizontal component of the magnetic field (B_H) creates an average force upward which makes the ring float by balancing its weight.



6. $\vec{\tau}_{CM} = 0$ means that the external force passes through the centre of mass, so rotation will not occur, but linear acceleration will increase $\vec{v}_{CM}$.

7. The total kinetic energy of translation, $\frac{1}{2}MC_{rms}^2 = \frac{3}{2}RT = \frac{3}{2}pV$, where M is the molar mass.

8. Inertia of rest.

9. Apply the principle of continuity $a_1v_1 = a_2v_2$. For the vertically upward motion, velocity decreases, so area a increases, which causes spread like a fountain. Same principle is applicable for the vertically downward motion.

10. The acceleration of a body rolling down an inclined plane is

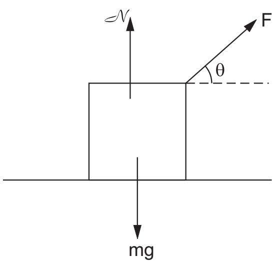
$$\left[a = \frac{g\sin\theta}{1 + \frac{K^2}{R^2}}\right]$$

under the condition $(a)_{solid} > (a)_{hollow}$, so the solid cylinder will reach the bottom first. Kinetic energy is gained by the loss in potential energy which is the same for both the cylinders.

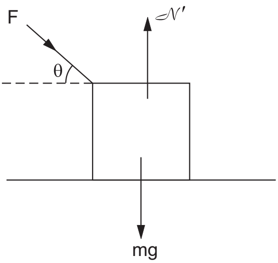
11. For the frame orbiting the earth (noninertial), the weight mg is balanced by the pseudo-force. Hence normal reaction is zero, which leads to apparent weightlessness. The situation is equivalent to the state of free fall.

12. Increase in temperature increases the resistance. Hence to keep the position of null points unchanged, the standard resistance has to be increased.

13. The two figures given below show the pulling force F and pushing force F.



(i)



(ii)

(i) During pull, $\mathcal{N} + F\sin\theta = mg$, so $\mathcal{N} = mg - F\sin\theta$.

(ii) During push, $\mathcal{N}' = (mg + F\sin\theta) > \mathcal{N}$

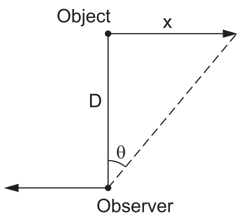
$$\Rightarrow \quad f' = \mu\mathcal{N}' > f(= \mu\mathcal{N}).$$

Hence, pull is easier than push.

14. Both the statements are independently true.

15. By placing a magnetic material of high permeability increases B, hence current sensitivity $\left(NB\frac{A}{C}\right)$ is increased. A magnetic substance with high permeability can be easily magnetised or demagnetised.

16. The relative displacement of the distant object (x) with respect to the observer subtends a small angle θ. Since $\tan\theta = \frac{x}{D}$, the distant objects appear to be at rest.



For statement 2, $\vec{v}_{2,1} = \vec{v}_2 - \vec{v}_1$ is true.

❑